AIRSHIP DESIGN

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EDITORIAL PREFACE

Rudyard Kipling has been quoted as saying of aerial navigation: "We are at the opening verse of the opening page of the chapter of endless possibilities." No longer is there doubt as to the practicability of flying. That was demonstrated by the United States Air Mail; the commercial airlines in Europe and South America; the crossings of the Atlantic Ocean by airplanes, seaplanes and airships; and finally the circumnavigation of the earth by airplane.

While the consequences to flow from man's new power cannot yet be estimated, of this we may be certain: As the development within a few generations of railway, steamship, and automobile has altered every relation of the world's life, so the possession at last of aircraft, enabling us to utilize the free and universal highway provided by nature, must lead to effects upon human activity no less wide and profound.

The need is widely felt already for a progressive literature of aerial navigation. We need technical information for designers, engineers, and pilots and for the growing army of students. We need also discussions of the practical implications of air navigation, for statesmen, economists, and representatives of industrial and commercial organizations whose interests and operations are affected by the new mode of transit. The Ronald Aeronautic Library, a series of volumes by specialists able to speak with authority, supplies this information. It is the purpose of the editor to keep the Library continually abreast of every phase of aerial development.

The division into separate volumes is governed by the needs of each branch of aeronautics. At the same time this permits of frequent revisions to keep pace with the progress of an expanding art. The arrangement of the text facilitates reference almost to the extent found in the standard engineering handbooks. Information is not limited to American experience; foreign sources are drawn upon freely.

C. DEF. CHANDLER,
Editor, Ronald Aeronautic Library

AUTHOR'S PREFACE

This volume is intended to fill the dual rôle of textbook for the student of airship design and handbook for the practical engineer. Much material which is necessary for the student may be needlessly elemental for the designer. The design of airships, particularly of the rigid type, is mainly a structural problem; and theoretical aerodynamics has nothing like the relative importance which it bears in airplane design. This is to be expected when we consider that the gross lift of an airship depends solely on the specific gravity of the gas and the bulk of the gas container, and not at all on shape or other aerodynamic characteristics which determine the lift of airplanes. Forthcoming volumes of the Ronald Aeronautic Library will deal with theoretical and applied aerodynamics; therefore a single chapter on aerodynamic forces is deemed sufficient for this book.

The literature on airship design is very meager, but on the other hand, there is a vast literature on the strength of materials and the design of structures. No one should attempt to master the art of airship design without a thorough grounding in the theory of beams, columns, and frame structures, including the elements of applied elasticity and the use of such elastic theorems as Castigliano's "Principle of Least Work" in solving statically indeterminate structures. Knowledge of structural theory being presupposed, only such theorems and methods of calculation as are peculiar to airship design are developed in this book.

The principal materials of airship construction are textile fabrics, duralumin, and steel. The textile fabrics are very fully described in Part III of "Free and Captive Balloons" by C. deF. Chandler, in the Ronald Aeronautic Library. Many volumes exist on steel, including the high tensile varieties used in aircraft construction. The literature on duralumin is also sufficiently extensive, so that there is no need to treat the subject in this volume.

The mensuration of areas and solids bounded by curves, and calculations of buoyancy, load, shear and bending moments are among the fundamentals of naval architecture; but a knowledge of naval architecture is not assumed for the student of airship design, although it is a very great help to him, and these subjects are therefore developed at some length in Chapters III and IV.

In the present state of the art of airship design, many matters are controversial, and in such cases the author has endeavored to present both sides of the question without bias or dogma, so that the reader may formulate his own theories and ideas, and perhaps arrive at new solutions for old and hitherto imperfectly solved problems.

The author gratefully acknowledges his indebtedness to his colleagues in the Bureau of Aeronautics, particularly to Commander J. C. Hunsaker, formerly Chief of the Design Section. Thanks are due also to the National Advisory Committee for Aeronautics and the Royal Aeronautical Society for kind permission to republish much valuable data.

C. P. Burgess

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CHAPTER I

THE TYPES OF AIRSHIPS

Classification. Airships are commonly classified into three types, known as nonrigid, semirigid, and rigid. The first two types have fabric envelopes dependent upon internal gas or air pressure for maintenance of form, while the rigid type has a structural framework of girders and wires over which a fabric cover is tightly drawn and maintained in shape independently of the gas pressure. In rigid airships the gas is contained within fabric cells which are normally flabby from being only partially inflated.

It has been proposed to classify all airships of the nonrigid and semirigid types as pressure airships, and the rigid type as pressureless. The metalclad airship, designed by the Aircraft Development Corporation, is unique in being both a pressure and a rigid airship, since it depends upon internal pressure to maintain its thin metal skin in tension and to avoid collapse in flight, while at the same time it is rigid in that it has a structural hull of practically invariable shape, which cannot be deflated and folded like a fabric envelope.

The nonrigid and semirigid types of airships are admirably described in "Pressure Airships" by Blakemore and Pagon (Ronald Aeronautic Library). This volume treats chiefly the largest and most important type, the rigid airship, except in so far as the principles and methods of design are common to all three classes. Some typical airships are shown in Figures 1 to 4.

The pressureless rigid airship. The modern rigid airship is an evolutionary development from the historic airship Zeppelin

I, built by Count Zeppelin in 1900. During the course of the quarter century of progress since the construction of the first Zeppelin, the Zeppelin Company has always been in the foreground, although other constructors in Germany, Great Britain, and the United States have contributed not a little to the evolution of the type.

The hull of the rigid airship consists of a framework of latticed girders running longitudinally and transversely, braced by steel wire tension members. There are from 17 to 25 rows of longitudinal girders, depending upon the size of the ship; and it is probable that the number will be increased in the larger ships of the future. These girders are usually spaced equally, to give a hull in which the cross-sections are regular polygons, although some departure from the regular polygonal form is common near the keel at the bottom of the ship. The function of the keel is to afford a longitudinal corridor through the ship, and to provide stowage space and a structure to carry the fuel, ballast, bombs, cargo, crew's quarters, etc. In the early Zeppelin airships, the keel was external to the hull; later it was placed internally, following the practice of the Schütte-Lanz Company, with great improvement to the aerodynamic characteristics, and a saving in the structural weight; but in the latest Zeppelin airship, the *Los Angeles* of the United States Navy, the bottom of the keel is dropped a little below the circumscribing circle through the other longitudinal girders.

The transverse girders form rings, designated frames, of which there are two kinds, known as main and intermediate frames. The main frames are cross-wired with a complex system of wiring between the joints; these frames give transverse rigidity to the hull structure, and form bulkheads between adjacent gas cells. The intermediate frames are necessarily without transverse wiring in order to avoid interference with the gas cells; and they have but little rigidity, although they assist the longitudinals to sustain gas pressure loads, and compressive forces resulting from the bending of the ship.

The hull structure is stiffened against shear and torsion by steel wires, designated shear wires, running diagonally in the quadrilateral panels bounded by the longitudinal and transverse girders.

The framework of the hull is covered by an envelope or outer cover of cloth, drawn smooth and tight, in order to minimize resistance, and made as nearly impervious as practicable to moisture and to the heat and light of the sun.

The gas from which the buoyancy is derived is contained within cells of cotton cloth lined with goldbeater's skin to provide gas-tightness. These cells press against the girders of the hull, and are restrained against excessive bulging between the girders by netting, termed the gas cell netting or wiring, secured to the inner edges of the longitudinal girders. The netting is sufficiently tight to insure a minimum clearance of about 3 to 5 in. between the gas cells and the outer cover. The gas cells are free to contract to no volume or to expand to the maximum limits of the structure and netting within which they are confined.

There is a very general impression that because the framing of a rigid airship lies in several planes, instead of in one plane like the side trussing of a bridge, the full diameter of the hull is not utilized to give girder strength. As a matter of fact, the transverse wires in the main frames, and the diagonal shear wires in the side panels, bind the girder structure so effectively that the entire hull acts practically as a homogeneous tube of high efficiency.

The metalclad airship. A rigid airship differing radically from the Zeppelin type is the metalclad airship, having a gas-tight envelope of light metal plating, supported by a metal framework. Within the metal envelope there is a fabric ballonet diaphragm to provide for variation in the gas volume. The first rigid airship ever constructed was of this type. It was designed and built by an Austrian, named Schwarz, in 1897. Its envelope was of aluminum plates, .008 in. in thickness, secured

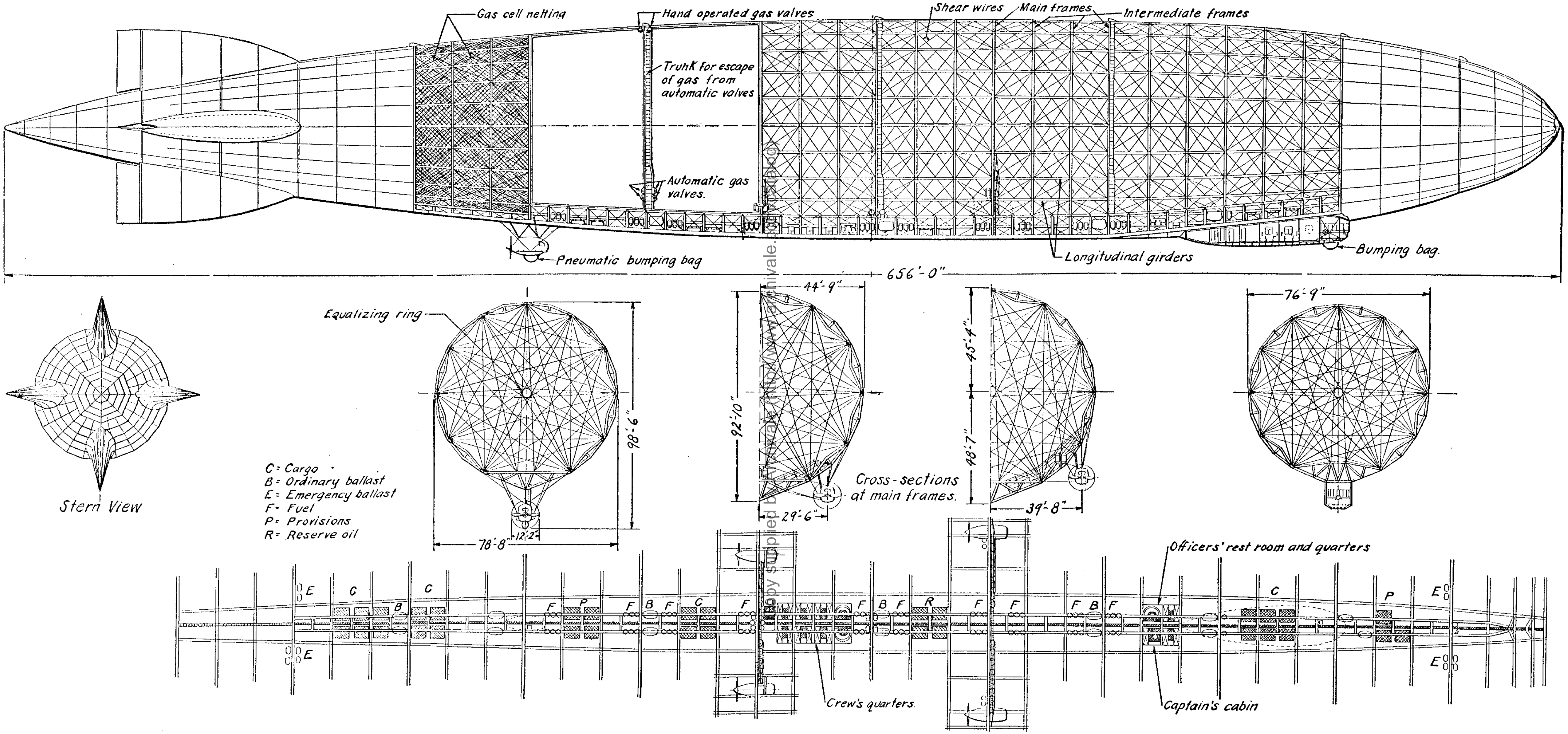


Figure 1. U.S.S. *Los Angeles*, General Arrangement and Cross-Sections of Ship and Layout of Keel

to a lattice framework of aluminum tubes. On its first and only flight the airship leaked so much gas at the joints of the plating that it crashed to the earth and became a total wreck only four miles from the starting point. The construction of an airship with a metal plate envelope has never been repeated; but at the present time (1927) the construction of such an airship is under way by the Aircraft Development Corporation of Detroit, Michigan.

Using an immensely superior technique to that which was possible to Major Schwarz thirty years ago, the faults and troubles of the first metalclad airship may be overcome; but the practicability of the type cannot yet be foreseen. The metalclad airship is of the pressure type, although rigid, and has the advantage over the conventional rigid type in that nearly all of its parts are in tension, and tensile forces can be taken care of by much lighter members than are required for equal compressive forces. The Zeppelin type is handicapped by a large weight of fabric in the gas cells and outer cover, contributing very little to the structural strength. In the pressure airship, the envelope not only provides the greater part of the strength, but also performs the functions of the gas cells and outer cover. Pressure airships, as hitherto constructed, have not been structurally lighter than the pressureless type, because rubberized balloon fabric [1] has a breaking length (i.e., the ultimate strength of a strip divided by the weight per foot of length) of only about 11,000 ft., as compared with about 45,000 ft. for duralumin, and 65,000 ft. for high tensile steel wire. Moreover, the strength of fabric diminishes so rapidly with age that a larger material factor of safety is required for it than for metal. The metalclad airship is a pressure airship constructed of duralumin instead of the much less efficient balloon fabric, and therefore has the possibility of the lightest construction yet attained, but it is

[1] Strength and weight of rubberized fabrics are tabulated in Part III of "Free and Captive Balloons," a volume of the Ronald Aeronautic Library.

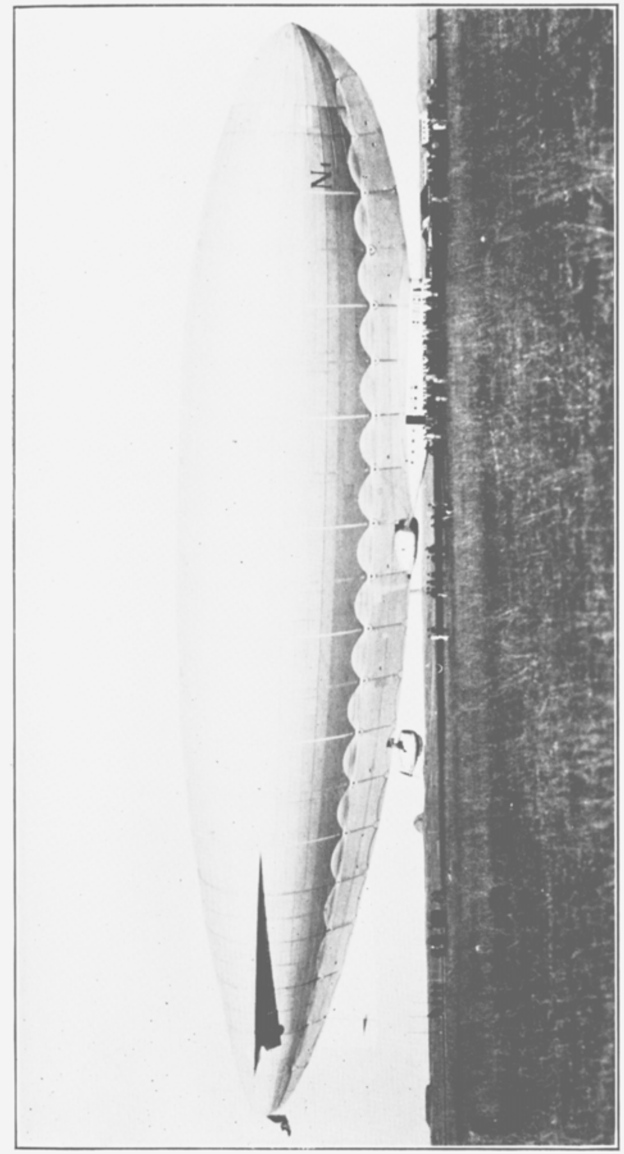


Figure 2. North Pole Semirigid Airship *Norge*

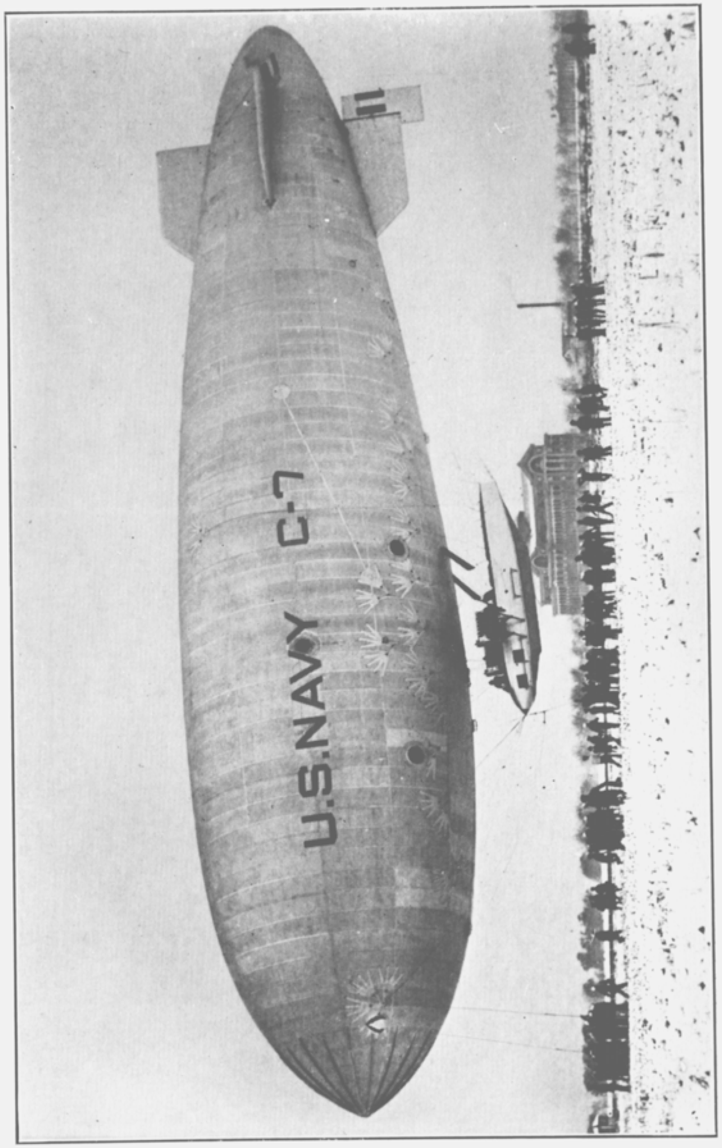


Figure 3. Nonrigid Type Airship, U. S. Navy C-7

not yet known what material factor of safety should be required to allow for the loss of strength through vibration and corrosion of such extremely thin metal.

Tests of small samples of the metal envelope, including riveted joints, have shown a gas-tightness from ten to a hundred times greater than rubberized fabric; but here again, experience with an actual ship is required to prove what can be done in practice over the great area of an airship envelope, exposed to the weather and varying stresses and vibrations.

The experimental metalclad airship being constructed by the Aircraft Development Corporation is of about 200,000 cu. ft. volume, 150 ft. in length, and 53 ft. maximum diameter. In appearance it closely resembles a nonrigid airship. It is also like a nonrigid airship in having no subdivision of the gas space. The size of this airship is too small to take advantage of the weight saving theoretically possible through the use of duralumin sheet instead of rubberized fabric. Its skin of .008 in. thickness is as thin as it is practicable to make, although its strength is ample for a much larger airship. Such an airship is much more liable to come to grief through lack of pressure than from excess of it.

The theoretical disadvantages of the metalclad rigid airship, apart from practical troubles which may yet develop in construction and operation, are the inaccessibility of the structure inside the gas space, and the dependence upon pressure to an even greater extent than the conventional pressure airships. Loss of pressure will cause a fabric envelope to wrinkle without damage to itself; but wrinkles in a metal envelope would probably produce serious tears, if not complete destruction.

Subdivision of the gas space can probably be effected without difficulty in large metalclad airships; but the subdivision will not be such a safeguard as in the pressureless ship, because structural integrity requires the maintenance of pressure even in a cell from which the gas is lost.

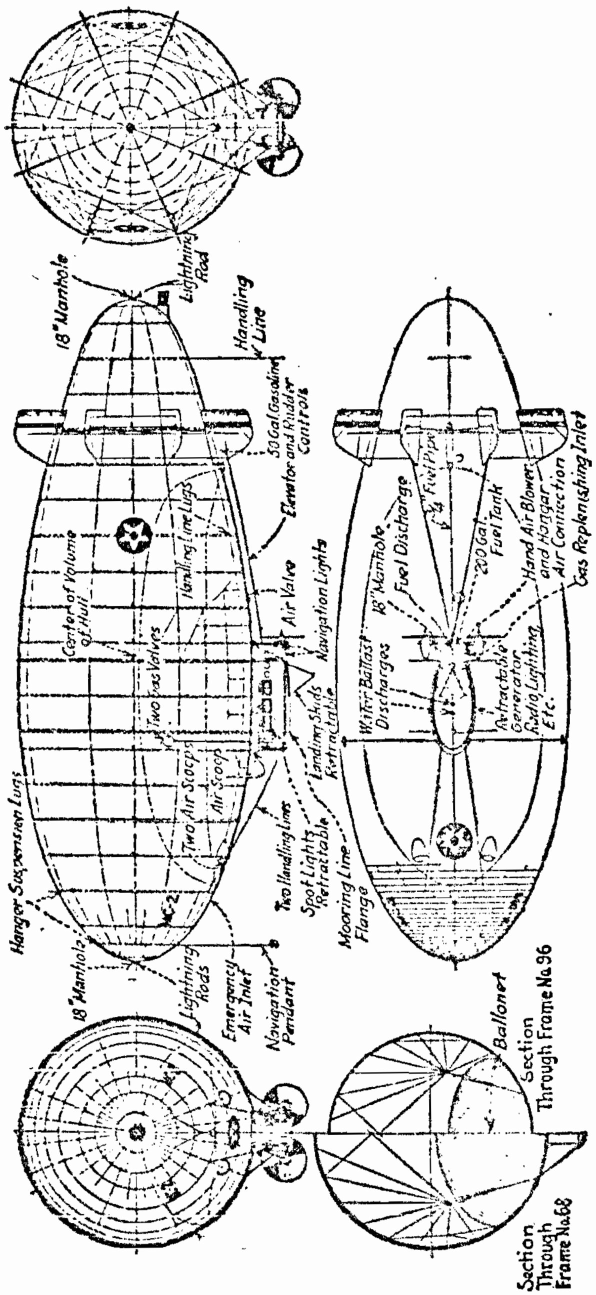


Figure 4. Metal-clad Airship *MC-2*

METALCLAD AIRSHIP *MC-2*

General Dimensions

Gas capacity	200,000 cu. ft.
~~Max. ballonet capacity~~	55,000 cu. ft.
Length of hull	150 ft.
Max. diam. of hull	53 ft.
Length of car	24 ft.
Width of car	8.5 ft.
Thickness of skin	.008 in.
Lateral fin area	250 sq. ft.
Total elevator area	180 sq. ft.
Top and bottom fin area	340 sq. ft.
Total rudder area	90 sq. ft.
Total fin and control surface	860 sq. ft.
Power (at 1,700 r.p.m.)	400 Hp.
Two Wright 200 Hp. "*J-4*" air-cooled radial engines	
Duralumin propellers (two-bladed)	

Estimated Performance

Gas Unit Lift	Helium (.062lb./ft.^{3})	Hydrogen (.068lb./ft.^{3})
Gross lift	12,600 lbs.	13,800 lbs.
Useful lift: Crew (4)	750 lbs.	750 lbs.
Passengers (6)	1,100 lbs.	(10) 1,900 lbs.
Fuel	1,500 lbs.	1,500 lbs.
Oil	200 lbs.	200 lbs.
Reserve	350 lbs.	750 lbs.
Weight empty	8,700 lbs.	8,700 lbs.
Commercial range (with 4 crew and passengers or cargo)	720 mi.	*1,200 mi.
Maximum range (with 4 crew only)	1,200 mi.	*2,200 mi.
Maximum speed (400 Hp.)	70 m.p.h.	
Cruising speed (200 Hp.)	53 m.p.h.	
Static ceiling	8,000 ft.	10,000 ft.

* Based on partial use of hydrogen fuel.

The future of rigid airships. The great outstanding merits of the pressureless rigid airship over all other types are:

(*a*) Permanence of form, independently of internal pressure.

(*b*) Subdivision of the gas space so that one or even more cells may be completely deflated without detriment to the airworthiness of the ship.

(*c*) Accessibility in flight to the structure and to the surface of the gas cells.

These three qualifications appear to be almost vitally necessary for large airships intended for long voyages; and there is at present no indication that they can be realized in any other type. Other advantages are:

(*d*) The ease with which gas cells and power cars may be removed and replaced.
(*e*) The strong structure of the bow, which makes the rigid airship suitable for high speed and for mast anchoring.
(*f*) The slow rate of loss of gas through holes in the fabric because of the low pressure.
(*g*) Safety in electrical storms through absence of rubber in the outer cover, and freedom from the electrical condenser effect which may occur in rubberized fabric.
(*h*) Minimum trouble from superheat because of the circulation of air between the outer cover and the gas cells.

Against these advantages may be set the following faults:

(*a*) High cost.
(*b*) Impossible to dismantle and store or transport for re-erection.
(*c*) Weakness of the single ply cloth used for the outer cover and gas cells.

The future development of airships is likely to show continually increasing dimensions, just as in the development of steamships. In both air and surface ships, the endurance and habitability improve with size. Strength to resist storms and squalls, such as destroyed the *Shenandoah*, seems certain to be attained; and undoubtedly smaller ratios of length to diameter will assist greatly in securing better longitudinal strength without increased weight.

The accessibility to all parts of the ship will be improved; and with increasing size, it will be possible to use heavier fabrics, more durable and less easily torn than those now employed.

If the practical difficulties of constructing and maintaining the metalclad airship can be successfully overcome, and its theoretical possibilities in the way of weight saving realized, it will undoubtedly have an important field of usefulness in

spite of its apparently inherent disadvantages of inaccessibility, and dependence on gas pressure.

Choice of type. The question of the best type of airship for any particular size or use is highly controversial; and every designer has his own convictions on the subject. It is generally conceded that the pressureless rigid airship is not efficient for sizes below about one million cubic feet volume. For smaller sizes, some type of pressure airship is required. It is a curious fact that the only thoroughly satisfactory fabric ever developed for the envelopes of pressure airships is three-ply rubberized cotton cloth weighing about 14 oz. per square yard, and having a mean breaking strength of about 80 lbs. per inch. This fabric is sufficiently strong for circular sectioned nonrigid airships up to about 200,000 cu. ft. volume. From 200,000 to 1,000,000 cu. ft., very nearly the same fabric is used, and in order to maintain the stresses constant as the dimensions increase, the designer resorts to increasing complexity of construction, substituting internal suspension and multi-lobe cross-sections for the simple circular sections of the smaller ships, and reducing the necessary gas pressure by increasing dependence on rigid members throughout the range of progress from the simple nonrigid types through the gradations of semirigids, approaching more and more nearly to the rigid type as the size increases.

CHAPTER II

SIZE AND PERFORMANCE

Procedure for Preliminary Estimates of the Size and Horsepower for a Given Performance

The initial step in designing an airship that will have the desired speed, endurance, and capacity for carrying a specified military or commercial load, is to make an approximate calculation of the size and horsepower required. The first published description of the following procedure was given in the National Advisory Committee for Aeronautics Technical Note No. 194.

The relation between horsepower, speed, and size may be conveniently stated by the expression:

$$\text{Hp.} = \frac{V^{2/3}\,\rho v^3}{550\,K}$$

where

Hp. = horsepower
v = speed in ft./sec.
ρ = density of the air in slugs/ft.^{3}
(1 slug = 32.2 lbs.)
V = air volume of the ship in ft.^{3}
550 = the number of ft.lbs./sec. developed by one Hp.
K = a non-dimensional coefficient depending upon the propeller efficiency and the design of the hull and its appendages

K is thus a measure of the overall propulsive efficiency of an airship, including both the resistance of the ship, and the propeller efficiency.

The total weight of an airship and its contents may be divided into five groups, as follows:

(1) Air and gas
(2) Fixed weights exclusive of power plant, power cars, and fuel system
(3) Crew, stores, and ballast
(4) Power plant, power cars, fuel system, and fuel.[1]
(5) Specified military or commercial load

The weight of a cubic foot of air in the "standard atmosphere"[2] (i.e., temperature 60° F, and 29.92 in. barometric pressure) at sea level is .07635 lbs.; and .07635 times the air volume of the airship is defined as the "standard displacement," *D*, in pounds. It follows that:

$$\text{Hp.} = \frac{D^{2/3}\,\rho v^3}{99\,K}$$

For the purpose of the preliminary estimates, items (1), (2), and (3) are assumed to vary as *D*, and are expressed as fractions of *D*. More accurate means of computing the variation of the weights under item (2) with size are described in the section on Normand's equation (page 20). Item (4) is assumed to be proportional to Hp., and is expressed as a fraction of $D^{2/3}$. Items (4) and (5) together take that fraction of *D* remaining over from items (1), (2), and (3).

Problem 1. Find the volume and horsepower of a rigid airship to carry a military load of 15,000 lbs. at 60 knots (101.3 ft./sec.) for 60 hours, 85% of the total volume being filled with helium lifting .064 lb./ft.^{3} (94% pure), in the standard atmosphere.

Since the hull is specified in the conditions of the problem to be 85% full of gas, the weight of the air in the hull is 15% *D*; and the weight of the gas is 85% *D* multiplied by the difference between the weight of air and the lift of gas per unit volume, and divided by the weight of air per unit volume. The total weight of air and gas is therefore given by:

[1] The weight, horsepower and other characteristics of American types of aircraft engines are set forth in "Aircraft Power Plants," a volume of the Ronald Aeronautic Library.

[2] Standard atmosphere and ascensional force of gases are treated fully in "Aerostatics," a volume of the Ronald Aeronautic Library.

$$\text{Air and gas} = [.15 + .85\,(.07635 - .064)/.07635] \times D$$
$$= .288\,D$$

From existing data, the fixed weights exclusive of power plants, power cars, and fuel system equal .3 D, and the crew, stores and ballast amount to .055 D.

There remains for power plant, fuel, and military load

$$(1 - .288 - .30 - .055)\,D = .357\,D$$

Assuming the weight of the power plant and its cars to be 8 lbs./Hp. and the weight of the fuel and fuel system to be .6 lb. per horsepower hour, the total weight of power plant, fuel system, and fuel is $[8 + (.6 \times 60)]$ Hp. = 44 (Hp.). Combining this with the military load,

$$15{,}000 + 44\,(\text{Hp.}) = .357\,D \qquad (1)$$

Suppose that from existing data on similar ships, $K = 63.5$.

$$\text{Hp.} = \frac{(101.3)^3 \times .00237 \times D^{2/3}}{99 \times 63.5}$$
$$= .39\,D^{2/3} \qquad (2)$$

Combining equations (1) and (2),

$$D - 48\,D^{2/3} = 42{,}000 \qquad (3)$$

The problem of the size required for a given performance will always reduce to an equation of the form of (3), which may be expressed generally:

$$D - AD^{2/3} = B$$

The solution for any particular values of A and B may be obtained from the charts (Figure 5 or 6) by laying a straight edge across these values of A and B on the scales at the left and right hand sides of the chart.

In this problem, $A = 48$ and $B = 42{,}000$, and from the chart (Figure 5), $D = 215{,}000$ lbs., and the air volume = 215,000/.07635 = 2,820,000 ft.³ $D^{2/3} = 3{,}600$, and Hp. = .39 × 3,600 = 1,410.

Effect of increasing the speed. *Problem 2.* Find the volume and horsepower of an airship to fulfill the same conditions

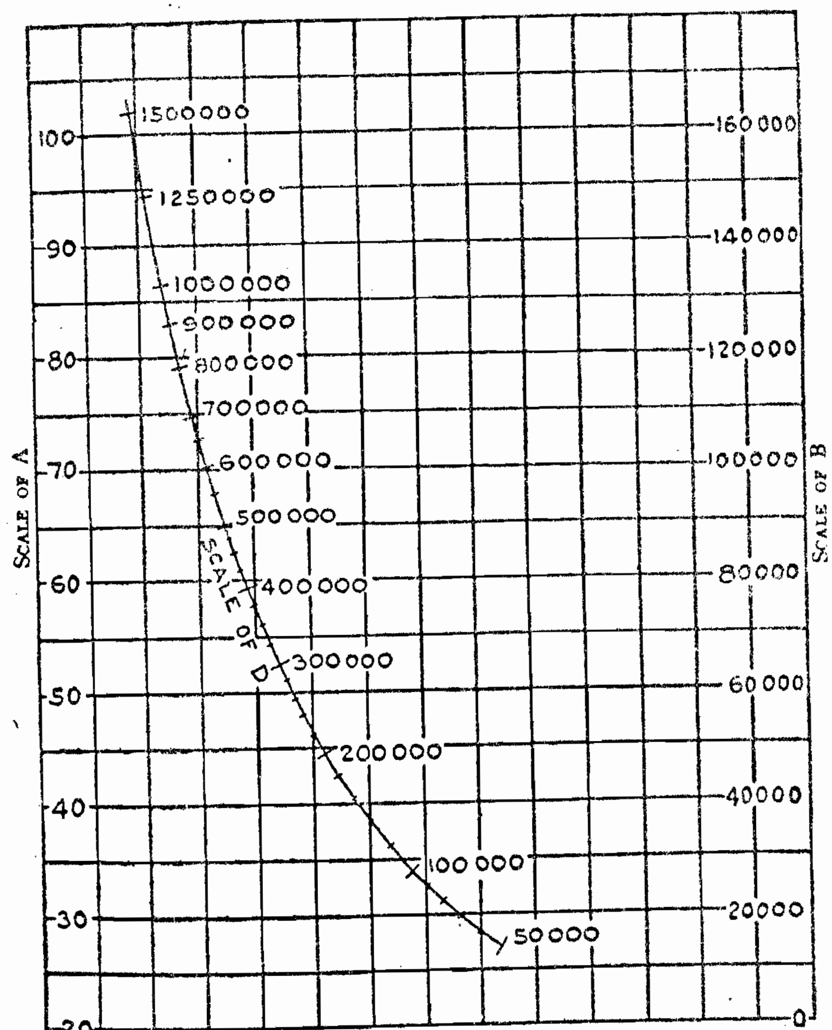


Figure 5. Curve of $D - AD^{2/3} = B$

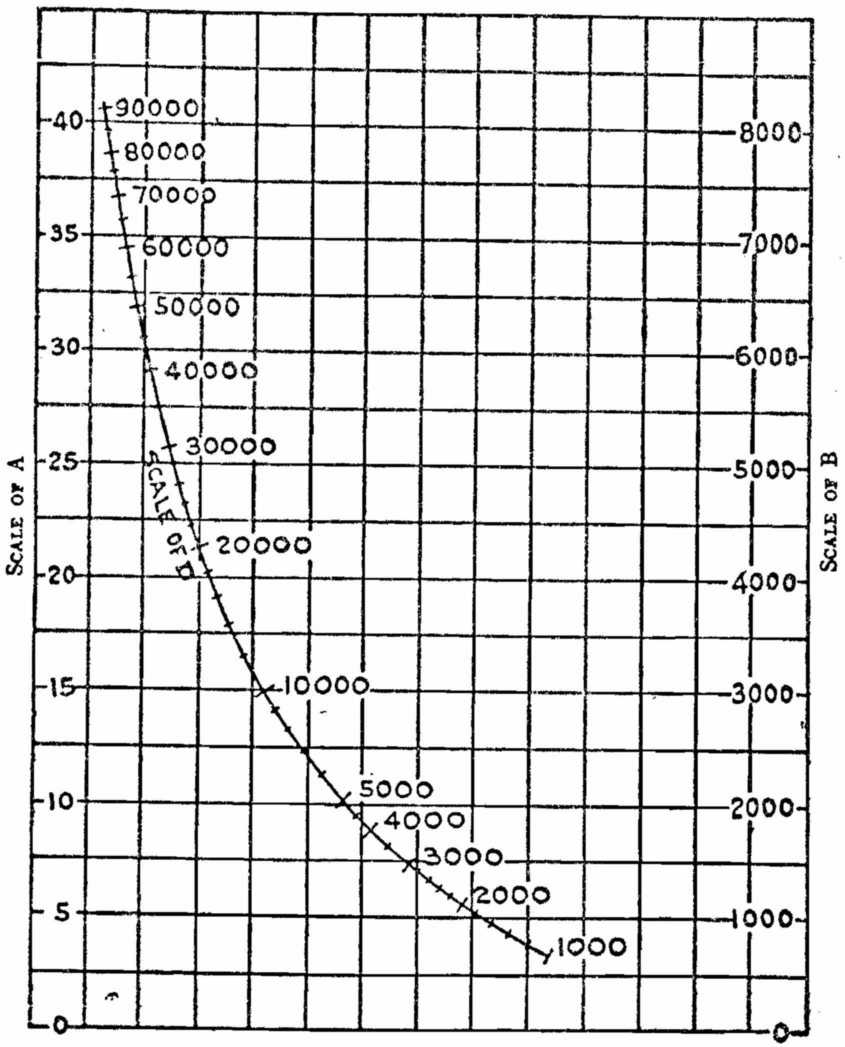


Figure 6. Curve of $D - AD^{2/3} = B$

as in Problem 1, except that the speed is to be 70 knots (118.2 ft./sec.) for 60 hours.

$$\text{Hp.} = \frac{(118.2)^3 \times .00237 \times D^{2/3}}{99 \times 63.5}$$

$$= .62\ D^{2/3}, \text{ and by combining with equation (1),}$$

$$D - 76.5\ D^{2/3} = 42{,}000$$

From Figure 5, $D = 570{,}000$ lbs. $D^{2/3} = 6{,}900$

Air volume $= 7{,}470{,}000$ ft.^{3}

Hp. $= .62 \times 6{,}900 = 4{,}290$

This example shows the great cost of increasing the speed while maintaining the same hours of endurance at the higher speed. On the other hand, the maximum speed could be quite easily increased to 70 knots, if the average cruising requirement remains 60 hrs. at 60 knots.

The horsepower at 60 knots is only 63% of that required for 70 knots, and the weight of power plant and fuel per horsepower is therefore $8 + (.63 \times .6 \times 60) = 30.7$ lbs. The fundamental equations are therefore:

$$15{,}000 + 30.7\ (\text{Hp.}) = .357\ D$$

and Hp. $= .62\ D^{2/3}$, whence

$$D - 53.3\ D^{2/3} = 42{,}000$$

$$D = 260{,}000 \text{ lbs.} \quad D^{2/3} = 4{,}080$$

$$\text{Hp.} = .62 \times 4{,}080 = 2{,}530$$

$$\text{Air volume} = 260{,}000/.07635 = 3{,}400{,}000 \text{ ft.}^3$$

Effect of reducing the structural weights. The method may be used also to find the extent to which the size may be reduced by structural improvements, as follows:

Problem 3. Suppose that by an improved design, the structural weight can be reduced 10% in comparison with the conditions assumed in Problem 1. The fixed weights exclusive of power plant, power cars, and fuel system become .27 D, and assuming as in Problem 1 that 60 knots is required for 60 hours,

$$15{,}000 + 44\,(\text{Hp.}) = .387\,D$$
$$\text{and Hp.} = .39\,D^{2/3}\text{, whence}$$
$$D - 44.4\,D^{2/3} = 38{,}800$$
$$D = 180{,}000 \text{ lbs.} \quad D^{2/3} = 3{,}200$$
$$\text{Air volume} = 180{,}000/.07635 = 2{,}360{,}000 \text{ ft.}^3$$
$$\text{Hp.} = .39 \times 3{,}200 = 1{,}250$$

By comparison with the results of Problem 1, it is found that a 10% saving in fixed weights exclusive of power plant, permits a 16% reduction in volume and 11.3% in horsepower.

A 10% improvement in the resistance coefficient may also be tested in a similar manner. Repeating the conditions of Problem 1, let K be increased by 10% from 63.5 to 69.9. Then:

$$\text{Hp.} = .355\,D^{2/3}$$
$$D - 43.75\,D^{2/3} = 42{,}000$$
$$D = 185{,}000 \text{ lbs.} \quad D^{2/3} = 3{,}250$$
$$\text{Air volume} = 185{,}000/.07635 = 2{,}420{,}000 \text{ ft.}^3$$
$$\text{Hp.} = .355 \times 3{,}250 = 1{,}150$$

Effect of altitude. If it is required to operate at a high altitude, the conversion factor, air volume $= D/.07635$, still holds, but in estimating the weights, the allowance made for air and gas must be increased, as in the following problem.

Problem 4. Determine the volume of a nonrigid airship to carry a military load of 1,000 lbs. at 4,000 ft. altitude at 50 knots (84.5 ft./sec.) for 10 hours. Ship to be inflated with hydrogen which has a lift of .068 lb./ft.^{3} in the standard atmosphere at sea level. Starting a flight at 4,000 ft. fully inflated, corresponds to 88.8% inflation at sea level; the weight of the air and gas at sea level is

$$[.112 + .888\,(.07635 - .068)/.07635]\,D = .209\,D$$

It may seem cumbersome to calculate the weight of air and gas in this manner instead of correcting D for the altitude, but it should be remembered that since the structural weights are to be taken as fractions of D, it is important that D should be taken as a constant fraction of the air volume.

From data on previous ships, the fixed weights exclusive of power plant and fuel system equal .4 D, and crew, stores, and ballast amount to .1 D. There remains for the power plant, fuel, and military load $(1 - .209 - .4 - .1)\,D = .291\,D$. Taking the weight of the power plant as 6 lb./Hp., and the weight of the fuel, oil, and containers as 0.6 lb./Hp. hr., the total weight of power plant, fuel, and fuel system is $[6 + (10 \times .6)]$ Hp. = 12 (Hp.), and the following equation is obtained:

$$1{,}000 + 12\,(\text{Hp.}) = .291\,D$$

From existing data, $K = 40.0$, and at 4,000 ft. altitude,

$$\rho = .888 \times .00237 = .0021 \text{ slugs; therefore,}$$

$$\text{Hp.} = \frac{(84.5)^3 \times .0021 \times D^{2/3}}{99 \times 40} = .32\,D^{2/3}$$

Combining these equations as before,

$$D - 13.2\,D^{2/3} = 3{,}340$$
$$\text{From Figure 6, } D = 9{,}100 \text{ lbs.} \quad D^{2/3} = 435$$
$$\text{Air volume} = 9{,}100/.07635 = 119{,}000 \text{ ft.}^3$$
$$\text{Hp.} = .32 \times 435 = 139$$

Commercial load on a given airship. The method may also be used to find the military or commercial load which can be carried by an airship of given size at a given speed and endurance, as follows:

Problem 5. Find the commercial load which can be carried by a rigid airship of 5,000,000 ft.^{3} air volume, at 4,000 ft. altitude, inflated with helium lifting .064 lb./ft.^{3} in standard atmosphere at sea level, and having a speed of 70 knots and an endurance of 24 hours. Given $K = 63.5$; and fixed weights exclusive of power plant $= .3\,D$; crew, stores, and ballast $= .05\,D$. Power plant = 8 lbs./Hp., fuel = 0.6 lb./Hp. hr., whence total weight of power plant and fuel $= [8 + (.6 \times 24)]$ Hp. = 22.4 (Hp.)

$$D = .07635 \times 5{,}000{,}000 = 382{,}000 \text{ lbs.}$$

Assuming that when fully inflated the gas cells occupy 95% of the air volume, the volume of gas when 88.8% inflated at sea level (to allow for 4,000 ft. altitude) is $.95 \times .888 = .844$ times the air volume, and the weight of the air and gas is

$$[.156 + .844\,(.07635 - .064)/.07635] \times D = .293\,D$$

$$\text{Hp.} = \frac{(118.2)^3 \times .0021 \times (5{,}000{,}000)^{2/3}}{550 \times 63.5}$$

$$= 2{,}900$$

The total weight of the power plant, power cars, fuel, and fuel system is $22.4 \times 2{,}900 = 65{,}000$ lbs. The weight available for commercial load is

$$382{,}000\,(1 - .293 - .3 - .05) - 65{,}000 = 71{,}500 \text{ lbs.}$$

Application of Normand's Equation to Estimating the Sizes and Weights of Airships

The previous section dealt with a method of first approximation to the required dimensions of an airship based upon the assumption that the total fixed weights, exclusive of the power plant, are a definite fraction of the air displacement. A more exact and detailed method will now be described. This method is based upon J. A. Normand's[3] approximate formulas for estimating the characteristics of vessels from a weight equation expressing the total weight of the ship as a sum of weight groups, each given in terms of the independent variables involved in the design.

The application of Normand's equation to airships was described by Comdr. J. C. Hunsaker in his Wilbur Wright Memorial Lecture to The Royal Aeronautical Society, 1920, published in *The Aeronautical Journal*, July, 1920.

Normand's equation is the sum of the weight groups, expressed as follows:

$$W = W_A + W_B + W_C + \text{etc.} = \Sigma W_Z,$$

where each term W_Z is expressed as a function of the principal

[3] Formules Approximative de Construction Navale, Paris, 1870.

elements of the design. The typical member of the weight equation is of the form

$$W_Z = KL^x D^y \alpha^z \beta^w, \text{ etc.},$$

where L and D are the length and diameter of the ship, and α, β, etc., are other variables entering with exponents, z, w, etc. By differentiation,

$$\Delta W_Z/W_Z = x\Delta L/L + y\Delta D/D + z\Delta\alpha/\alpha + \text{etc.}$$

In the problem of determining the characteristics of a new airship from data on existing ships, it is usually most convenient to begin by assuming variable volume with constant form, and then, if desired, the effect of changing the ratio L/D with constant volume can be investigated. When L/D is constant, D may be eliminated from all expressions for weight, and the typical member of the weight equation becomes

$$W_Z = KL^x\alpha^z\beta^w, \text{ etc.} \tag{4}$$

And $\Delta W_Z/W_Z = x\Delta L/L + z\Delta\alpha/\alpha + \text{etc.}$

But $W = KL^3$, and

$$\Delta W/W = 3\Delta L/L$$

Whence,

$$\Delta W_Z = W_Z(x\Delta W/3W + z\Delta\alpha/\alpha + \text{etc.}) \tag{5}$$

and $\Delta W = \Sigma\Delta W_Z = 1/(1 - \Sigma x W_Z/3W)\Sigma W_Z(z\Delta\alpha/\alpha + \text{etc.}$

$$= N\Sigma W_Z\,(z\Delta\alpha/\alpha + \text{etc.})$$

where $N = 1/(1 - \Sigma x W_Z/3W)$ (6)

When L/D is varied with constant volume,

$$\Delta W_Z/W_Z = x\Delta L/L + y\Delta D/D, \text{ and}$$

$$\Delta W/W = \Delta L/L + 2\Delta D/D = 0, \text{ whence}$$

$$\Delta W = \Sigma\Delta W_Z = (\Delta L/L)\Sigma W_Z(x - y/2) \tag{7}$$

Variation of the weight items with linear dimensions. The manner of dividing the total weight W into the W_Z groups is a matter of the individual choice of the designer. One method is to consider the total weight in the air as composed of the following weight groups:

A. Outer cover; complete with fastening, seams, dope, lacing, and local stiffening

B. Gas bags; complete with sleeves, appendages and fastenings, but without valves

C. Longitudinal girders; complete including mains and intermediates, corridor structure, local bow stiffening, mooring attachments, stern piece, but not attachments to transverse members

D. Transverse framing; main and intermediate transverses, together with connections to longitudinals, wire terminals and attachments for car suspension

E. Shear wires; diagonal strength wiring to resist shear between hull girders

F. Transverse wires; wiring of main transverses

G. Netting; mesh wiring and netting to distribute pressure of gas bags to hull framing, also axial wire, if any

H. Fins and rudders; complete in place with covering, hinges and quadrants, but not operating gear in ship

I. Hull fittings; mooring and handling tackle, valves, valve controls, gas exhaust, and filling trunks, access ladders and walkways, look-out stations, ventilation, rudder operating mechanism outside control car, reserve ballast

J. Machinery; all machinery weights and power cars including engines, gearing, clutches, propellers, radiators, water and auxiliaries not in the hull of the ship, empty oil and gasoline tanks in cars, and piping in cars

K. Fuel; gasoline and oil necessary for endurance specified, gasoline and oil tanks in hull, piping in hull, special supports for tanks

L. Miscellaneous fixed weights:

L_1. Electrical; all electrical equipment except radio, but including lighting, signal and interior communication sets

L_2. Crew; the normal crew necessary to operate the ship with their food, clothing and conveniences

M. Gas and air in the ship

Θ. Independent weights; cargo, armament, radio, control car with all gear installed, stores, ballast or passengers, margin.

Considering the weight groups, as above described, for airships of similar general construction, that is, with the same number of gas bags and longitudinals, we may establish approximate relations between the weights and the principal dimensions and performance characteristics taken as independent variables.

The following independent variables are fixed by the desired military or commercial characteristics:

V = maximum trial speed
v = cruising speed
t = hours endurance at cruising speed
θ = independent weights arbitrarily specified to be carried, such as passengers, bombs, or cargo

In addition, the following independent variables must be assumed by the designer as his estimate of the state of the art:

a = unit weight of outer cover
b = unit weight of gas bags
c = unit consumption of engines at speed v'
e = efficiency of propellers and gears
f = allowed fibre stress in structural material
m = unit weight of machinery installation
r = resistance coefficient

The dependent variables are:

L = length of ship
D = diameter of ship
W = total weight of ship (displacement)
M = bending moment

The variation of each group with the linear dimensions and the independent variables must be considered in detail.

A. Outer cover. The weight of outer cover complete with seams and lacing is proportional to the unit weight of material and to the area covered. The area is determined by the lines of the ship and the principal dimensions, L and D. The total area of the outer cover is proportional to kLD where k is a coefficient depending on the form of the ship, and very nearly constant for ships of good form. The weight of outer cover is, therefore, $kaLD$, or considering k truly constant:

$$W_A \sim aLD$$

B. Gas bags. The gas bags, assumed of constant number, require an area of fabric of weight W_{B1} for the circumferential portions proportional to the area of outer cover or to LD. In addition the ends require an area of weight W_{B2} proportional to D^2. Examination of returned weights from actual ships shows the two portions of the total area of gas bags are very nearly equal. Then:

$$W_{B1} \sim bLD \text{ and } W_{B2} \sim bD^2, \text{ where}$$
$$W_B = W_{B1} + W_{B2}$$

C. Longitudinal girders. The weight of the longitudinal girders may be divided into four parts, corresponding to four different functions of these members. The division is as follows:

W_{C1} = that part of the weight of the longitudinals devoted to resisting the static bending moment of the ship

W_{C2} = the part of the weight devoted to resisting the aerodynamic bending moment

The end load may be either tension or compression, but the girders are always amply strong for any tensile force which may come upon them, and the maximum compressive load must be taken as the criterion for design purposes. The girders are of such low slenderness ratio that failure as long columns need not be considered.

P = end load in any girder
A = sectional area of girder
$f = My/I$, and
$P = fA = MAy/\Sigma Ay^2$
Static shear $\sim LD^2$, and
Static $M \sim L^2D^2$, and $y \sim D$
A for static $M \sim L^2D/f$
$W_{C1} \sim L^3D/f$

Aerodynamic shear $\sim L^{2/3} D^{4/3} V^2$
Aerodynamic $M \sim L^{5/3} D^{4/3} V^2$
A for aerodynamic $M \sim L^{5/3}D^{1/3} V^2/f$
$W_{C2} \sim L^{8/3} D^{1/3} V^2/f$

For the gas pressure loads on the longitudinals, let:

w = load per running foot on any girder
i = moment of inertia of girder section
d = depth of girder section
p = gas pressure at girder
m = maximum bending moment in girder
A = sectional area of girder

Then:

$$w \sim pD \sim D^2$$
$$m \sim wL^2 \sim D^2L^2$$
$$f \sim md/i$$

Suppose $d \sim D$, preserving geometrical similarity. Then:

$$\frac{d}{i} \sim \frac{D}{AD^2} \sim \frac{1}{AD}$$
$$A \sim \frac{DL^2}{f}$$

The weight $\sim AL$, or $W_{C3} \sim L^3D/f$

A fourth function of the longitudinal girders occurs in the corridor which acts as a bridge to carry the concentrated loads of fuel and ballast between the main frames. This function may be considered as involving loads varying in the same way as those due to the static bending moments, or:

$$W_{C4} \sim L^3D/f$$

Summarizing the weight of the longitudinal girders,

$$W_C = W_{C1} + W_{C3} + W_{C4} + W_{C2}$$
$$(W_{C1} + W_{C3} + W_{C4}) \sim L^3D/f$$
$$W_{C2} \sim L^{8/3}D^{1/3}V^2/f$$

In practical problems it may be necessary to make some estimate or assumption as to the magnitude of the fraction W_{C2}/W_C (i.e., the fraction of the total weight of the longitudinals devoted to resisting the aerodynamic bending moment).

D. Transverse framing. The most severe load on a main transverse frame is due to the tension in the transverse wiring caused by a deflated gas bag or excess pressure in one bag. The mean intensity of effective gas pressure is proportional to D, the area of gas bag supported by one wire to D^2, and the total side load ϕ on that wire to D^3.

Let a = sectional area of wire
E = modulus of elasticity of wire
T = tension in wire
P = compression in frame

Since $T^3 = Ea\phi^2/24$, where T/a and E are constant[4],

$$\phi \sim D^3 \text{ and } P \sim T$$

and since $W_{D1} \sim PD/f$

$$W_{D1} \sim D^4/f$$

The intermediate frames, if any are fitted, are put in compression by the shear wires, and in bending by gas pressure. Consider the former as the more important; then the shearing force at any section is proportional to the volume up to that section or to LD^2. The tension in the shear wires is proportional to the shear, and the load P thrown into the transverse frame by these wires is expressed by

$$P \sim LD^2$$
$$W_{D2} \sim PD/f \sim LD^3/f$$
$$\text{and } W_D = W_{D1} + W_{D2}$$

E. Shear wires. The area of shear wires cut by any cross-section of the ship should be proportional to the shear or to LD^2. Their weight will then be:

$$W_E \sim L^2D^2$$

F. Transverse wires. These have been considered with W_D. Therefore, $W_F \sim D^4$

The unit stress in the material of the wires is assumed constant and does not appear.

[4] Proof of this formula is given in Chapter VIII.

G. Netting. The gas bag netting and mesh wiring support, per foot length of ship, a unit gas pressure proportional to D, and an area of bag between longitudinals proportional to D, or a load per foot run of ship proportional to D^2. The weight of netting per foot length of ship should then vary as D^3, and the weight of netting for the whole ship:

$$W_G \sim LD^3$$

H. Fins and rudders. The necessary area of fins and rudders is found from experience to vary as LD for ships of usual form. The unit air pressure on these surfaces varies as V^2, and the weight (for similar structures) is therefore:

$$W_H \sim (LD)^{3/2}V^2$$

I. Hull fittings and reserve ballast. Assume these weights vary as the displacement. Then:

$$W_I \sim LD^2$$

J. Machinery. To a first approximation the resistance to propulsion is represented by:

$$R = rL^{2/3}D^{4/3}V^2$$

where r is an aerodynamic coefficient dependent on the lines of the ship, and but little changed by small variations in the ratio L/D. The power required is then:

$$J = RV/e = (r/e)L^{2/3}D^{4/3}V^3$$

and the total weight of machinery:

$$W_J = mJ = (mr/e)L^{2/3}D^{4/3}V^3$$

K. Fuel. The weight of fuel and oil, including necessary tanks, is directly dependent on the consumption of the engines, the power and endurance (at cruising speed) assumed.

$$W_K = cJt = (ctr/e)L^{2/3}D^{4/3}v^3$$

L. Miscellaneous. Under this head for convenience are grouped weights necessary for the ship as a useful and manageable craft, but not altogether proportional to its size, comprising:

L_1. Electrical apparatus for lighting and signaling
L_2. Crew and their effects

The above weights are naturally greater for larger ships, but do not increase so rapidly as the displacement. They are only partially under the control of the designer. It seems reasonable to assume that:

$$W_L \sim LD$$

M. Gas and air.

$$W_M \sim LD^2$$

Independent load. Finally, we have those weights which may be arbitrarily fixed without reference to any other characteristic of the ship, such as cargo, radio, control car, passengers, bombs, margin, etc.

$$W_\theta \sim (LD)^\circ = \Theta, \text{ a constant}$$

When L/D is constant, the foregoing expressions for the variation of the weight groups may be reduced to the following simplified forms:

$$W_A \sim aL^2 \qquad W_H \sim L^3V^2$$
$$W_B \sim bL^2 \qquad W_I \sim L^3$$
$$(W_{C1} + W_{C3} + W_{C4}) \sim L^4/f \qquad W_J \sim (mr/e)\, L^2V^3$$
$$W_{C2} \sim L^3V^2/f \qquad W_K \sim (ctr/e)\, L^2v^3$$
$$W_D \sim L^4/f \qquad W_L \sim L^2$$
$$W_E \sim L^4 \qquad W_M \sim L^3$$
$$W_F \sim L^4 \qquad W_\theta \sim \theta$$
$$W_G \sim L^4$$

The coefficient N determines the amount by which the size and weight of the ship will be changed by variation of any one weight or performance characteristic, when keeping the other characteristics constant.

Numerical example on Normand's equation. The practical application of Normand's equation may be illustrated by the following example:

With a design of an airship of the Zeppelin *L*-49 type as a basis, it is desired to find the effect of the following proposed alterations:

(*a*) Increase the bomb load by 2,000 lbs.
(*b*) Use a stronger outer cover 25% heavier
(*c*) Reduce weight of transverses and longitudinals 15% by a new type of construction
(*d*) Increase the speed 5%
(*e*) Reduce L/D from 8 to about 7

The principal dimensions and characteristics of the given airship are as follows:

L-49

Length	643 ft.
Diameter	78 ft. 8 in.
Volume	1,940,000 cu. ft.
Gross lift	129,800 lbs.
Air displacement	164,000 lbs.
Maximum speed	60 m.p.h.
Cruising speed	45 m.p.h.
Total B.Hp.	1,200
B.Hp. at cruising speed	600
Fuel capacity	55,000 lbs.

Calculation of the coefficient N. The first step in solving this problem is to calculate the coefficient N, defined in equation (6). The total change in displacement for any given modification in the performance characteristics is proportional to N.

From equation (6) and Table 1,

$$N = 1/(1 - \Sigma x W_x/3W)$$
$$= 1/(1 - .8266)$$
$$= 5.76$$

To carry the additional 2,000 lbs. of bombs without loss in other characteristics, the displacement must be increased by

$$\Delta W = N\Delta\theta = 5.76 \times 2{,}000 = 11{,}520 \text{ lbs.}$$

TABLE 1. CALCULATION OF N FOR A RIGID AIRSHIP OF THE TYPE OF THE L-49

Item	Symbol	Weight lbs.	x	$xW_x/3W$
Outer cover	W_A	3,900	2	.0158
Gas cells	W_B	8,860	2	.0360
Longitudinals	$W_{C1}+W_{C3}$	5,330	4	.0434
Longitudinals	W_{C2}	2,670	3	.0163
Corridor	W_{C4}	3,000	4	.0244
Transverses, main	W_{D1}	5,200	4	.0423
Transverses, intermediate	W_{D2}	2,450	4	.0200
Shear wires	W_E	1,140	4	.0093
Transverse wires	W_F	1,100	4	.0090
Netting and axial wire	W_G	2,260	4	.0184
Tail surfaces	W_H	1,800	3	.0110
Ballast, fitting, and controls	W_I	9,500	3	.0550
Power plant	W_J	12,000	2	.0488
Fuel and tanks	W_K	58,990	2	.2400
Miscellaneous	W_L	6,000	2	.0244
Air and gas	W_M	34,200	3	.2085
Armament, radio, and control car	W_Θ	5,600	0	0
		164,000	$\Sigma xW_x/3W$	= .8266

Similarly, the heavier outer cover requires that the displacement be increased by

$$\Delta W = NW_A\,\Delta a/a$$
$$= 5.76 \times 3{,}900 \times .25 = 5{,}620 \text{ lbs.}$$

The saving due to 15% less weight in the longitudinal and transverse structural framing is given by

$$\Delta W = -.15\,N\,(W_C + W_D)$$
$$= -.15 \times 5.76\,(5{,}330 + 2{,}670 + 3{,}000 + 5{,}200 + 2{,}450)$$
$$= -16{,}100 \text{ lbs.}$$

The 5% increase in speed requires more powerful engines and stronger longitudinals and tail surfaces. It is assumed that the cruising speed remains unchanged, so that the weight of fuel is changed only as required by the increased size. The increase of displacement is given by

$$\Delta W = N(\Delta V/V)\,(2W_{C2} + 2W_H + 3W_J)$$

Note that: $W_{C2} \sim V^2$

$$W_H \sim V^2$$
$$W_J \sim V^3$$
$$\Delta W = 5.76 \times .05\,[(2 \times 2{,}670) + (2 \times 1{,}800) + (3 \times 12{,}000)]$$
$$= 12{,}960 \text{ lbs.}$$

The total increase of displacement required by the four items a, b, c, and d is

$$11{,}520 + 5{,}620 - 16{,}100 + 12{,}960 = 14{,}000 \text{ lbs.}$$

The new air displacement is 164,000 + 14,000 = 178,000 lbs.

The weight statement for the new ship is obtained by operating on each weight group in accordance with equation (5). The following table is obtained by these operations.

NEW WEIGHT STATEMENT AFTER CHANGES

Item	Symbol	Change of Weight lbs.	New Weight lbs.
Outer cover	W_A	1,198	5,098
Gas cells	W_B	505	9,365
Longitudinals	$W_{C1}+W_{C3}$	− 193	5,137
Longitudinals	W_{C2}	− 94	2,764
Corridor	W_{C4}	− 109	2,891
Transverses, main	W_{D1}	− 188	5,012
Transverses, intermediate	W_{D2}	− 89	2,361
Shear wires	W_E	130	1,270
Transverse wires	W_F	125	1,225
Netting and axial wires	W_G	258	2,518
Tail surfaces	W_H	334	2,134
Ballast, fittings, and controls	W_I	810	10,310
Power plant	W_J	2,480	14,480
Fuel and tank	W_K	3,300	62,350
Miscellaneous	W_L	342	6,342
Gas and air	W_M	2,920	37,120
Armament, radio, and control car	W_Θ	2,000	7,600
		13,977	177,977 178,000
		Discrepancy	−23

The change in length or any other linear dimension is given by

$$\Delta L/L = \Delta W/3W = .0284$$

The new length is 661.3 ft., and the new diameter 80.9 ft.

Effect of changing the form. In the foregoing example the slenderness ratio, L/D, was assumed to be constant. Consider now the effect of reducing L/D from 8 to 7 by taking 10% off L, and adding 5% to D. The first step is the computation of $\Sigma W_x (x - y/2)$ as in the following table.

CALCULATION OF $\Sigma W_x\ (x - y/2)$ FOR L-49 TYPE

Item	Weight lbs.	x	y	$W_x (x - y/2)$ lbs.
Outer cover	3,900	1	1	1,950
Gas bag sides	4,430	1	1	2,215
Gas bag ends	4,430	0	2	− 4,430
Longitudinals	5,330	3	1	13,325
Longitudinals	2,670	8/3	1/3	6,675
Corridor	3,000	3	1	7,500
Transverses, main	5,200	0	4	− 10,400
Transverses, intermediate	2,450	1	3	− 1,225
Shear wires	1,140	2	2	1,140
Transverse wires	1,100	0	4	− 2,200
Netting and axial wire	2,260	1	3	− 1,130
Tail surfaces	1,800	1¼	1½	1,350
Ballast, fittings, and controls	9,500	1	2	0
Power plant	12,000	2/3	4/3	0
Fuel and tank	58,990	2/3	4/3	0
Miscellaneous	6,000	2/3	4/3	0
Armament, etc.	5,600	0	0	0
Gas and air	34,200	1	2	0
	164,000			14,770

By equation (7), the saving in weight due to the change of form is given by:

$$\Delta W = (\Delta L/L)\Sigma W_x(x - y/2)$$
$$= -.10 \times 14{,}770 = -1{,}477 \text{ lbs.}$$

Suppose that in addition the change of form reduces the resistance coefficient by 2%. This permits the displacement to be reduced by

$$\Delta W = -.02\, N\, (W_J + W_K)$$
$$= -.02 \times 5.76\, (14{,}480 + 62{,}350)$$
$$= -8{,}850 \text{ lbs.}$$

The new weight statement for the airship of reduced fineness ratio may be prepared in a manner analogous to the procedure in the previous example with constant form.

Limitations to the use of Normand's equation. It may be noted from these examples on the use of Normand's equation that the value of N is assumed constant throughout the calculations of the changes involved; whereas actually N must change to some extent with changing characteristics of the ship. For this reason, the method is not reliable when the change of displacement is more than 25%. For greater changes in displacement, the method is not only inherently inaccurate, but there are always such changes in design that the assumptions of proportionality on which the computations of N are based can no longer be considered to hold good.

Increase of size necessary to obtain the same performance with helium as with hydrogen. If helium lifting .060 lb./ft.^{3} is substituted for hydrogen lifting .068 lb./ft.^{3}, the change is equivalent to increasing the load by .008 lb./ft.^{3} of gas volume. In the airship of the L-49 type considered in the foregoing examples on Normand's equation, this volume is 1,940,000 ft.^{3} and the increase in load is 15,500 lbs. Given $N = 5.76$, the necessary increase in displacement to maintain the performance is theoretically $5.76 \times 15{,}500 = 89{,}280$ lbs., or 54.5%. This increase is so great as to be beyond the range in which Normand's equation is accurate.

Practical modification of Normand's equation. In the foregoing examples of Normand's equation, $N = 5.76$, whereas for most types of surface craft, including such widely different types as battleships, cruisers, and ocean liners, N varies from about 2.0 to 2.5. The comparatively large value of N indicates that in the airship the performance improves less rapidly with increasing size than in surface ships. On the other hand, it is to be remembered that in this comparison, the airship is handicapped by such a large proportion of the useful load being devoted to fuel which must be increased with increasing size. If the fuel carried by the *L*-49 type ship were reduced from 58,990 lbs. to 28,990 lbs., and the military or commercial load correspondingly increased to 35,600 lbs., N would be reduced to 3.39.

Another point to be remembered in favor of the airship is that values of N for surface ships are based largely upon experience, and the greater part of the structural weights are assumed in accordance with experience to be proportional to L^3, although by theory the longitudinal strength requires the weight of the longitudinal structure to vary as L^4. There is not yet sufficient data available on the weights of airships of closely similar design to determine N by short-cuts based on experience. Undoubtedly, many of the weight groups which in airships are theoretically proportional to L^4, would in practice vary according to some lower exponent of L, because of the opportunities with increasing size to use materials to better advantage.

Attempts have sometimes been made to show by curves based on data from actual ships that the ratio of useful lift, sometimes called the "static efficiency" increases quite rapidly with size. This implies low values of N. The trouble with such estimates from actual data is that they usually compare small ships with larger ones of later date, so that the improvement in static efficiency is not by any means all due to increasing size. The small post-war Zeppelins *Bodensee* and *Nordstern* prove that small airships of modern design can be given very excellent performance.

Efficiency Formulas

Weight and propulsive coefficients. There are only two performance factors in airships which can conveniently be given numerical values suitable for incorporation in formulas expressing relative efficiency. One of these factors is the relation between volume, speed and horsepower, and the other is a function of fixed weight and air displacement. The first factor may be expressed by the coefficient K, defined on page 12.

The static efficiency, or the ratio of the useful to the gross lift, is a fraction very commonly stated in performance data. An objection to its use is that it depends upon the lift of the gas used for inflation. It is proposed to overcome this objection by substituting the closely similar coefficient given by

$$C = \frac{D - W}{D}$$

where D is the standard displacement as defined on page 13, and W is the fixed weight, or weight of the ship empty.

The overall efficiency of the airship is expressed by:

$$E = CK$$

Values of E, C, and K for various types of airships are given in Table 2.

Caution on the use of efficiency coefficients. It is much to be regretted that the performance data on airships is very frequently unreliable. The reported speeds are especially uncertain, and since E and K are proportional to the cube of the speed, other things remaining constant, it is obvious that errors which easily occur in the observation of the speed either by airspeed meters or runs on a measured course produce serious errors in the efficiency coefficients.

Aside from inaccuracies due to errors in the speed determination, the efficiency coefficient cannot be considered as a true measure of the overall efficiency of an airship. Reduction of the weight of the structure or machinery increases the coefficient, although the apparent gain in performance may be a real loss due to reducing the structural strength or to increased fuel consumption. Increasing the speed and power will usually reduce the efficiency coefficient because the additional weight of machinery diminishes C without increasing K.

In spite of the frequently misleading character of the performance or efficiency coefficients, they are undoubtedly of value to the airship designer if taken with due caution and regard to the other characteristics of the airships concerned.

TABLE 2. EFFICIENCY COEFFICIENTS OF AIRSHIPS

Type	Designation	Nationality	When Built	Air Volume ft.³	Lgth. ft.	Length/(vol.)$^{1/3}$	Horsepower	Speed ft./sec.	(vol.)$^{2/3}$ ft.²	Displacement lbs.	Weight Empty lbs.	*C*	*K*	*E*
Non-rigid	S.S.P.	British	1916	70,000	143	3.46	76	73.5	1,700	5,350	2,880	.46	38.2	17.6
"	N.S.	British	1918	360,000	262	3.69	500	76.5	5,030	27,400	13,900	.49	19.4	9.5
"	*C*	U. S.	1918	180,000	192	3.41	250	87.0	3,200	13,700	7,900	.42	30.3	15.2
"	Zodiac	French	1918	328,000	262	3.80	500	72.0	4,800	25,000	11,900	.52	15.4	8.0
Semi-rigid	*M*	Italian	1917	441,000	269	3.54	440	67.5	5,800	33,600	18,100	.46	17.5	8.1
"	*O*	Italian	1918	127,000	177	3.53	240	77.5	2,520	9,700	5,200	.46	21.0	9.7
"	P.V.	Italian	1917	176,000	203	3.69	450	82.0	3,140	13,400	8,300	.38	16.6	6.3
"	*Roma*	Italian	1921	1,250,000	410	3.81	900	76.0	11,600	95,500	44,500	.53	24.4	12.9
"	*N-1*	Italian	1924	700,000	348	3.93	750	91.0	7,900	53,500	24,200	.55	34.1	18.8
Rigid	*R-9*	British	1916	930,000	526	5.40	600	66.0	9,500	71,000	42,100	.41	19.5	8.0
"	*R-23*	British	1917	1,040,000	535	5.28	1,000	81.0	10,260	79,500	39,900	.50	23.4	11.7
"	*R-31*	British	1918	1,610,000	615	5.24	1,500	103.0	13,740	123,000	68,300	.44	43.0	18.9
"	*R-33*	British	1919	2,100,000	643	5.03	1,250	89.0	16,400	160,000	81,700	.49	39.8	19.5
"	*L-33*	German	1916	2,100,000	643	5.03	1,440	92.0	16,400	160,000	67,200	.58	38.2	22.2
"	*L-49*	German	1917	2,100,000	643	5.03	1,200	97.0	16,400	160,000	58,200	.64	53.6	34.4
"	*L-70*	German	1918	2,340,000	692	5.20	1,900	115.0	17,600	178,000	61,800	.65	60.5	39.3
"	*Bodensee*	German	1919	875,000	429	4.50	1,040	121.0	9,200	66,800	28,700	.57	67.2	38.3
"	*L-100*	German	Proposed	4,075,000	781	4.88	2,900	121.0	25,600	311,000	78,600	.75	67.2	50.5
"	*ZR-1*	U. S.	1922	2,290,000	676	5.15	1,610	92.0	17,350	175,000	80,200	.54	36.2	19.6
"	*ZR-3*	U. S.	1924	2,760,000	656	4.68	2,200	115.0	19,700	210,000	91,000	.57	38.5	33.4

NOTE: All horsepowers are based on performance in standard atmosphere at sea level, ρ = .00237 slug/ft.³

Effect of the Lift of the Gas Upon the Performance of Airships

There is much misunderstanding and confusion regarding the loss in performance of airships resulting from decrease in the lift of the gas, especially from the use of helium instead of hydrogen. It is common practice in America to take the unit lifts of hydrogen and helium as .068 and .060 lb./ft.³ in the standard atmosphere at sea level. Both units are conservative. From these figures, 11.8% of the gross lift is lost by the use of helium instead of hydrogen; but the percentage losses of useful and military or commercial load are much greater because the weight of the ship empty is a fixed quantity, and the absolute losses of gross and useful lifts are therefore equal.

Let q = ratio of unit lift of helium to unit lift of hydrogen
u = ratio of useful to gross lift with hydrogen
m = ratio of military or commercial load to gross lift with hydrogen

Then the fraction of useful load lost by the use of helium instead of hydrogen is $(1 - q)/u$; and similarly, the fraction of military or commercial load lost is $(1 - q)/m$.

Example. In an airship inflated with hydrogen, it is given that

$$u = .38$$
$$m = .20$$

Find the percentage losses of useful and commercial loads due to the substitution of helium having 88.2% as much lift as hydrogen.

Loss of useful load is $(1 - .882)/.38 = 31.0\%$
Loss of commercial load is $(1 - .882)/.20 = 59.0\%$

A further loss in performance from the use of helium follows from the necessity of starting flight with only partial inflation in order to avoid valving the costly gas as the altitude is increased. With hydrogen, it is customary to start a long voyage fully inflated, and gas is valved as fuel is consumed and the ship gains altitude. This disadvantage of helium may be overcome through the use of coal-gas in ballonets filling the waste air space, and used as fuel in flight.

Influence of altitude upon airship performance. Both the gross lift and the resistance of airships are directly proportional to the density of the air. The propeller efficiency is independent of the air density, and therefore of the altitude, because at a given r.p.m. the propeller thrust also varies directly as the density, so that the relation between speed and r.p.m. is constant.

Within the limits of altitude at which airships ordinarily operate, say under 10,000 ft., the specific fuel consumption of most engines with properly adjusted carbureters is nearly constant, although the total power output, like the resistance of the ship, is proportional to the air density with constant speed and r.p.m. It follows that to a good approximation, the total fuel consumption is directly proportional to the air density.

The simple direct proportionality of the gross lift and the fuel consumption to the air density makes the calculation of the variation of performance with altitude a very simple matter, depending only upon the variation of the air density with altitude. For very accurate computations, regard must be had to the fact that the specific fuel consumption at a given r.p.m. is not quite independent of the air density.

Although at a given r.p.m. the speed is independent of the altitude, many airship engines are designed not to be run with wide open throttle below a certain altitude, and airships fitted with such engines cannot attain full speed below that altitude. Above the altitude at which wide open throttle is permissible, the maximum speed attainable is practically constant.

Since both the total fuel consumption and the aerodynamic forces upon the airship vary as the air density at a given speed, it is desirable to fly as nearly as possible at the altitude at which the gas space is 100% full, unless more favorable winds are to be found at a lower altitude.

Example. Given an airship in which the gross lift at sea level is 125,000 lbs., the lift available for fuel at sea level is 30,000 lbs., the speed and fuel consumption near sea level, 60 knots and 500 lbs. per hour, respectively; find the endurance at 60 knots speed at 6,000 ft. altitude.

At 6,000 ft., the air density is 83.7% as much as at sea level. The gross lift at that altitude is reduced by $125{,}000\ (1 - .837) = 20{,}400$ lbs., and the fuel capacity is reduced to $30{,}000 - 20{,}400 = 9{,}600$ lbs. The fuel consumption at 60 knots at 6,000 ft. altitude is $.837 \times 500 = 418$ lbs. per hour. The maximum endurance is therefore $9{,}600/418 = 23$ hours, or 1,380 nautical miles.

Relation Between Fullness of Gas Space and Altitude to be Attained

It is shown in another volume of the Ronald Aeronautic Library[5] that aside from the effects of superheating, the volume of the gas is inversely proportional to the density of the surrounding air. It follows that to attain any desired altitude without loss of gas, the percentage fullness of the gas space at the start of a flight must not exceed the ratio of the air density at the maximum altitude to the density at the point of departure.

Example. Find the maximum fullness of the gas cells which will permit attaining an altitude of 10,000 ft., starting from sea level, without loss of gas.

The ratio of air densities at 10,000 ft. and sea level is found from the tables of "standard atmosphere" to be .738. The gas fullness at the start is therefore limited to 73.8%

Static ceiling of airships. The static ceiling of an airship in any given condition of loading is the altitude at which the gas space is 100% full, with the ship in static equilibrium. The maximum static ceiling which an airship can ever attain is determined by the ratio of the dischargeable weight to the gross lift at sea level. Let this ratio be x. Then the maximum static

[5] "Aerostatics," by E. P. Warner.

ceiling is the altitude at which the ratio of the air density to the density at sea level is $1 - x$, for at that altitude the gas space will be completely full, even though no dischargeable weights remain aboard the ship.

Example. Find the maximum static ceiling in the standard atmosphere of an airship in which the ratio of the dischargeable weight to the gross lift at sea level is .35.

From the tables of standard atmosphere, the altitude at which the air density is $(1 - .35)$ times the density at sea level is 14,000 ft., and this is the maximum static ceiling of this airship in the standard atmosphere.

Ballonet capacity of nonrigid airships. Nonrigid airships must have sufficient ballonet capacity to permit of descent from the maximum ceiling without loss of the internal pressure upon which maintenance of the form depends. Descent from the maximum static ceiling requires that the ratio of the ballonet volume to the total gas space equals the ratio of the dischargeable weights to the gross lift at sea level. This is the ratio designated x in the previous section on "Static ceiling." The ballonet capacity should be a little larger than necessary from static considerations alone in order to permit of safe descent from an altitude reached dynamically above the static ceiling.

If the ballonet volume is less than required by static or dynamic ceiling conditions, the airship should never be permitted to rise higher than the altitude at which the ratio of air density to the density at the ground is less than one minus the ratio of ballonet volume to the total gas space.

Example. Find the ballonet capacity required for safe descent to sea level from the maximum static ceiling of a nonrigid airship in which the ratio of dischargeable weight to gross lift at sea level is .25.

The ballonet capacity required is 25% of the total volume of the gas space when fully inflated.

Example. Find the maximum altitude from which a nonrigid airship may safely descend to sea level, given the ballonet capacity is 20% of the total gas volume.

The altitude at which the air density is $(1 - .20)$ times the density at sea level is 7,500 ft., and this is the maximum altitude from which the airship can descend without loss of pressure.

CHAPTER III

VOLUMES, AREAS, AND LINEAR DIMENSIONS

Displacement or volume. It is customary to state the size of an airship in terms of the maximum gas volume, or sometimes a nominal 95% or 90% of that volume. The gas volume is obviously a most important quantity, because it determines the gross lift of the airship. Of equal importance is the air volume, by which is meant the total volume over the outside of the hull or envelope and its appendages, including the cars and tail surfaces. As a measure of the size of an airship, the air volume has the advantage that it is a fixed quantity instead of a variable like the gas volume. Aerodynamic calculations, including speed and power, are properly based upon air volume, and not upon gas volume.

In nonrigid airships, the maximum gas volume may be practically equal to the air volume of the envelope; and the cars and tail surfaces will usually not have a volume of more than about one, or at most two, per cent of the total air volume. In rigid airships, there are fairly large air spaces within the hull, and the maximum gas volume is between 90% and 96% of the total air volume.

Areas of cross-sections. The first step in the calculation of the air or gas volume of an airship is to determine the cross-sectional areas of the air or gas space.

Circular, or approximately circular, cross-sections are common in airships, and the area A, of a section of radius R is given by the familiar formula, $A = \pi R^2$. The cross-sections of rigid airships are usually regular inscribed polygons of from 17 to 24 sides; the area of such a polygon of n sides inscribed in a circle of radius R is given by:

$$A = nR^2 (\sin \pi/n)(\cos \pi/n)$$

The following table shows the ratio of the areas of regular polygons of various numbers of sides to the area of the circumscribing circle.

AREAS OF REGULAR INSCRIBED POLYGONS

n	π/n	$\sin \pi/n$	$\cos \pi/n$	A	$A/\pi R^2$
5	36.0°	.5878	.8090	2.36 R^2	.752
9	20.0°	.3420	.9397	2.88 R^2	.916
13	13.84°	.2392	.9709	3.02 R^2	.962
17	10.6°	.1840	.9829	3.07 R^2	.977
19	9.48°	.1646	.9852	3.08 R^2	.980
21	8.57°	.1490	.9888	3.09 R^2	.983
23	7.83°	.1362	.9907	3.10 R^2	.986
25	7.20°	.1253	.9921	3.11 R^2	.989
30	6.0°	.1045	.9945	3.12 R^2	.993

The areas of irregular cross-sections, such as are commonly found in semirigid airships, should be measured by a planimeter, or if such an instrument is not available, one of the rules described in the next section for finding the area of the curve of cross-sectional areas may be applied.

Curve of cross-sectional areas. A convenient graphical representation of the distribution of volume or buoyancy along the hull of an airship is given by the curve of cross-sectional areas (Figure 7), in which the ordinates are the areas of the cross-sections plotted on a base line representing the length of the airship. The area bounded by this curve and the base line, divided by the scale of length along the base and the scale of areas represented by the ordinates, gives the volume of the envelope or gas space for which the curve is drawn.

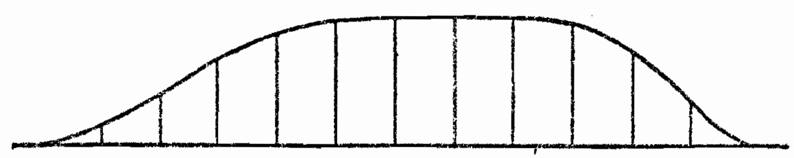
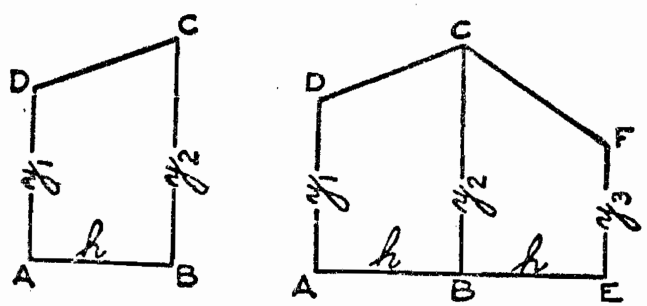


Figure 7. Curve of Sectional Areas

Computation of areas bounded by curved lines. Areas bounded by curved lines are most easily measured mechanically by use of a planimeter. Rules for arithmetical computation commonly used by naval architects are: (1) the trapezoidal rule, (2) Simpson's rules, (3) Tchibyscheff's rule.

The trapezoidal rule. In this method of computation it is assumed that the ends of the chosen ordinates of the curve



Figures 8–9. Illustrating Trapezoidal Rule for Measurement of Areas

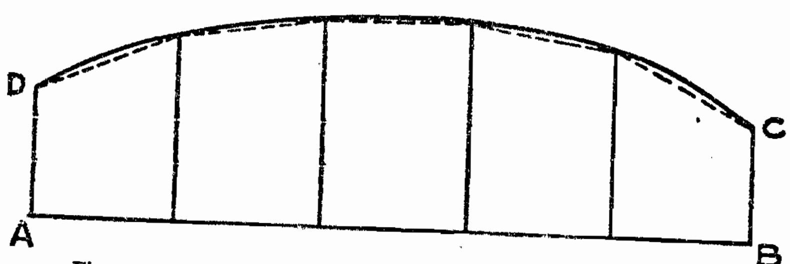


Figure 10. Illustrating Trapezoidal Rule for Measurement of Areas

are connected by straight lines, so that the entire area is reduced to a series of trapezoids. It is obvious that this assumption slightly reduces the measured area of the curve, but the error is on the safe side in underestimating rather than overestimating the volume of the ship.

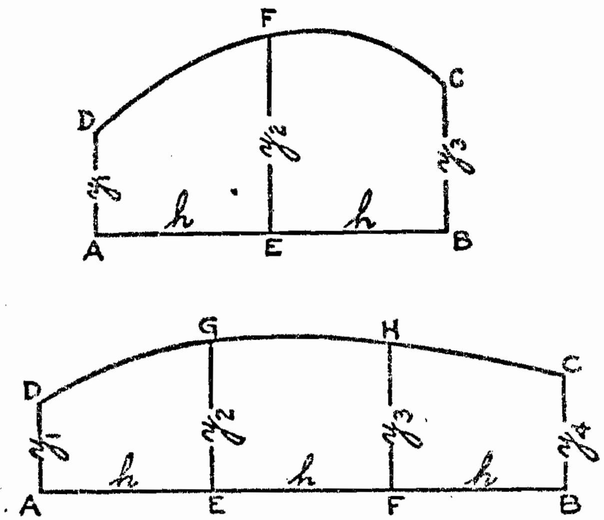
The area of a trapezoid, as *ABCD* (Figure 8), is given by

$$\text{Area} = (h/2)\,(y_1 + y_2)$$

And the area of two adjacent trapezoids of equal base length as *ABEFCD* (Figure 9), is given by

$$(h/2)\,(y_1 + 2y_2 + y_3)$$

To find the area of a curvilinear figure, as *ABCD* (Figure 10), by the trapezoidal rule, divide the base into any number of equal parts, and erect perpendiculars from the base to the curve; then



Figures 11–12. Illustrating Simpson's Rules for Measurement of Areas

to half the sum of the first and last of these ordinates add the sum of all the intermediate ones; the result multiplied by the common interval between the ordinates gives the area required.

Simpson's first rule. More accurate than the trapezoidal rule are Simpson's rules which are frequently used to compute the area of the curve of cross-sectional areas, and hence to find the volume of the ship.

Simpson's first rule is based upon the assumption that the curved line *DC*, forming one boundary of the curvilinear area *ABCD* (Figure 11) is a portion of a parabola of the second order, i.e., a curve represented by an equation of the form, $y = ax^2 +$

$bx + c$. In Figure 11, let AD, EF and BC be ordinates at equal intervals, perpendicular to the base AB. Then if the curved line DFC is a portion of a parabola of the second order, the area bounded by the figure is by integration equal to

$$(h/3)\,(y_1 + 4y_2 + y_3)$$

A long curvilinear area of continuous curvature, such as is represented approximately by a curve of sectional areas of an airship envelope, may be divided into a number of portions similar to the above, to each of which the rule will apply, and the total area with equal intervals, h, between the ordinates is $(h/3)(y_1 + 4y_2 + 2y_3 + 4y_4 + \cdots + 4y_{n-1} + y_n)$

Simpson's second rule. This rule assumes that the curved line DC (Figure 12) is a portion of a parabola of the third order, given by an equation of the form, $y = ax^3 + bx^2 + cx + d$. With equal intervals between stations, the area bounded by the figure is given by

$$\text{Area} = (3h/8)\,(y_1 + 3y_2 + 3y_3 + y_4)$$

With seven equally spaced stations, the area is

$$(3h/8)\,(y_1 + 3y_2 + 3y_3 + 2y_4 + 3y_5 + 3y_6 + y_7)$$

It is frequently desirable to halve the ordinate spacing near the bow and stern, where the change of cross-sectional area is more rapid than amidships; and if this is done, the multipliers of the ordinates in the way of the halved spacing must also be halved.

The multipliers are termed "Simpson's multipliers," and the products of the ordinates and the multipliers are termed "Simpson's functions."

Tchibyscheff's rule. So far as is known, Tchibyscheff's rule is rarely, if ever, used in airship calculations, but it has the advantage in many cases of requiring fewer figures than Simpson's rules, and requiring fewer ordinates to obtain accurate results. Its disadvantage is the use of an unequal and peculiar spacing of the ordinates, in accordance with the following table.

Number of Ordinates	Distance of Ordinates from Middle of Base in Fractions of Half the Base Length
2	.5773
3	0, .7071
4	.1876, .7947
5	0, .3745, .8325
6	.2666, .4225, .8662
7	0, .3239, .5297, .8839
9	0, .1679, .5288, .6010, .9116
10	.0838, .3127, .5000, .6873, .9162

Having selected the desired number of ordinates, and located them by the above table, the area of the curve of cross-sectional areas is given by:

$$\text{Area} = \frac{\text{Sum of ordinates}}{\text{No. of ordinates}} \times \text{length of base}$$

Center of volume or buoyancy. The center of buoyancy of an airship, designated by the letters C.B., coincides with the center of gravity of the gas. The longitudinal location of this center is of great importance; and for an airship to trim level when at rest, it is necessary that the center of gravity, C.G., of all fixed and movable weights be vertically below the C. G. of the gas.

Longitudinal C. B. The trapezoidal rule may be applied to the computation of the longitudinal position of the center of buoyancy in the following manner. Divide the curve of areas into stations at equal intervals, and multiply each station by the distance from some convenient station, preferably at or near the mid-length; add all these moments, counting the moments of the fore-body as positive, and those of the after-body as negative, and divide the algebraic sum of the moments by the sum of the sectional areas. The result gives the distance of the C. B. from the station about which moments were taken, forward if the sum of the moments is positive, and aft if the sum is negative.

More accurate results by Simpson's or Tchibyscheff's rules are obtained by taking the areas of the sections and putting them through the multipliers; these functions of the areas are in turn multiplied by moment arms from some chosen station, and the algebraic sum of the moments of the functions is divided by the sum of the functions, and the resulting quotient is the distance of the C. B. from the chosen station.

Vertical C. B. When the cross-sections of the inflated gas space are circular, the C. B. obviously lies on the longitudinal geometrical axis. For cross-sections of non-circular form, the vertical position of the C. B. may be calculated by dividing the gas space by a number of horizontal planes, and multiplying the volumes between these planes by their height from some chosen horizontal datum line. These moments, divided by the total gas volume, give the height of the C. B. from the datum line.

Relation between linear dimensions and volume. Two important ratios or coefficients are used in determining the dimensions of an airship for which a given volume is required. These are the slenderness, elongation, or fineness ratio, defined as the overall length divided by the maximum diameter; and the prismatic coefficient, defined as the actual volume divided by the volume of a prism having the same length and maximum cross-section as the airship. When the section is a circle, the prism is a cylinder, and the coefficient is sometimes called the cylindric coefficient.

The effects of the slenderness ratio and the prismatic coefficient upon resistance are discussed in Chapter V. There is usually no reason why nonrigid airships should not have the dimensions which will give the least resistance; but in rigid airships, there are apt to be other considerations calling for some departure from a shape selected by purely aerodynamic con-

siderations. A slenderness ratio of about 4.5 to 5.0 and a prismatic coefficient of about .60 to .65 appear to give the least resistance; although it should be remembered that these matters are still controversial. The Zeppelin Company claims that the least resistance is obtained with a slenderness ratio of six or more; while the Aircraft Development Corporation insists that about three gives the best results.

In large rigid airships, a slenderness ratio of not less than six is usually desirable in order to avoid excessive weight in the transverse frames, to facilitate handling on the ground, and to give length for effective spacing of the power cars. Increasing the prismatic coefficient by incorporating some parallel middle body in the lines of the hull gives the practical advantage of reducing the linear dimensions for a given volume, and reducing the cost of construction. If not carried too far, it increases the resistance only a very little.

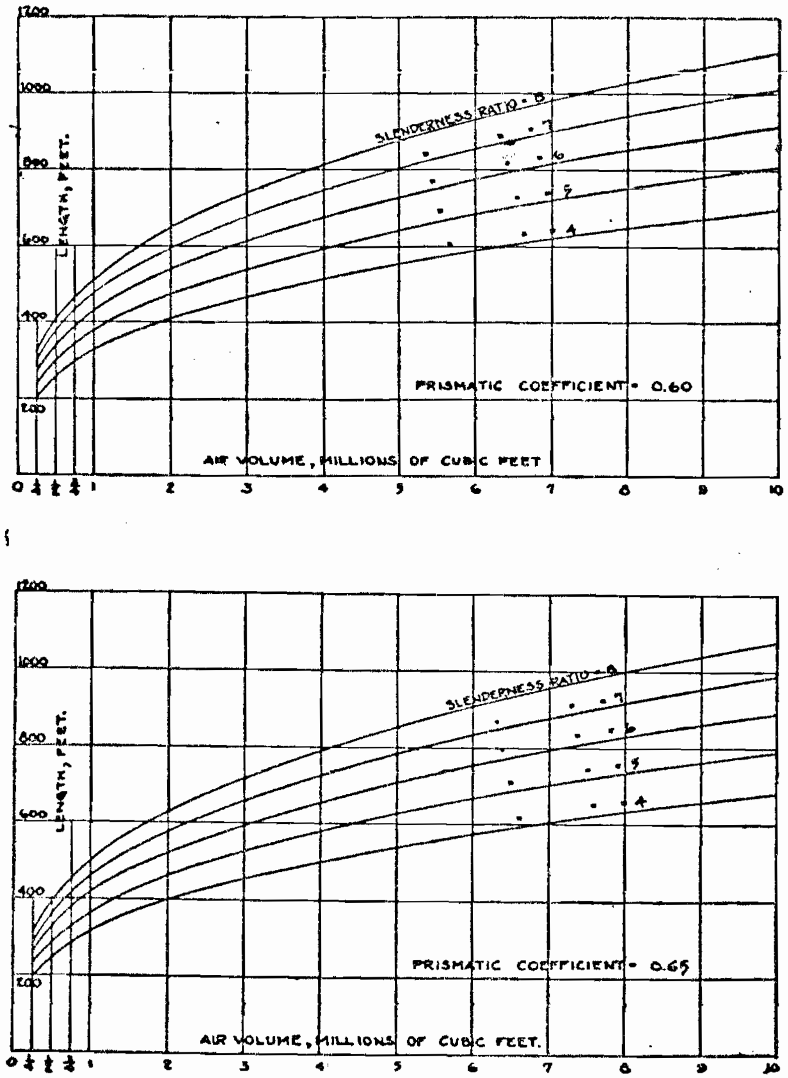
The relation between volume, length, diameter, slenderness ratio, and prismatic coefficient may be expressed by the equation:

$$V = C_V L D^2 \pi/4$$
$$= C_V F D^3 \pi/4$$

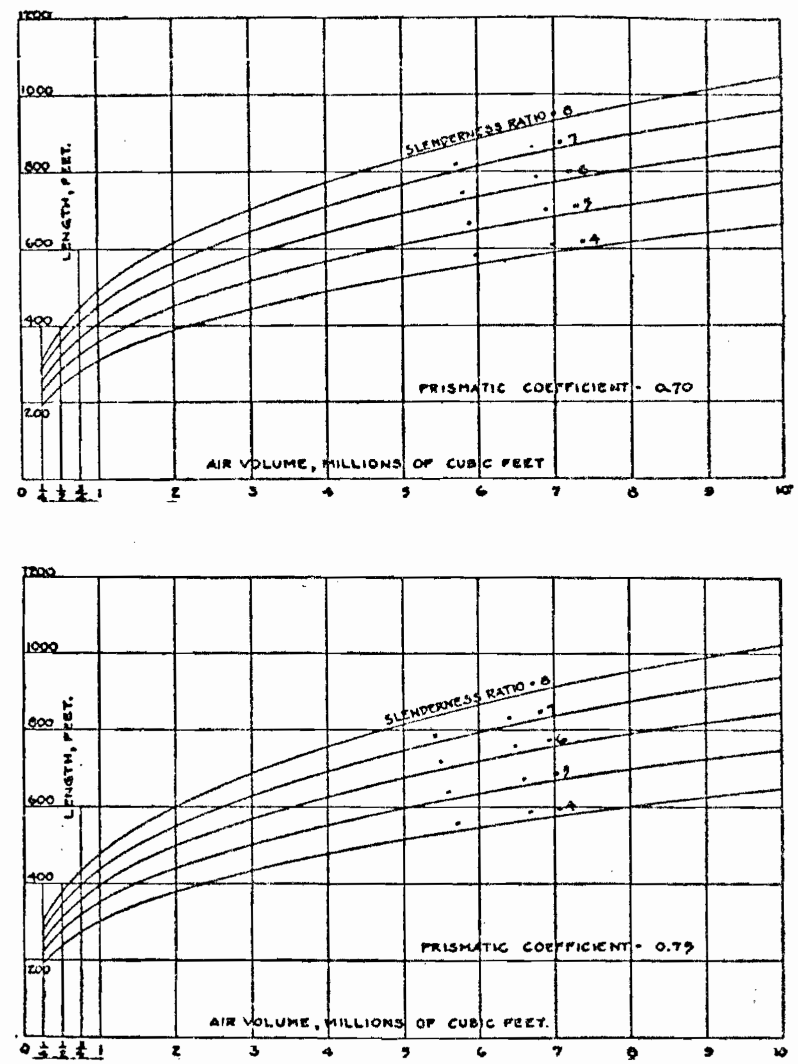
where V = volume
L = length
D = diameter
C_V = prismatic coefficient
F = slenderness ratio

Example. Find the length and diameter of a rigid airship to have an air volume of 5,000,000 cu. ft., given F = 6.0, and $C_V = .65$.

$$D^3 = \frac{4V}{\pi F C_V}$$
$$= \frac{4 \times 5,000,000}{\pi \times 6.0 \times .65}$$
$$= 1,630,000 \text{ ft.}^3$$
$$D = 117.5 \text{ ft.}$$
$$L = FD = 6 \times 117.5 = 705 \text{ ft.}$$



Figures 13–14. Curves of Length vs. Volume



Figures 15–16. Curves of Length vs. Volume

The length and diameter of airships up to 10,000,000 cu. ft. volume, with slenderness ratio between 5 and 8, and prismatic coefficient between .6 and .8 may be picked off the curves in Figures 13 to 16.

Table 3. Data on Airship Envelopes

Surface $= C_S\sqrt{VL} = C_S'DL$
Volume $= C_V AL$

Designation	Length L	Diameter D	Volume V	Surface S	C_S	C_S'	C_V
L. E.	2.99	.641	.506	4.59	3.44	2.40	.617
B	3.53	.696	.831	5.80	3.39	2.36	.618
F	3.12	.641	.670	5.01	3.46	2.50	.661
E. P.	3.09	.641	.589	4.59	3.40	2.32	.590
C	2.97	.641	.624	4.70	3.45	2.47	.651
P	3.21	.641	.580	4.53	3.30	2.20	.575
Goodyear No. 4	3.39	.687	.784	5.47	3.46	2.50	.663
P-AA'	3.89	.641	.750	5.70	3.34	2.29	.597
P-AD'	3.50	.641	.742	5.57	3.45	2.48	.658
P-BA'	3.83	.641	.749	5.69	3.35	2.32	.616
P-BC'	3.58	.641	.745	5.61	3.43	2.45	.646
P-CB'	3.64	.641	.744	5.62	3.41	2.41	.633
P-CD'	3.39	.641	.737	5.51	3.48	2.53	.673
Gottingen No. 1	3.79	.638	.643	5.16	3.30	2.13	.531
Gottingen No. 2	3.41	.639	.643	5.16	3.48	2.37	.588
Gottingen No. 3	3.47	.589	.643	5.16	3.45	2.52	.681
Gottingen No. 4	3.79	.617	.643	5.16	3.30	2.20	.567
Gottingen No. 5	3.46	.597	.643	5.16	3.46	2.50	.664
Gottingen No. 6	3.80	.630	.643	5.16	3.26	2.10	.531
P^1	1.59	.390	.0970	1.275	3.25	2.06	.510
P^5	1.23	.387	.0714	.953	3.22	2.00	.494
E^1	1.37	.392	.0972	1.214	3.33	2.24	.586
C-25	3.13	.641	.677	5.023	3.45	2.50	.669
C-50	3.29	.641	.729	5.345	3.45	2.54	.687
C1-o	3.61	.641	.833	5.99	3.46	2.59	.713
C2-o	4.25	.641	1.040	7.28	3.46	2.68	.758
C3-o	4.89	.641	1.248	8.57	3.47	2.73	.792
C4-o	5.53	.641	1.455	9.86	3.47	2.78	.815
C5-o	6.17	.641	1.663	11.15	3.48	2.83	.834
3C	1.925	.641	.404	3.07	3.48	2.49	.651
6C	3.85	.641	.807	6.03	3.42	2.45	.651
8C	5.133	.641	1.077	8.00	3.41	2.43	.651
2321	4.00	.286	.1600	2.70	3.38	2.36	.613
2322	4.00	.286	.1823	2.91	3.41	2.54	.715
2323	4.00	.286	.2059	3.12	3.44	2.73	.803
2324	4.00	.286	.2265	3.33	3.50	2.91	.883
2325	4.00	.348	.2360	3.29	3.39	2.36	.621
2326	4.00	.348	.2685	3.54	3.42	2.54	.706
2327	4.00	.348	.301	3.79	3.45	2.72	.793
2328	4.00	.348	.334	4.06	3.51	2.92	.878
2329	4.00	.444	.382	4.21	3.40	2.37	.616
2330	4.00	.444	.436	4.53	3.43	2.55	.704
2331	4.00	.444	.488	4.85	3.47	2.73	.788
2332	4.00	.444	.541	5.17	3.51	2.91	.873
2333	4.00	.615	.726	5.82	3.41	2.37	.611
2334	4.00	.615	.827	6.28	3.45	2.55	.696
2335	4.00	.615	.930	6.72	3.48	2.73	.782
2336	4.00	.615	1.030	7.18	3.54	2.92	.866
2337	4.00	1.000	1.805	9.45	3.41	2.36	.595
2338	4.00	1.000	2.135	10.19	3.49	2.55	.680
2339	4.00	1.000	2.405	10.92	3.52	2.73	.766
2340	4.00	1.000	2.680	11.65	3.54	2.91	.853

Surface area coefficients. Two coefficients, C_S and C_S', connecting the surface area, S, of the hull with the length and volume are defined by the expressions:

$$S = C_S\sqrt{VL}$$
and $S = C_S'DL$

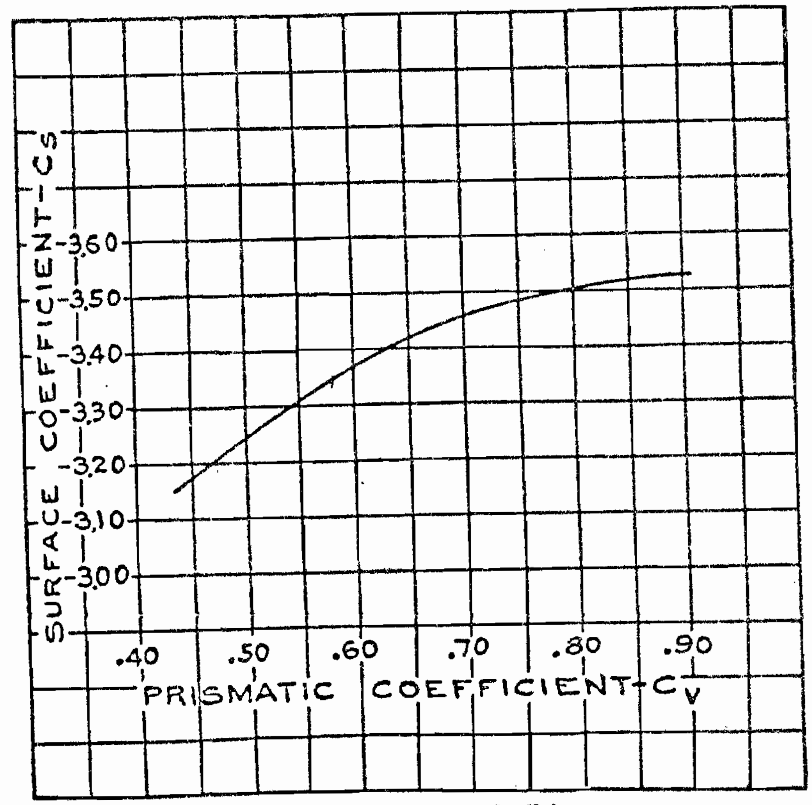


Figure 17. Surface Area Coefficients

The values of C_S and C_S' have been calculated from the actual dimensions, surfaces, and volumes of 52 streamline bodies, which form a series covering the entire range of shapes used in the present aeronautical practice, and are shown in Table 3. Although both C_S and C_S' are nondimensional, it is found that neither is constant. Each depends to a certain extent upon the prismatic coefficient. C_s and C_s' have been plotted against C_v in Figures 17 and 18. It is to be noted that while neither of the coefficients is constant, C_s may be considered constant with a

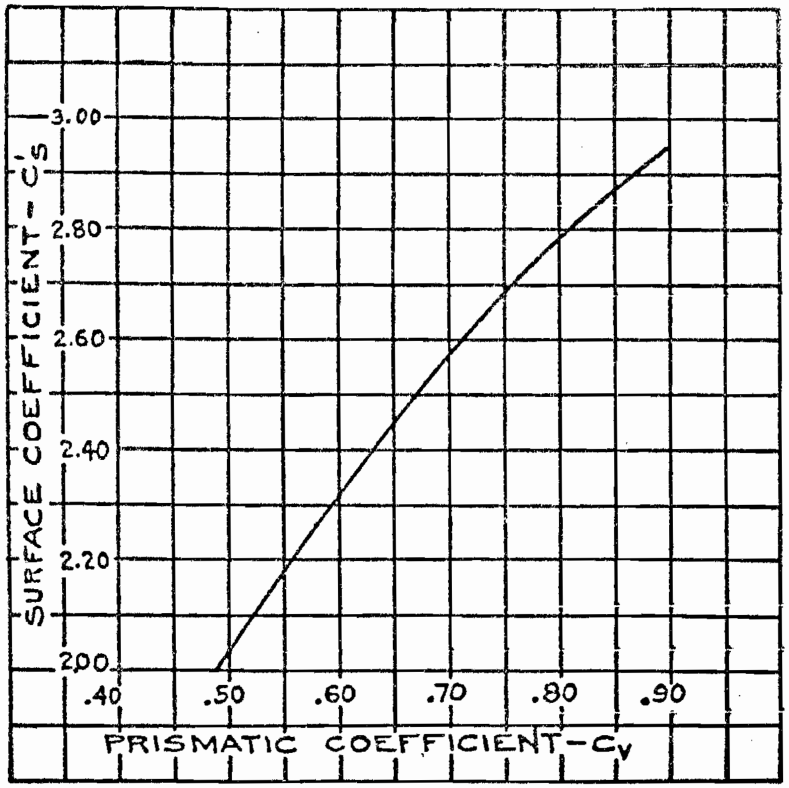


Figure 18. Surface Area Coefficients

probable maximum error of less than 3%. The value recommended for use under these conditions is

$$C_s = 3.45$$

and it applies equally well to the average nonrigid or rigid airship shape.

Theoretical limits to C_s. It is of interest to define the limits of the coefficient C_s. Obviously the maximum and mini-

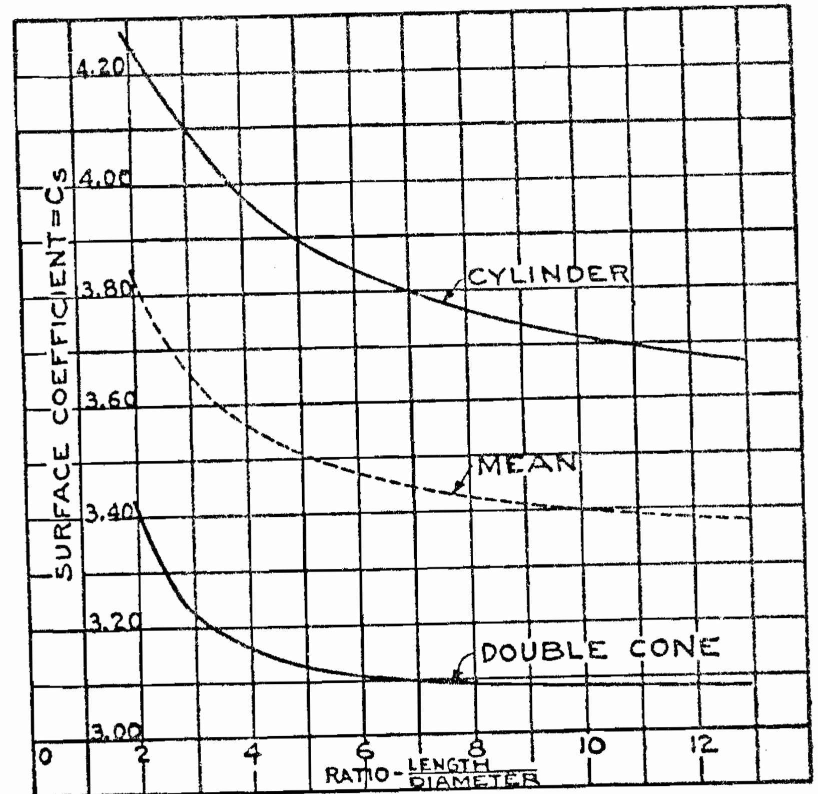


Figure 19. Surface Area Coefficients

mum values will be found from a cylinder and a double cone. The expressions for C_s in these cases are:

$$\text{Cylinder } C_s = 2\sqrt{\pi}\left(\frac{D + 2L}{2L}\right)$$

$$\text{Double cone } C_s = \sqrt{3\pi}\left(\frac{\sqrt{D^2 + L^2}}{L}\right)$$

These have been evaluated and are plotted in Figure 19. It will be noted that the mean value of C_s at $L/D = 7.0$ is C_s

= 3.45. This is the general mean value for streamline bodies and is due to the fact that in order to obtain the least resistance per unit volume it has been found necessary to increase or decrease the fullness of the lines as the ratio L/D is increased or decreased.

Coefficients for design of the tail surfaces. In Chapter V, criteria are derived for the efficiency and adequacy of the tail surfaces for stability and control, based upon wind tunnel test data. These criteria are useful in conjunction with wind tunnel

TABLE 4. COEFFICIENTS OF AIRSHIP TAIL SURFACES

Type	Designation	Nationality	Volume (Air) ft.³	Length ft.	$\frac{\text{Length}}{(\text{vol.})^{1/3}}$	Area of Surface: Vertical A_v ft.²	Area of Surface: Horizontal A_h ft.²	$\frac{A_v}{(\text{vol.})^{2/3}}$	$\frac{A_h}{(\text{vol.})^{2/3}}$
Nonrigid	S.S.Z.	British	70,000	143	3.46	230	284	.135	.167
"	N.S.	British	360,000	262	3.69	743	1,124	.148	.226
"	*C*	United States	180,000	192	3.41	424	538	.133	.168
"	*J*	United States	175,000	168	3.00	455	492	.146	.158
"	Zodiac	French	328,000	262	3.80	881	838	.184	.175
"	Astra	French	340,000	262	3.75	862	1,293	.178	.267
Semirigid	*M*	Italian	441,000	269	3.61	1,120	647	.193	.111
"	*O*	Italian	127,000	177	3.62	494	263	.195	.104
"	P.V.	Italian	176,000	203	3.69	598	617	.191	.197
"	*Roma*	Italian	1,250,000	410	3.80	1,015	1,446	.088	.125
Rigid	*R*-9	British	930,000	526	5.40	1,676	2,620	.176	.276
"	*R*-23	British	1,040,000	535	5.28	1,880	2,280	.183	.222
"	*R*-31	British	1,610,000	615	5.24	2,060	2,191	.150	.159
"	*R*-38	British	2,860,000	695	4.90	2,617	2,938	.130	.146
"	*L*-33	German	2,100,000	643	5.03	1,876	2,505	.115	.153
"	*L*-49	German	2,100,000	643	5.03	1,864	2,456	.114	.150
"	*ZR*-1	United States	2,290,000	676	5.15	2,335	2,870	.135	.166
"	*ZR*-3	United States	2,760,000	656	4.68	2,510	2,510	.128	.128

tests in establishing the relative merits of alternative designs of surfaces, or to ascertain if a given design is likely to have the desired stability; but they do not afford the designer much help in making a preliminary estimate of the tail surface area likely to be required for a new design. For this purpose, coefficients based on data on existing ships are most useful. Coefficients of the areas of the vertical and horizontal tail surfaces divided by (air volume)$^{2/3}$ are given in Table 4. Some designers have used coefficients of tail surface areas based upon the maximum meridian area or cross-section area of the ship; but the magnitudes of such coefficients vary widely with the fineness

ratio, while coefficients based on (volume)$^{2/3}$ are found to be reasonably constant throughout the maximum range of fineness ratios. In Table 4 the usual fineness ratio L/D is replaced by the ratio $L/(\text{vol.})^{1/3}$ in order to take account of ships having non-circular cross-sections, such as the Astra-Torres and most semirigid airships.

The percentage of the total tail surface area allotted to the movable control surfaces has varied widely in the past. Experience has shown that the effectiveness of the control surfaces varies but slowly with the depth of their chords. Modern practice is to extend the fins and rudders laterally to nearly the maximum diameter of the hull, and make the chord of each control surface about one third of the span. The areas of the control surfaces are rather less than 20% of the total tail surface area.

For preliminary design purposes, until corrected by stability criteria derived from wind tunnel tests, it is usually safe to base the area A of either vertical or horizontal tail surfaces upon the empirical expression:

$$A = .13\ V^{2/3}$$

CHAPTER IV

LOAD, SHEAR, AND MOMENTS

Moment to trim an airship. A simple conception of the static stability of an airship may be obtained by comparing it to a pendulum, the center of buoyancy (C. B.) corresponding to the point of suspension of the pendulum. In some cases the C. B. or the C. G., or both, move when the airship inclines, and then the static stability becomes more complicated.

When the positions of the C. B. and C. G. are not affected by inclination of the ship, the moment required to trim the ship from its position of equilibrium to the angle α, either longitudinally or transversely, is given by:

$$M = WH \sin \alpha$$

where M = the moment
W = the gross weight of the ship
H = the vertical distance between the C. B. and C. G.

Effect of unrestrained gas surface upon the static stability of airships. The bottom of a partially inflated gas cell or the top of a partially inflated air ballonet is normally a horizontal surface except in so far as it is distorted by the weight of the fabric. It follows that when the airship inclines, either longitudinally or transversely, there is a movement of the bottom surface of the gas space and of the center of buoyancy relatively to the ship. The movement of the C. B. reduces the static stability of the airship to an extent which may be determined by the procedure of naval architects in dealing with the effect of free water in the hold of a vessel.[1]

The virtual lowering of the position of the C. B. of an airship due to an unrestrained bottom surface of a gas cell is equal

[1] See Attwood's "Text Book of Theoretical Naval Architecture."

to i/V, where i is the moment of inertia of the surface about its neutral axis normal to the direction of inclination, and V is the total volume of gas in the ship.

Example. Find the extent to which the C. B. is lowered by the unrestrained bottom surface of a partially inflated gas cell; given that the gas volume of the ship as inflated is 3,000,000 cu. ft., and the lower surface of the gas cell is a rectangle 60 ft. in length by 80 ft. in width, and the longitudinal stability of the ship is under consideration.

The moment of inertia i of the bottom surface of the gas cell about its transverse neutral axis is given by:

$$i = 1/12 \times 80 \times 60^3$$
$$= 1{,}440{,}000 \text{ ft.}^4$$
$$i/V = 1{,}440{,}000/3{,}000{,}000 = .48 \text{ ft.}$$

Twelve unrestrained surfaces of this size would lower the virtual C. B. 12 × .48 = 5.7 ft., but as the distance between the C. B. and C. G. of an airship this size will be of the order of 30 ft., the static stability is reduced less than 20%, even in this extreme case.

Load, shear, and bending moments. In the primary stress calculations, the hull of an airship is regarded as a loaded beam subjected to bending moments and longitudinal forces from gas pressure, also shearing and bending forces from the unequal longitudinal distribution of the weight, buoyancy, and aerodynamic forces. The static shear and bending result from the weight and buoyancy. Aerodynamic shear and bending result from the transverse components of the air pressure on the hull and tail surfaces, opposed by the weight or inertia of the ship. Considering the ship as a beam, it is obvious that the static forces produce shear and bending only in the vertical longitudinal plane; but the aerodynamic forces may act in any longitudinal plane.

Curves of weight, buoyancy, and load. The first step in the calculation of the shear and bending is to determine the longitudinal distribution of the weight and buoyancy. The exact distribution of the weight can only be found from complete and accurate weight estimates, and these are not available until a design has reached an advanced stage towards completion. In preliminary calculations the designer is forced to make some intelligent guesses, based mainly upon weight data from previous ships. The importance of such data can scarcely be overestimated. Every designer endeavors to accumulate them for his private stock in trade. Weight data from the U. S. S. *Los Angeles* are shown in Tables 5 and 6.

Weight estimating is facilitated by beginning with a rough calculation of the weight of the hull per running foot, varying as the diameter, and then adding allowances for the principal concentrated loads, such as the cars, engines, bow cap, tail surfaces, crew, fuel, cargo, ballast, bombs, etc. In a nonrigid airship, the designer can control the loads on the envelope to a considerable degree by an arbitrary choice of tensions in the car suspension cables.

The computation of the curve of longitudinal distribution of buoyancy is easy, since it is identical with the curve of sectional areas of the gas space (see page 43) multiplied by the unit lift of the gas.

If the longitudinal distribution of weight and buoyancy are represented graphically by curves, these curves should be plotted to equal scales of pounds per running foot, or to other convenient units. The load curve is then the difference between the weight and buoyancy curves, counting excess buoyancy as positive load (upward force), and excess weight as negative load (downward force).

If the ship is in static equilibrium of weight and buoyancy and floating in level trim, the weight and buoyancy curves must enclose equal areas, and the longitudinal centers of gravity of these areas must coincide. It follows that the positive and negative areas of the load curve must be equal to each other.

TABLE 5. WEIGHTS OF THE AIRSHIP *Los Angeles* (*ZR-3*)

Items	Weight, lbs.	Items	Weight, lbs.
Longitudinals	9,350	Exhaust trunks	302
Transverse frame	14,880	Climbing shaft and platform	120
Wiring	2,560	Mooring equipment	1,030
Outer cover	4,200*	Trail ropes and handling lines	326
Netting	642	Ballast bags, controls and instruments	2,380
Gas cells	9,000	Quarters and compartments in keel	870
Gas valves	385	Equipment for passenger quarters	1,403
Fins	1,810*	Radio, furniture, and equipment	3,340
Rudders and elevators	1,382*		
Corridor	2,820		
Machinery	19,800		
Control car	2,520		
5 Power cars	3,310		
Miscellaneous stiffening	1,510	Total weight of ship, empty	83,940

* Weight of cover on fins, rudders, and elevators included in the item "outer cover."

TABLE 6. WEIGHT OF AIRSHIP *Los Angeles*, FRAME BY FRAME

Frame No.	Hull, Fins and Rudders lbs.	Accessories lbs.	Power Plant lbs.	Accommodations lbs.	Total Weight lbs.
−10	367	....	....	....	367
0	2,900	4	....	....	2,904
10	2,940	7	....	4	2,951
25	2,765	356	....	150	3,271
40	3,370	136	....	181	3,687
55	3,810	119	4,000	128	8,057
70	4,170	86	595	150	5,001
85	4,450	95	8,230	572	13,347
100	4,440	154	550	680	5,824
115	4,480	106	8,190	66	12,842
130	4,290	114	595	414	5,413
145	4,010	985	286	3,370	8,651
160	3,800	1,560	....	2,895	8,255
175	2,930	567	....	88	3,585
187	1,228	446	....	....	1,674
	49,950	4,735	22,446	8,698	85,829

NOTE: Frames are numbered in meters from the sternpost.

The shear force and bending moment curves. By a well-known principle, the transverse shear at any cross-section is equal to the integration of the area of the load curve up to that point, having due regard to the scales of the ordinates and the base line, and working from either end. The curve of shearing forces may therefore be plotted by integrating the load curve, starting at either the bow or stern. The integration is most readily performed with a mechanical integrator; but if such an instrument is not available, the integration may be effected by dividing the load curve into intervals, and summing the areas between successive stations, having regard to the signs of the areas. Since the positive and negative areas of the load curve are equal, the shear curve must return to zero at the opposite end of the ship from which it was begun. If this condition is not fulfilled, the work must contain an error somewhere. Another check to make at this point, before commencing to plot the bending moment curve, is to determine if the positive and negative areas enclosed by the shearing force curve are equal. They will be equal if the longitudinal positions of the centers of gravity of the weight and buoyancy curves coincide and the integration is correctly performed.

The next step is to plot the bending moment curve. By a well-known principle, the ordinates of this curve are obtained by integration of the shear curve, just as the ordinates of the latter were obtained by integration of the load curve. Since the positive and negative areas of the shear curve are equal, the bending moment curve must also return to zero at the opposite end of the ship from which it was commenced.

Calculations of shear and bending when the lift and weights are assumed to be concentrated. In calculations of the shearing forces and bending moments in rigid airships, it is customary to assume that both the buoyancy and weight forces are concentrated at the main frames, instead of being distributed along the hull. It follows that with these assumptions there can be no curves of buoyancy, weight, or load, since these quantities are zero everywhere except at the main frames. The shear is

Table 7. Loads, Shear, and Bending Moments in U.S.S. *Shenandoah* when the Gross Lift is 136,634 lbs.

Station meters.	Gross Lift lbs.	Fixed Weight lbs.	Disposable Weight lbs.	Total Weight lbs.	Load lbs.	Shear lbs.	Bending Moment m. lbs.
0	307	2,618	0	2,618	−2,311		0
						−2,311	
10	1,453	1,877	0	1,877	−424		−23,110
						−2,735	
20	2,812	1,902	0	1,902	910		−50,460
						−1,825	
30	4,496	1,991	2,276	4,267	229		−68,710
						−1,596	
40	5,789	2,328	2,200	4,528	1,261		−84,670
						−335	
50	7,128	2,389	5,182	7,571	−443		−88,020
						−778	
60	8,218	5,858	1,512	7,370	848		−95,800
						70	
70	8,985	2,708	2,378	5,086	3,899		−95,100
						3,969	
80	9,402	3,091	5,656	8,747	655		−55,410
						4,624	
90	9,510	9,483	6,100	15,583	−6,073		−9,170
						−1,449	
100	9,540	3,224	6,055	9,279	261		−24,660
						−1,188	
110	9,584	3,069	5,704	8,773	811		−36,540
						−377	
120	9,560	8,183	5,016	13,199	−3,639		−40,310
						−4,016	
130	9,536	3,096	1,790	4,886	4,650		−80,470
						634	
140	9,417	3,064	5,562	8,626	791		−74,130
						1,425	
150	9,003	2,712	5,406	8,118	885		−59,880
						2,310	
160	8,169	8,057	2,259	10,316	−2,147		−36,780
						163	
170	6,778	3,076	2,653	5,729	1,049		−35,150
						1,212	
180	4,467	3,212	1,227	4,439	28		−23,030
						1,240	
188	2,222	1,520	0	1,520	702		−13,110
						1,942	
194.75	258	1,100	1,100	2,200	−1,942		0
	136,634	74,558	62,076	136,634			

then constant between the frames, changing at each frame by the amount of the load at the frame. The shear curve is thus a series of steps; and the bending moment curve obtained by integration of the shear curve is of constant slope between consecutive main frames, but the ordinates do not change abruptly at the frames.

When the buoyancy and weights are assumed to be concentrated in this manner, the shear and bending may be computed very readily by arithmetical methods instead of by graphical procedure. The lift, weight, and load at the main frames are tabulated; and the shear between the consecutive frames is then the algebraic sum of the loads at the frames down to the frame space under consideration. The bending moment at any frame is the algebraic sum of the shear in the frame spaces multiplied by the frame spacing down to the frame under consideration.

An example of the computation of the shear and bending in a rigid airship by this method is given in Table 7.

Calculation of the shear and bending due to a load upon the tail surfaces. The motion of an airship due to a transverse force upon the tail surfaces opposed only by the inertia of the virtual mass of the airship may be regarded as a pure rotation about the center of percussion corresponding to the point of application of the resultant load upon the tail surfaces, or it may be regarded as a rotation about the C. B. combined with a lateral translation of the C. B. The latter way of regarding the motion is the more convenient in the computations of the shear and bending.

The fundamental equations expressing the relations between the resultant force upon the tail surfaces and the product of the mass and the acceleration of the ship in translation and rotation are:

$$F = \alpha \Sigma w \qquad (8)$$

and $$Fl = \beta \Sigma w x^2 \qquad (9)$$

where F = the resultant force upon the tail surfaces
l = the distance from the point of application of F to the C. B.
w = the weight at a main frame, and all weights are assumed to be concentrated at these frames
x = the distance of a main frame from the C. B.
α = the translational acceleration
β = the angular acceleration

The inertia force at each frame is $w\alpha + wx\beta$

It is customary to assume in these calculations that the weight w is distributed in proportion to the cross-sectional area A. Equations (8) and (9) may then be replaced by:

$$F = C_1 \Sigma A \qquad (10)$$

and $$Fl = C_2 \Sigma A x^2 \qquad (11)$$

TABLE 8. 1ST AND 2ND MOMENTS OF MAIN FRAME AREAS OF 4,100,000 FT.^{3} RIGID AIRSHIP

Station	Area ft.^{2}	Arm ft.	1st Moments ft.^{3}	2nd Moments ft.^{4}
− 50	100	− 350	− 35,000	12,250,000
0	1,360	− 300	− 408,000	122,400,000
50	3,410	− 250	− 852,500	213,000,000
100	5,550	− 200	− 1,110,000	222,000,000
150	7,270	− 150	− 1,090,500	163,500,000
200	8,250	− 100	− 825,000	82,500,000
250	8,340	− 50	− 417,000	20,850,000
300	8,340	0	0	0
350	8,340	50	417,000	20,850,000
400	8,340	100	834,000	83,400,000
450	8,250	150	1,237,500	185,500,000
500	7,250	200	1,450,000	290,000,000
550	5,200	250	1,322,500	331,000,000
600	2,330	300	699,000	209,700,000
	82,420		5,960,000	1,956,950,000
			− 4,738,000	
			1,222,000	
		C. B.	$\frac{1,222,000}{82,420}$	= 14.83 ft. forward of frame 300.

Moment of inertia about C. B. = 1,965,950,000 − (82,420 × 14.83^2) = 1,938,-800,000 ft.^{4} × 50 ft.

The inertia force at each frame becomes

$$f = C_1A + C_2Ax \tag{12}$$

The first step in the calculations of the shear and bending is to compute ΣA and ΣAx^2; and from these quantities, C_1 and C_2 for a given magnitude of F are found by equations (10) and (11). The force at each main frame is then calculated by equation (12); and the shearing forces are computed in the usual way by summation of these forces or loads along the ship; the bending moment is found in a similar manner by integration of the shear. Tables 8 and 9 show the calculations of the shear and bending due to a force of 10,000 lbs. on the tail surfaces of a rigid airship of 4,100,000 cu. ft. volume. In this example the stations are numbered in feet from the sternport.

TABLE 9. 4,100,000 CU. FT. AIRSHIP. SHEAR AND BENDING WITH 10,000 LBS. APPLIED AT STERNPOST, OPPOSED ONLY BY INERTIA

Station	$\frac{Ax}{1,000}$	C_1A	C_2Ax	Load lbs.	Shear lbs.	ΣS lbs.	Bending Moment ft. lbs.
− 50	− 36	− 12	− 58	− 5		0	0
					− 5		
0	− 428	− 165	− 695	9,075		− 5	− 250
					9,070		
50	− 903	− 414	− 1,468	− 1,882		9,065	453,250
					7,188		
100	− 1,192	− 674	− 1,940	− 2,614		16,253	812,650
					4,574		
150	− 1,200	− 883	− 1,950	− 2,833		20,827	1,041,350
					1,741		
200	− 946	− 1,002	− 1,538	− 2,540		22,568	1,128,400
					− 799		
250	− 540	− 1,011	− 877	− 1,888		21,769	1,088,450
					− 2,687		
300	− 124	− 1,011	− 202	− 1,213		19,082	954,100
					− 3,900		
350	294	− 1,011	478	− 533		15,182	759,100
					− 4,433		
400	710	− 1,011	1,153	142		10,749	537,450
					− 4,291		
450	1,115	− 1,002	1,810	808		6,458	322,900
					− 3,483		
500	1,342	− 881	2,185	1,305		2,975	148,750
					− 2,178		
550	1,242	− 641	2,022	1,381		797	39,850
					− 797		
600	665	− 283	1,080	797		0	0
		− 10,000	0	0	0		

$$C_1 = F/\Sigma A = 10,000/82,420 = .121$$

$$C_2 = Fl/\Sigma Ax^2 = \frac{10,000 \times 314.8}{1,938,800,000} = .001625$$

CHAPTER V

AERODYNAMIC FORCES

The aerodynamic forces acting on airships may conveniently be divided into drag and transverse forces. The drag or resistance forces are obviously of extreme importance; every effort is made to reduce them to the smallest practicable magnitude in order to improve the speed and economize in engine power and fuel consumption. The drag forces produce structural strains of very little importance, except around the extreme bow. The important aerodynamic forces from a structural point of view are almost entirely normal to the longitudinal axis of the ship. Transverse forces may be divided into two kinds: (a) forces imposed through the rudders and elevators to control the direction and altitude, and to balance inequalities of weight and buoyancy; (b) forces resulting from gusts when flying in rough air.

The Resistance of Airships

An airship designer makes every effort to secure the least possible resistance consistent with structurally economic form. Fortunately, there is rarely much conflict between these requirements. The form of least resistance per unit volume for rigid airships is about the lightest structure per unit volume. Pressure airships without subdivision of the gas space would be structurally most economical if the form were spherical. The best practical compromise for such airships is to choose a form of slightly less slenderness ratio than would be chosen from purely aerodynamic considerations alone. In rigid airships, the use of rather more parallel middle body than is aerodynamically

most efficient permits some saving in overall dimensions and in cost of construction, but makes little or no difference in the weight. Limitations of shed dimensions and ease of handling upon the ground may also influence form.

The resistance of the air to the motion of an airship is conveniently resolved into two parts; (a) pressure difference, and (b) skin friction.

The air pressure acts normally to the surface of the body, and in an ideal, incompressible and frictionless fluid, the sum of the forward acting components of the pressure upon the afterbody is the same as the rearward components upon the forebody. In the real fluid, air, the forward acting components are less than the rearward acting ones, and hence there is the resistance known as *pressure difference.*

Skin friction is the tangential force of the air acting upon the surface of the body.

Theoretical considerations which have been confirmed in part by experiment show that for geometrically similar forms at equal speeds, the pressure difference resistance varies as L^2, where L is a linear dimension. Experiments on skin friction resistance show that it varies approximately as $L^{1.86}$. This difference between the rates of variation of the two resistances leads to an important difference between model and full scale results. Suppose, for example, that in a model at a certain air speed, the resistances due to pressure difference and skin friction are equal to each other. Then in a full size ship in which the linear dimensions are 120 times greater than in the model, the pressure difference will be $120^2 = 14{,}400$ times greater than in the model at the same speed; but the frictional resistance will be only $120^{1.86} = 7{,}367$ times greater than in the model. In other words, in the actual ship, the resistance due to pressure difference is almost twice that due to friction, instead of the two being equal to each other as in the model.

The greater the slenderness ratio of airship hulls, the greater the surface area per unit volume, and hence the greater the frictional resistance, but within limits, the less the pressure difference. It follows that since the pressure difference increases more rapidly than the frictional resistance with increasing dimensions, the slenderness ratio should increase to some extent with size in order to obtain the form of least total resistance. The extent to which the slenderness ratio should be varied with size is one of the questions involved in the vexed problem of scale effect.

Pressure difference and frictional resistance are not wholly independent of each other. Pressure difference is the result of turbulence in the flow of air around the body and depends mainly upon the shape of the body, for which reason it is frequently called "form resistance"; but the turbulence may also be to some extent the result of frictional resistance. The surface area and the speed are the principal determining factors in skin friction; but the form also influences the rate of flow of air over the surface, and hence has its effect upon the frictional resistance.

Coefficients of resistance. Two coefficients of resistance, commonly used in design work for estimating the resistance of airship hulls and their appendages, are the drag coefficient:

$$K_d = \frac{R}{qA}$$

and the shape coefficient:

$$C = \frac{R}{qV^{2/3}}$$

where R = resistance of the body
A = area of the largest cross-section
V = the volume
$q = \rho v^2/2$, i.e., the velocity head of the air stream, or the pressure upon a flat plate normal to the stream.

The drag coefficient K_d expresses the resistance per unit area of the maximum cross-section, and is most useful as a measure of the resistance of appendages, such as cars, where a certain cross-sectional area must be provided. The shape coefficient

C expresses the resistance per unit volume; and is therefore most valuable for measuring the resistance of the hull where the minimum drag in proportion to the air displacement or lift is required.

Both coefficients are based on the assumption that the resistance of geometrically similar bodies is proportional to the squares of the velocity and the linear dimensions. We have already seen that the first assumption is not strictly true for skin friction; and the second is subject to correction for "scale effect."

Some shape and drag coefficients obtained in wind tunnel tests are shown in Table 10.

Coefficient of skin friction. From the experiments of Froude, Zahm, and Stanton, the frictional resistance of airship envelopes may be estimated by the expression:

$$R_f = .0035\rho\, S^{.93}\, v^{1.86}$$

Example. Find the resistance due to skin friction on the hull of an airship 500 ft. in length and 90 ft. maximum diameter, at a speed of 88 ft./sec. in air having a density of .0021 slugs/ft.^{3} Let it be assumed that the coefficient of surface area, C_S' (see page 53) is 2.40. We then have:

$$\begin{aligned} S &= C_S'LD \\ &= 2.40 \times 500 \times 90 = 108{,}000 \text{ ft.}^2 \\ R_f &= .0035 \times .0021 \times 108{,}000^{.93} \times 88^{1.86} \\ &= 255 \text{ lbs.} \end{aligned}$$

Owing to the difficulty of separating frictional resistance from pressure difference, the experimental determination of skin friction has been confined to flat boards from 2 to 16 ft. in length, and 6 to 25.5 in. in width. As in the case of form resistance, the precise scale effect between the experimental surfaces and the actual airship is somewhat conjectural.

For minimum skin friction, the surface must be smooth; but Zahm found that the friction was the same for all smooth surfaces, such as dry or wet smooth wood, hard varnish, sticky

TABLE 10. CHARACTERISTICS OF VARIOUS HULL FORMS TESTED IN NAVY WIND TUNNEL, WASHINGTON, D. C.

Name of Model	Date of Test	Length L ft.	Diameter D ft.	Surface S ft.²	Area Max. Section A ft.²	Volume V ft.³	Slenderness Ratio $=L/D$	Distance Max. Dia. from Nose % L	Distance C. B. from Nose % L	Prismatic Coefficient $\frac{V}{AL}$	Surface Area Coefficients $\frac{S}{\sqrt{VL}}$	Disc Ratio	Shape Coefficient 20 m.p.h.	Shape Coefficient 40 m.p.h.	Shape Coefficient 60 m.p.h.	Drag Coefficient 20 m.p.h.	Drag Coefficient 40 m.p.h.	Drag Coefficient 60 m.p.h.
Navy *B*.....	11-18-19	3.527	.6967	5.800	.381	.8304	5.060	37.80	....	.6176	3.389	15.3	.0336	.0308	.0296	.0788	.0724	.0693
Navy *C*.....	3-26-21	2.949	.6417	4.750	.323	.6259	4.620	30.00	46.37	.6562	3.496	16.8	.0318	.0288	.0272	.0724	.0667	.0621
Navy *F*.....	8-26-10	3.125	.6417	5.007	.323	.6690	4.870	36.25	48.64	.6621	3.463	15.5	.0336	.0292	.0284	.0803	.0716	.0677
E.P.........	10-23-19	3.092	.6417	4.507	.323	.5890	4.820	41.50	43.92	.5891	3.406	17.2	.0332	.0294	.0276	.0728	.0645	.0605
Parseval *P*-I.	4-17-19	3.942	.6417	5.465	.323	.7240	6.140	38.75	43.19	.5079	3.235	12.6	.037	.0348	.033	.0935	.0875	.0827
Parseval *P*-II.......	4-17-19	3.208	.6417	4.528	.323	.5891	4.990	38.90	44.46	.5677	3.204	15.6	.0362	.034	.0328	.0796	.0749	.072
Parseval *P*-III.......	4-17-19	3.208	.6417	4.750	.323	.6331	4.990	47.33	45.85	.6095	3.333	14.2	.0358	.0338	.0322	.0823	.078	.074
AA.........	10-1-19	1.902	.5833	2.760	.207	.3196	3.410	42.50	46.00	.6003	3.460	12.3	.041	.051	.0554	.0725	.090	.0978
C Class Cylindric midships:																		
¼ diameter	3-26-21	3.100	.6417	5.073	.323	.6777	4.850	....	....	.6749	3.495	16.4	.0308	.028	.0264	.0744	.0673	.0633
½ diameter.	3-26-21	3.270	.6417	5.398	.323	.7297	5.100	....	....	.6909	3.495	15.5	.0306	.0282	.027	.0771	.0718	.0784
1 diameter..	3-26-21	3.590	.6417	6.043	.323	.8330	5.570	....	....	.7184	3.464	13.7	.0328	.0292	.0272	.0017	.0808	.0752
2 diameters	3-26-21	4.232	.6417	7.337	.323	1.0404	6.600	....	....	.7611	3.497	11.6	.035	.030	.0272	.112	.0958	.0875
3 diameters.	3-26-21	4.872	.6417	8.627	.323	1.2471	7.590	....	....	.7025	3.500	9.9	.0346	.0312	.0296	.1215	.112	.106
4 diameters.	3-26-21	5.515	.6417	9.022	.323	1.4548	8.590	....	....	.8167	3.503	8.8	.035	.0314	.0292	.1402	.1255	.118
5 diameters.	3-26-21	6.158	.6417	11.218	.323	1.6625	9.600	....	....	.8358	3.506	8.2	.0328	.0308	.0296	.148	.135	.130
Shenandoah.	7-22-22	5.645	.656	9.27	.335	1.3275	8.60	32.0	46.6	.700	3.38	10.16	.0342	.0308	.0290	.1225	.1105	.104

varnish, glazed cambric, calendered paper, sheet zinc, etc. Coarse buckram and unglazed fabric with a slight down was found to have about 15% higher coefficient than smooth surfaces, and the resistance varied nearly as v^2, instead of as $v^{1.86}$.

Cases have been known in which the total resistance of a model of an airship hull tested in a wind tunnel was less than the calculated frictional resistance alone. It follows that if the expression for frictional or tangential resistance is correct, the resistance due to pressure difference must have been negative in these cases. At first sight, this seems impossible; but further consideration indicates the bare possibility that the retardation of the air flow by skin friction might result in greater total pressure upon the after-body than if there were no friction. The condition of negative pressure difference has only been found at certain critical velocities. It almost certainly never occurs in an actual airship.

Effect of slenderness ratio and parallel middle-body. In rigid airships, it is usually desirable to increase the length beyond five diameters in order to space the cars properly, and to facilitate handling upon the ground, and to prevent excessive weight of the gas cells and transverse frames. Frequently the limitations of shed accommodations also require rather larger elongation ratios than would otherwise be selected.

The designer has the choice of inserting some parallel middle-body, or retaining a continuously curved form by increasing the station spacing. To determine the relative merits of these two possibilities, two interesting series of tests were carried out in the wind tunnel of the Bureau of Construction and Repair at the Navy Yard, Washington, D. C. In the first series of tests various lengths of parallel middle-body were inserted between the bow and stern of a model of the *C* class nonrigid airship. The results have been published in the National Advisory Committee for Aeronautics Report No. 138, entitled "The Drag of *C* Class Airship Hull with Varying Length of

Cylindric Midships." The tests showed that a length of parallel middle-body up to one diameter made very little change in the shape coefficient; and three diameters of parallel middle-body increased the coefficient by only 7.6%. The other series of tests was on models of the *C* class airship with various spacing of the ordinates, giving lengths of from 2 to 10 diameters, without parallel middle-body. It was found that increasing the

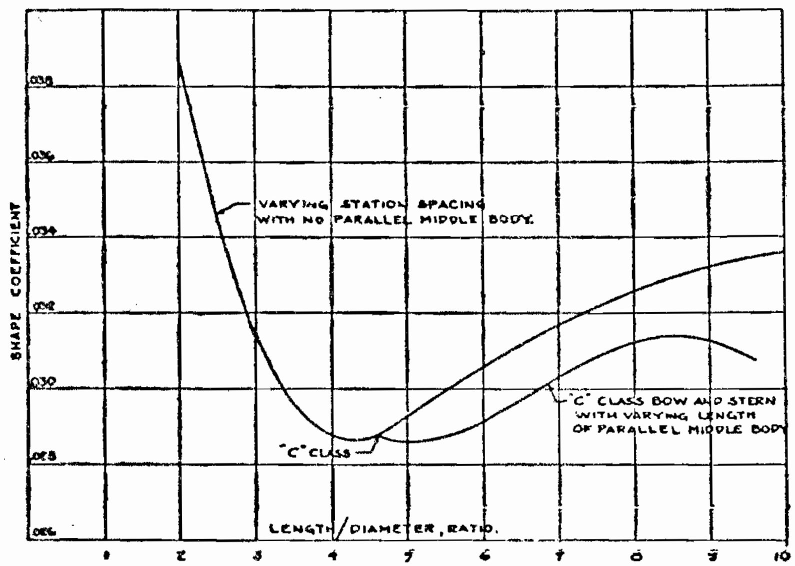


Figure 20. Shape Coefficients of *C*-Class Form with Varying Slenderness Ratio and Parallel Middle-Body

length/diameter ratio from the 4.6 of the *C* class to 8.0 resulted in an increase of 13.2% in the shape coefficient. The curves of the shape coefficient were found to pass through minimum values at a length/diameter ratio of about 4.3 without parallel middle-body, and at 5.2 with a length of middle-body equal to .6 diameter.

The shape coefficients found in both series of tests are shown in Figure 20.

Comparison of the coefficients shows that when it is desired

to increase the slenderness ratio beyond about 4.5, the additional length is obtained with less resistance by inserting parallel middle-body rather than by increasing the spacing of the ordinates. It should be remembered, however, that this conclusion is based upon only two series of wind tunnel tests; and the reliability of such tests as a basis of prediction of full scale

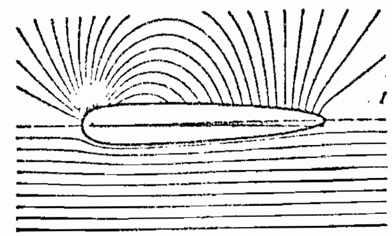


FIG. 21

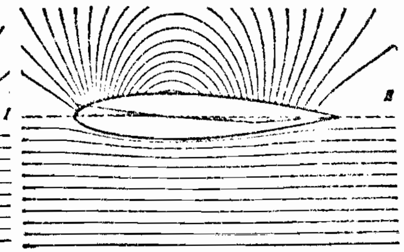


FIG. 22

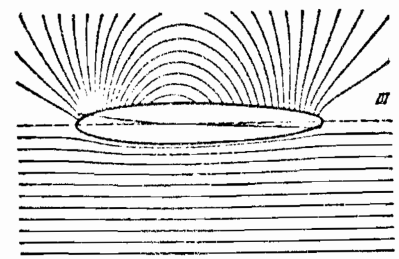


FIG. 23

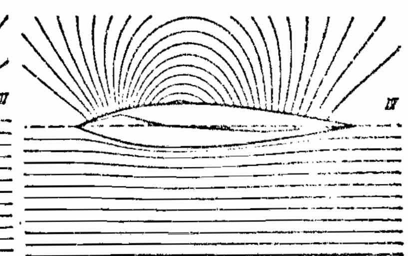


FIG. 24

Figures 21–24. Airship Shapes Derived by the Mathematical Theory of Sources and Sinks

Distribution of sources and sinks indicated on axis. Upper half: streamlines relative to undisturbed air. Lower half: streamlines relative to airship.

resistance is very controversial. The airship *Los Angeles* has a length/diameter ratio of 7.25, and the Zeppelin Company chose a shape entirely without parallel middle-body, claiming that this gives less resistance than a form of similar proportions with some parallel body.

It is obvious that the practical merits of the problem, aside from purely aerodynamic considerations, are very much in favor of parallel middle-body; the construction will be simpler and less expensive, and the overall dimensions of the ship will be less for a given volume and elongation ratio.

Mathematically Developed Forms for Airship Hulls

Most airship forms have been developed by cut-and-try methods, resorting to wind tunnel tests to determine the resistance of the various forms proposed.

Streamline shapes have sometimes been developed mathematically from the theoretical flow resulting from arbitrarily assumed systems of sources and sinks. A discussion of the forms developed in this manner by Dr. G. Fuhrmann, and tested in the aerodynamical laboratory at Göttingen, Germany, is contained in Report No. 116 of the National Advisory Committee for Aeronautics, entitled "Applications of Modern Hydrodynamics to Aeronautics," by L. Prandtl, who was associated with Dr. Fuhrmann before the latter's death early in the war. Figures 21 to 24 show four of Fuhrmann's envelope shapes, and also the sources and sinks upon which they are based, and the theoretical streamlines. The shaded areas above and below the longitudinal axis of the body in the figures represent the sources and sinks respectively. The upper halves of the figures show the streamlines for a reference system at rest with respect to the undisturbed air, and the lower halves show the streamlines for a reference system moving with the body.

The shape coefficients obtained in the Göttingen wind tunnel for these shapes are as follows:

Model	I	II	III	IV
C	0.0340	0.0220	0.0246	0.0248

The computations of streamline forms corresponding to arbitrary sources and sinks are very complicated and will not be described here. Reference is made to a brief account in the National Advisory Committee for Aeronautics Report No. 116, or a longer one in German in the Jahrbuch der Motorluftschiff-

Studiengesellschaft, Fünfter Band, 1911–1912. Wind tunnel tests have not indicated any outstanding virtues in forms developed by these complex methods; and indeed there is no theoretical reason for expecting that any particular source and sink should give an outstandingly good form. Shapes drawn by designers with a natural aptitude for these matters have shown up as well as any. The U. S. Navy's *C*-class airship form is an example of a very good shape designed in this manner; its offsets are given in Table 11.

A form of low resistance obtained from a simple mathematical expression has been developed by the National Physical Laboratory, Teddington, England, and is composed of an ellipsoid and a paraboloid. The fore-body is generated by the revolution about its *X*-axis of a semi-ellipse having the equation:

$$\frac{x^2}{a^2} + \frac{4y^2}{D^2} = 1$$

where D is the maximum diameter, and a is the length of the fore-body.

The after-body is generated by the revolution about its *X*-axis of a semi-parabola having the equation:

$$y = \frac{D}{2} - \frac{Dx^2}{4a^2}$$

From this equation, $y = 0$, when $x = a\sqrt{2}$, so that the after-body is $\sqrt{2}$ times longer than the fore-body. When there is no parallel middle-body, the fore-and after-bodies fit together perfectly, both having the same radius of curvature at their junction at the maximum section. It may be noted also that the radius of curvature is increasing all the way from the bow to the stern.

This form is the one designated E. P. (ellipsoid-paraboloid) in Table 3 (page 52). In that particular model the length of the fore-body was twice the maximum diameter, i.e., $a = 2D$. In addition to the data and coefficients given in Table 3, the following may be useful:

Center of buoyancy is at 44% L from the bow.

Center of gravity of surface is at 45% L from the bow.

Without detriment to the aerodynamic qualities of the form, the point of the stern may be rounded off, effecting considerable savings of weight and overall length. Conversely, the extreme bow may be lengthened to a blunt point in order to facilitate stiffening against external air pressure, or for better attachment of the bow mooring gear. A variation on the ellipsoid-paraboloid

is an ellipsoid-hyperboloid designed by the Aircraft Development Corporation, Detroit, Michigan. In this form the fore-body is again an ellipsoid, but the after-body is an hyperboloid given by the equation:

$$\frac{4(D-y)^2}{D^2} - \frac{x^2}{a^2} = 1 \text{ (referred to same axis as ellipse)}$$

A wind tunnel model of the ellipsoid-hyperboloid form has been constructed and tested in the wind tunnel of the Wash-

TABLE 11. OFFSETS OF VARIOUS HULL FORMS

Navy *B* (Goodrich)		Navy *C*		Navy *F*		E.P.		Parseval *P*-I		Parseval *P*-II		Parseval *P*-III		S.S.T.		*AA*		*Shenandoah*		*Los Angeles*	
Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D	Distance from Nose % L	Diameter % D
3.36	24.16	2.81	32.47	1.23	23.12	0.13	24.88	1.25	27.37	1.25	27.27	1.25	21.56	1.24	21.41	2.00	20.58	0.49	18.47	6.25	63.00
4.73	41.27	5.62	55.06	2.45	35.06	2.59	34.00	2.50	37.92	2.50	37.92	2.50	32.99	2.51	32.98	4.19	35.49	2.18	41.25	13.75	84.63
7.09	55.14	8.43	69.61	3.68	43.90	5.19	48.44	5.00	51.95	5.00	51.95	5.00	47.79	4.99	47.38	8.38	54.65	3.88	53.95	21.25	94.70
9.45	66.27	11.24	79.22	4.91	50.61	10.37	66.10	10.00	71.17	10.00	71.17	10.00	66.23	9.99	66.07	12.57	67.71	7.76	72.21	28.75	98.90
11.81	75.36	16.86	91.17	7.36	62.73	15.56	78.12	14.99	83.38	15.00	83.36	15.00	78.70	14.98	78.89	16.75	77.50	12.62	85.42	36.25	99.99
14.18	81.94	22.48	97.40	9.81	72.08	20.75	86.60	19.98	91.17	20.00	91.17	20.00	88.05	19.97	88.07	20.94	84.60	17.46	93.12	43.75	100.00
18.90	90.31	28.11	100.00	12.26	79.57	25.94	92.73	24.98	96.10	25.00	96.10	25.00	94.03	24.97	94.04	25.13	89.09	22.30	97.38	51.25	98.75
23.63	94.98	33.73	100.00	14.71	84.93	31.12	96.75	29.98	98.96	30.00	98.96	30.00	97.40	29.96	97.32	29.32	94.18	27.15	99.12	58.75	94.39
28.35	98.00	42.16	98.18	19.62	93.51	36.31	99.40	34.97	100.00	35.00	100.00	35.00	99.22	34.97	99.11	33.51	97.23	32.00	100.00	66.25	86.88
33.09	99.64	50.59	94.29	24.54	98.05	41.50	100.00	39.96	99.48	40.00	99.48	40.00	100.00	39.98	99.80	37.70	99.01	51.36	100.00	73.75	76.16
37.82	100.00	59.02	88.63	29.45	99.61	48.81	98.44	44.96	98.18	45.00	98.18	45.00	100.00	44.99	100.00	41.88	100.00	56.20	99.29	81.25	62.13
47.25	98.44	67.45	81.56	34.35	100.00	56.12	93.77	49.96	94.81	50.00	94.81	50.00	98.96	50.00	98.75	46.07	99.43	61.04	97.28	88.75	43.95
56.70	93.06	75.89	71.69	39.27	99.74	63.43	86.23	54.96	89.87	55.00	89.87	55.00	95.86	54.99	95.87	50.26	98.09	65.88	93.12	93.75	27.93
66.15	83.25	84.32	59.48	44.17	98.96	70.74	75.32	59.96	83.90	60.00	83.90	60.00	91.69	59.97	91.75	54.45	95.88	70.72	86.97	100.00	0
70.88	76.91	90.94	48.57	49.07	97.53	78.05	60.52	64.95	76.36	65.00	76.36	65.00	85.97	64.96	86.24	58.64	93.47	75.56	78.61		
75.60	69.38	92.75	41.56	53.98	95.15	85.36	44.16	69.95	67.53	70.00	67.53	70.00	78.96	69.94	79.14	62.83	89.64	80.40	68.17		
80.33	61.00	95.56	31.95	58.78	92.34	92.68	23.90	74.94	57.66	75.00	57.66	75.00	70.91	74.93	70.34	67.02	84.81	85.24	55.58		
85.05	51.44	98.37	18.96	63.69	88.31	100.00	0	79.94	47.01	80.00	47.01	80.00	59.74	79.91	59.76	71.20	78.42	90.08	40.74		
89.78	39.35	100.00	0	68.69	83.25			84.93	35.84	85.00	35.84	85.00	47.27	84.89	47.39	75.40	71.04	94.91	22.89		
92.14	31.94			73.60	77.27			89.92	24.16	90.00	24.16	90.00	23.25	89.87	32.99	79.58	63.52				
94.50	23.44			78.51	70.26			94.92	12.21	95.00	12.21	95.00	17.14	94.86	10.83	83.76	54.65				
96.86	14.00			83.41	62.38			100.00	0	100.00	0	100.00	0	100.00	0	87.96	45.78				
98.14	8.97			88.32	52.47											92.14	35.49				
100.00	0			93.22	40.52											96.34	22.21				
				94.45	36.75											100.00	0				
				95.68	33.12																
				96.91	28.31																
				98.13	22.47																
				99.36	12.26																
				100.00	0																

ington Navy Yard. In this model, $a = 1.2\,D$, and the afterbody was cut off at the point where its diameter equaled .485 times the maximum diameter, and was closed with a very nearly hemispherical stern. The length/diameter ratio of this model was 2.82, and its shape coefficient was only .0264, which is the lowest ever found on any model tested in the wind tunnel of the Washington Navy Yard.

Scale Effect

The term "scale effect" is often a blessed phrase to cover any discrepancies between model and full size performance, when the true explanation lies in instrumental errors, incorrect

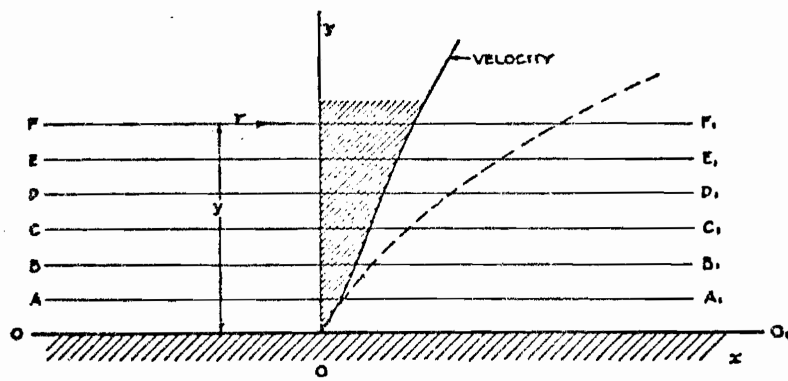


Figure 25. Laminar Flow of Viscous Fluids

observation, or faulty methods of testing. Nevertheless, scale effect is a reality wrapped in much mystery which still remains to be unveiled.

To explain the nature of the problem of scale effect, it is necessary to begin with the fundamentals of motion in viscous fluids. In Figure 25, oo_1 is a flat surface over which a very viscous fluid, such as glycerine, is flowing as the result of a pressure gradient from left to right. Observation shows that

the velocity of the flow is zero all along oo_1 and increases directly as the distance from the flat surface. The tangential force f per unit area of the surface oo_1 is given by the expression:

$$f = \mu v/y \tag{13}$$

where v is the velocity at distance y from the plate, and μ is a constant for the fluid, defined as its "coefficient of viscosity." Expression (13) shows that the skin friction or tangential force in very viscous fluids is directly proportional to the velocity. This law is also found to hold for only slightly viscous fluids, such as gases, when the velocity is very low; but at high speeds, the tangential force varies nearly as the square of the speed.

The explanation of this curious difference between slow and fast motions was first presented by Professor Osborne Reynolds, who illustrated his discovery by experiments in glass tubes. Water from a tank was allowed to flow slowly through a tube, into which a streak of colored fluid was also introduced. So long as the speed was kept below a certain critical value, the colored streak remained clear cut and distinct in the center of the tube. Beyond the critical speed, the color broke up quite suddenly into a confused mass and became mixed with the rest of the water, indicating the formation of eddies which destroyed the laminar flow and gave a new character to the fluid motion.

Professor Reynolds showed that the change of motion always occurred at a certain value of $\rho vd/\mu$, where ρ is the density of the fluid, v the velocity, d the diameter of the pipe in which the experiment was conducted, and μ the coefficient of viscosity defined by (13). The expression $\rho vd/\mu$ is usually written as vL/ν and is known as the "Reynolds' number."

Lord Rayleigh has shown by dimensional theory that there must be a certain definite relation between the variables entering into any expression for the fluid resistance of a series of geometrically similar bodies. The changes of air pressure due to the motion of an airship are so small in proportion to the total atmospheric pressure that the compressibility of the air may be neglected; and on that assumption, the general expression for resistance must have the form:

$$R = \rho L^2 v^2 f\left(\frac{vL}{\nu}\right)$$

where L is some linear dimension of the body, and f denotes some unknown function of the Reynolds' number. To eliminate scale effect, we must have the same Reynolds' number for the bodies compared.

The Reynolds' number depends on ρ, L, v, and ν. The first three quantities are variable; the last is a constant for the fluid; in our case, air. The quantities ρ and v are usually the same for model and airship, but L is very different. If we could make ρ inversely proportional to L, the Reynolds' numbers would be the same for model and ship, and scale effect would be eliminated. The Langley Memorial Laboratory of the National Advisory Committee for Aeronautics has a very wonderful wind tunnel in which models can be tested at varying pressures up to twenty atmospheres. This permits full size Reynolds' numbers to be used for models of airships of about 200,000 cu. ft. volume or less. There is still some discrepancy for large rigid airships, but nothing like so great as in the conventional atmospheric pressure tunnels. Little has been done as yet in using this tunnel to study scale effect in airships, but a great advance in our knowledge of the subject may be expected in the next few years.

External Drag

The resistance of the appendages to the hull, such as the cars, tail surfaces, etc., is sometimes called "external drag." For many years it was thought that the total drag of an airship could be computed by summing the bare hull drag and the various items of external drag, obtained separately; but it is now known that this method gives very erroneous results, because it takes no account of mutual interference. Wind tunnel

tests of fully rigged models have shown twice the resistance of the bare hull, when the drag of the appendages alone was only 40% to 60% of the bare hull drag. Appendages forward of the maximum diameter are found to increase the resistance of the hull much more than similar appendages placed aft.

Deceleration Tests

Deceleration tests to determine the resistance of an airship are easily carried out, but it is not always so easy to put an accurate interpretation on the results. The theory of the deceleration test is that the resistance of an airship in static equilibrium and moving without power through air at rest or in uniform motion is given by:

$$R = m\,dv/dt = \rho V_m\,dv/dt \tag{14}$$

where R = the resistance
V_m = the virtual mass
dv/dt = the rate of loss of speed, i.e., the deceleration

The virtual mass is the actual mass of the ship and its contents, including air and gas plus the additional mass of the air carried along with the ship. The additional mass may be computed by multiplying the actual mass by the coefficient k_1 given in Table 12 for ellipsoids of various slenderness ratios. Alternatively, it may be taken as equal to $\rho\pi r^3/3$, where r = the radius of the largest cross-section, as suggested by Munk in National Advisory Committee for Aeronautics Report No. 117. Both methods assume that the additional mass is the same as it would be in an ideal, nonviscous fluid. It seems possible that the skin friction of an airship creates a frictional wake, or a mass of air moving with the ship, considerably greater than calculated by either of the above methods.

The resistance is also given approximately by an expression of the form:

$$R = A\,\rho\,v^2/2 \tag{15}$$

A has the dimensions of an area, and is called by Munk "the area of drag." Combining expressions (14) and (15),

$$dt/dv = s/v^2 \qquad (16)$$

where s is a length, and is constant when R is proportional to v^2. By integration of (16),

$$t = -s/v + B$$

In a deceleration test starting at $v = v_0$, when $t = 0$, $B = s/v_0$, whence at time t and speed v,

$$s = \frac{t}{1/v_0 - 1/v} \qquad (17)$$

The length s is equal to the slope of $1/v$ plotted against t.

By combining equations (14), (15) and (16), we have:

$$A = 2V_m/s \qquad (18)$$

The area of drag found in this way includes the resistance of the propellers. Wind tunnel experiments on the drag of non-rotating propellers have shown that their area of drag is approximately .8 times their projected area on the transverse plane. It is usually inconvenient to stop the rotation of the propellers entirely, and in most deceleration tests they are per-

TABLE 12. COEFFICIENTS OF ADDITIONAL MASS OF ELLIPSOIDS OF VARIOUS LENGTH/DIAMETER RATIOS

Length Diameters	k_1 Longitudinal	k_2 Transverse	$k_2 - k_1$	k' Rotation
1.0	.500	.500	0	0
1.50	.305	.621	.316	.094
2.00	.209	.702	.493	.240
2.51	.156	.763	.607	.367
2.99	.122	.803	.681	.465
3.99	.082	.860	.778	.608
4.99	.059	.895	.836	.701
6.01	.045	.918	.873	.764
6.97	.036	.933	.897	.805
8.01	.029	.945	.916	.840
9.02	.024	.954	.930	.865
9.97	.021	.960	.939	.883
	0	1.000	1.000	1.000

mitted to idle with the engines, or to revolve freely, disconnected from the engines. It used to be thought that a freely revolving propeller had less drag than a fixed one; but the contrary is now known to be true. However, in a series of deceleration tests of the U.S.S. *Los Angeles* with fixed and free propellers, no appreciable difference could be detected. There is no known means of correction for the increased resistance due to either fixed or idling propellers interfering with the flow of air around the hull.

The corrections for additional mass and propeller resistance are opposed to each other and both are probably of the order of 4% to 10%. In view of the uncertainty of their magnitudes, it is permissible in practice to neglect both.

In most deceleration tests, the pitot tube has been used to measure the speed. In tests carried out on the U.S.S. *Shenandoah* and U.S.S. *Los Angeles*, the pitot tube was checked by an electric air speed meter of the windmill type suspended well below the ship. At any constant speed, the two instruments agreed very well; but during deceleration, the electric air speed meter showed a much higher rate of deceleration than the pitot tube. The electric instrument could not anticipate the motion of the ship, and consequently the discrepancy must have been due to lag in the pitot tube, which is therefore not a suitable instrument for deceleration tests.

From the foregoing discussion of deceleration tests, it should be clear that they provide at best only a rough and ready means of estimating full scale resistance. Trials with hub dynamometers showing the actual thrust in flight appear to offer the only really accurate means of determining the resistance. Practically nothing has been done in this direction.

Because of the uncertainty of the scale effect, full scale data on the resistance of airships are especially important. Owing to errors of observation and inaccuracies of instruments, the reported data are often conflicting, and must be carefully analyzed and corrected by all available means in order to obtain consistent and trustworthy results. It is believed that the most

TABLE 13. PRINCIPAL DATA AND CONSISTENT RESULTS OF AIRSHIP PERFORMANCE ANALYZED BY LIEUT. C. H. HAVILL, U. S. N.

Item	Ship	Principal Data							Consistent Results							
		Gas Volume cu. ft.	Length ft.	Max. Diam. of Hull ft.	Air Volume of Hull cu. ft.	No. of Engines	Fineness Ratio	Total Surface Area of Hull sq. ft.	Maximum Speed ft./sec.	Maximum Speed mi./hr.	Maximum Speed knots	$Hp._{max.}$	K* Propulsive Coeff.	E* Propeller Eff.	A_p* Drag Area Power On	C* (Whole Ship)
	CONTINUOUS CURVATURE SERIES															
1	U.S.N. *B*	84,000	183	36.5	84,000	1	5.06	13,950	69.00	47.04	40.86	99.46	27.63	.622	87.40	.045582
2	U.S.N. *C*	180,000	196	42	180,000	2	4.62	21,424	88.00	60.00	52.11	290.42	31.59	.631	127.40	.039963
3	U.S.N. *D*	190,000	198	42	190,000	2	4.72	23,061	83.11	56.66	49.21	250.00	32.78	.650	131.49	.039807
4	U.S.N. *E*	95,000	162	33.5	95,000	1	4.84	20,404	82.20	56.04	48.67	153.10	32.55	.610	78.00	.037561
5	U.S.N. *F*	95,000	162	33.5	95,000	1	4.84	20,404	77.30	52.70	45.77	126.80	32.68	.620	79.01	.037945
6	*LZ*-120 & 121, *Bodensee* (after lengthening), *Nordstern*	706,200	427	61.8	797,000	4	6.70	69,990	119.99	81.81	71.05	958.89	66.76	.660	170.01	.019776
7	*LZ*-126, U.S.S. *Los Angeles* (no water recovery)	2,599,110	658.3	90.7	2,764,461	5	7.25	147,027	115.00	78.41	68.09	2017.00	64.03	.580	356.99	.018125
	PARALLEL SECTION SERIES															
8	*LZ*-1	375,000	428	38.2	400,000	2	10.21	41,100	26.40	18.00	15.63	27.63	15.58	.380	251.00	.046234
9	*LZ*-4 & 5	530,000	446	42.7	572,000	2	10.50	48,180	41.00	27.95	24.28	202.93	10.08	.280	383.01	.055578
10	*LZ*-7 & 8	680,200	486	45.9	734,000	3	10.60	56,300	51.08	35.44	30.78	343.34	14.36	.330	374.00	.045073
11	*LZ*-10 & 12	592,000	460	45.9	631,000	3	10.00	53,400	62.40	42.54	36.95	347.88	18.44	.658	563.00	.071371
12	*LZ*-15 & 16	688,000	466	48.8	744,000	3	9.55	57,600	67.50	46.02	39.97	479.89	21.45	.570	412.90	.050191
13	*LZ*-22 & 23	728,000	512	48.8	787,000	3	10.48	62,650	66.69	45.47	39.49	502.71	18.42	.520	482.00	.056547
14	*LZ*-24 to 35	794,000	519	48.8	858,000	3	10.61	64,000	70.89	48.33	41.97	592.68	23.40	.530	409.00	.054298
15	*LZ*-36	830,000	530	52.6	884,000	3	10.08	70,300	77.09	52.56	45.65	591.40	30.51	.540	333.98	.036262
16	*LZ*-42 to 50	1,130,000	536	61.8	1,220,000	4	8.68	82,400	81.61	55.64	48.32	836.58	31.96	.550	393.00	.034422
17	*LZ*-59 to 61 & 64 to 71 (except 60 & 70)	1,262,000	586	61.8	1,363,000	4	9.50	91,000	86.00	58.64	50.92	959.39	35.25	.530	369.49	.030068
18	*LZ*-72 to 90 (except 73, 77 & 81)	1,945,000	645	78.3	2,149,000	6	8.24	125,200	92.40	63.00	54.71	1465.80	38.52	.550	474.00	.028540
19	*LZ*-91 to 94	1,965,298	645	78.3	2,140,000	5	8.24	125,430	92.41	63.01	54.72	1215.81	46.65	.594	423.00	.025505
20	*LZ*-95 to 99	1,970,000	645	78.3	2,140,000	5	8.24	125,436	96.19	65.58	56.96	1189.30	53.57	.600	372.00	.022402
21	*LZ*-100 & 101	1,972,000	645	78.3	2,141,000	5	8.24	125,500	97.40	66.41	57.67	1191.30	55.51	.620	371.00	.022340
22	*LZ*-102	2,420,000	745	78.3	2,640,000	5	9.52	146,200	94.29	64.29	55.83	1201.60	57.44	.630	419.00	.021935
23	*LZ*-104	2,427,000	745	78.3	2,640,000	5	9.52	146,200	94.19	64.22	55.77	1193.21	57.66	.640	424.00	.022198
24	*LZ*-106 to 111	1,972,000	645	78.3	2,141,000	5	8.24	125,436	104.81	71.46	62.06	1441.39	57.14	.640	372.00	.022400
25	*LZ*-112 to 114	2,190,000	745	78.3	2,400,000	5	9.52	144,173	113.12	77.13	66.98	1998.81	55.91	.630	404.00	.022538
26	*ZR*-1, U.S.S. *Shenandoah*	2,151,174	680.2	78.7	2,289,861	5	8.64	132,889	91.00	62.04	53.88	1559.71	36.34	.425	402.51	.023172

* At maximum speed and horsepower.

Note: C = Shape coefficient

complete digest that has ever been made of the experimental data available on resistance is the work of Lieut. C. H. Havill, U. S. N., published by the National Advisory Committee for Aeronautics.[1] Lieut. Havill applied Lipka's method of digesting inconsistent data to the following quantities: maximum speed, maximum horsepower, propulsive coefficient at maximum speed, area of drag, and shape coefficient for the whole ship. This was done not only for the data on each individual ship, but included also smoothing out the inconsistencies between similar ships in respect to shape coefficient and propeller efficiency upon which many quantities depended. The consistent results are shown in Table 13.

Transverse Aerodynamic Forces

In airships constructed before 1916, the speed was low, and the principal weights were concentrated at or near one or two cars. Consequently, the shearing forces and bending moment due to weight and buoyancy greatly exceeded those due to aerodynamic forces on the hull and tail surfaces; and if the hull had a fair margin of strength against static forces, the aerodynamic forces might safely be neglected. As airships became larger and faster, with the power plant divided among more cars, the static forces became less important, while the aerodynamic forces, being proportional to the square of the speed, rose rapidly in magnitude and importance.

Effect of sudden application of the rudders and elevators. Several investigators, working independently in the early stages of the development of the problem of aerodynamic forces, came to the conclusion that the most severe aerodynamic condition occurs when the rudders or elevators are first put over. It was urged that this is the worst condition because a transverse force applied to the hull through the center of pressure on the tail surfaces produces a turning moment opposed by the rotational

[1] Technical Notes Nos. 247 and 248, "The Drag of Airships."

inertia of the airship; and as the angular motion increases, the swinging of the stern reduces the force produced on the tail surfaces by the rudders or elevators. The weak point in the argument is that the airship is assumed to be at no angle of yaw or pitch, whereas the actual flight path is always a series of curves about the mean direction, and it frequently occurs that the ship is at an angle of yaw or pitch against the positions of the rudders or elevators, thus making the total force on the fins and control surfaces considerably greater than with the same helm angles at no yaw or pitch. There is much room for argument as to the maximum instantaneous angles of pitch to be assumed. It is the practice of some successful designers arbitrarily to double the force upon the tail surfaces, as determined by wind tunnel experiments with full rudder and elevator angles simultaneously, and no yaw or pitch.

A method of calculating the shear and bending resulting from an arbitrary force opposed only by the inertia of the ship is described in Chapter IV.

Aerodynamic forces at fixed angle of pitch. When an airship is flown either light or heavy, use is made of aerodynamic force to balance the excess of buoyancy or of weight, as the case may be. The required positive or negative aerodynamic lift is obtained by flying with the longitudinal axis at an angle of pitch to the flight path; and the elevators are manipulated as necessary to maintain this angle. It is a curious fact that when an airship is at a fixed angle of yaw or pitch the aerodynamic forces on the tapered after-body of the hull act from the leeward toward the windward side, instead of in the opposite direction as would be expected. The upsetting moment on the ship due to the aerodynamic force acting toward the leeward side on the fore-body and toward the windward side on the after-body is so great that when flying with the bow up it is usually necessary to hold the elevators down instead of up as would be expected; and conversely, when flying with the bow down, it is necessary

to have the elevators turned up. An airship when flying light appears, therefore, to be tail light, and to be tail heavy when flying heavy.

Dr. M. M. Munk, of the National Advisory Committee for Aeronautics, has proposed a method of calculating the distribution of the transverse aerodynamic forces upon an airship moving at a fixed angle of pitch or yaw in an ideal, incompressible, frictionless fluid. He derived the following expression:

$$\frac{dF}{dx} = f = \frac{dS}{dx}\frac{\rho v^2}{2}(k_2 - k_1)\sin 2\theta \qquad (19)$$

where
- F = the transverse force
- x = distance along the longitudinal axis of the airship
- S = cross-sectional area of the hull
- v = air speed
- ρ = density of the air in absolute units
- θ = angle of yaw or pitch
- k_2 and k_1 = the coefficients of additional mass of air transversely and longitudinally, respectively

The basis of Munk's theory is that the additional mass of air carried along by the motion of the airship is the same around any short length of the hull as around an equal length of an infinite cylinder of the same cross-sectional area, with a correction factor equal to $k_2 - k_1$ to allow for the difference between an infinite cylinder and an ellipsoid of finite L/D. Where the cross-sectional area of the hull is increasing in the direction of the air flow, the additional mass of air is also increasing, and the resultant transverse air force on the hull from the windward toward the leeward side is such as to impart the momentum to the increasing mass of air. Conversely, where the cross-sectional area is decreasing, the diminishing momentum of the surrounding mass of air imposes a transverse force on the hull from the leeward toward the windward side.

The assumption of a frictionless fluid involves a considerable departure from reality; but pressure plotting experiments on wind tunnel models have shown that on the fore-body of the ship the actual forces agree quite well with Munk's theory. On the after-body the real force acting from the leeward toward the windward side is very much less than the theoretical one; but the actual forces on this portion of the bare hull are relatively unimportant, because if the real forces acting toward the windward side of the bare hull are less than by theory, the real forces acting in the opposite direction on the tail surfaces in the same region are also less than required by theory, because the resultant of the forces on the after-body and on the tail surfaces must give equilibrium of moments with the forces on the fore-body. The integration of equation (19) all the way along the ship shows the net transverse force on the bare hull to be zero; and it follows that the force assumed to be applied at the center of pressure on the tail surfaces is equal to the total aerodynamic lift. The moment of the theoretical force on the bare hull is

$$V \frac{\rho}{2} v^2 (k_2 - k_1) \sin 2\theta \qquad (20)$$

Where V is the volume of the hull, and the other symbols are as in equation (19). It follows that the angle of pitch required to obtain any desired aerodynamic lift is given by

$$\sin 2\theta = \frac{2Fa}{V \rho v^2 (k_2 - k_1)} \qquad (21)$$

where F = the dynamic lift = the force assumed to be applied at the center of pressure of the tail surfaces, and a = the distance between that center and the center of buoyancy.

The force F is not only the resultant of the pressure on the tail surfaces, but includes also the difference between the theoretical and the actual forces on the after-body of the hull. It acts a little forward of the center of area of the tail surfaces.

In the calculation of the shearing force and bending moment for any given excess of weight or buoyancy it is customary to

TABLE 14. SHEAR AND B. M. IN U. S. S. *Shenandoah* AT 85 FT./SEC. AND 6° 42′ PITCH

Station meters	$f(\Delta S)$ lbs.	$f(S\Delta x)$ lbs.	F lbs.	Load lbs.	Shear lbs.	Bending Moment m. lbs.
0	− 820	− 57		− 877		0
					− 877	
10	− 1,032	− 179	2,300	1,089		− 8,760
					212	
20	− 1,200	− 334	2,300	766		− 6,650
					978	
30	− 1,228	− 500	3,754	2,026		+ 3,130
					3,004	
40	− 1,180	− 665	4,937	3,092		33,170
					6,096	
50	− 985	− 816	2,300	499		94,130
					6,595	
60	− 755	− 933		− 1,688		160,080
					4,907	
70	− 494	− 1,020		− 1,514		209,150
					3,393	
80	− 151	− 1,067		− 1,218		243,080
					2,175	
90	− 55	− 1,077		− 1,132		264,830
					1,043	
100	0	− 1,077		− 1,077		275,260
					− 34	
110	0	− 1,077		− 1,077		274,920
					− 1,111	
120	0	− 1,077		− 1,077		263,810
					− 2,188	
130	41	− 1,077		− 1,036		241,930
					− 3,224	
140	151	− 1,067		− 916		209,690
					− 4,140	
150	494	− 1,030		− 536		168,290
					− 4,676	
160	851	− 933		− 82		121,530
					− 4,758	
170	1,346	− 783		563		73,950
					− 4,195	
180	1,891	− 563		1,328		32,000
					− 2,867	
188	1,780	− 250		1,530		9,020
					− 1,337	
194.75	1,346	− 9		1,337		0
		− 15,591	15,591			

See Note on next page.

NOTES TO TABLE 14

$$f(\Delta S) = [(\rho v^2/2)(k_2 - k_1)\sin 2\theta]\,\Delta S$$
$$= (.0021 \times 85^2/2) \times .924 \times .2317)\,\Delta S$$
$$= 1.625\,\Delta S$$

$$F = \frac{\rho v^2}{2a} V (k_2 - k_1) \sin 2\theta$$
$$= \frac{.0021 \times 85^2}{2 \times 238} \times 2{,}290{,}000 \times .924 \times .2317$$
$$= 15{,}590 \text{ lbs.}$$
$$\Sigma f(S\Delta x) = F, \text{ and } \Sigma S\Delta x = V$$
$$\text{Therefore } f(S\Delta x) = \frac{F}{V} S\Delta x = \frac{15{,}590}{2{,}290{,}000} S\Delta x$$
$$= .0068\, S\Delta x$$

assume that the excess of weight or buoyancy, as the case may be, is distributed along the hull in the same manner as the cross-sectional areas of the hull. This will not in general be strictly true, but any shearing or bending introduced from a different distribution of weight or buoyancy can most conveniently be regarded as a static force to be superposed upon the aerodynamic shearing forces and bending moments. A shear and bending calculation based on equation (19) is shown in Table 14.

Aerodynamic forces in a steady turn. The experimental determination of the aerodynamic forces upon an airship in a steady turn is extremely difficult. The conditions cannot be imitated in any ordinary wind tunnel; and the only practicable research with models appears to be pressure distribution experiments on a model secured to a whirling arm in still air. This has never been done.

Full scale experiments on pressure distribution have been carried out upon the airship *C*-7, but the transverse forces upon the hull were so small that no law of their distribution could be derived or even checked. The principal conclusion from the full scale tests was that the force on the tail surfaces and the hull in their neighborhood is the chief factor in determining the bending moments in the hull, and this force can most conveniently be considered as opposed by inertia.

Munk has extended his analysis of the aerodynamic forces

on an airship moving in an ideal frictionless and incompressible fluid to include steady turning. This is much more complicated than fixed angle of pitch or yaw, and the theoretical results have not been checked by model experiments as has been done in the simpler case; but the good agreement between theory and experiment on the fore-body at fixed angle of pitch, gives confidence that theory is also reasonably applicable to the actual forces on the fore-body of an airship in a steady turn. Whatever discrepancy may exist as regards the after-body, is again unimportant because of the pressure on the fins and rudders which must give equilibrium with the forces on the fore-body. Munk's equation for the distribution of air pressure forces along the hull in a steady turn is as follows:

$$\frac{dF}{dx} = (k_2 - k_1)\frac{dS}{dx} v^2 \frac{\rho}{2} \sin 2\theta + k' v^2 \frac{\rho}{R} S \cos\theta + k' v^2 \frac{\rho x}{R} \frac{dS}{dx} \cos\theta \qquad (22)$$

The only symbol introduced into this equation, not entering into equation (19) for the forces at a fixed angle of yaw or pitch, is k', the coefficient of the additional mass due to rotation. Values of this coefficient are given in Table 12. It should be noted also that x, the distance along the hull, is measured from the center of volume, and θ is the angle of yaw at that center.

The first term in equation (22) corresponds to the force along the hull at a fixed angle of yaw, and is inward toward the center of the turn on the fore-body and outward on the after-body. It produces a turning moment balanced by an inward force assumed to be applied at the center of area of the fins. The sum of the second and third terms gives no resultant force or moment. The second term alone gives a transverse force, almost equal in magnitude to the centrifugal force of the displaced air, and distributed in the same manner, but acting inward instead of outward. The third term represents outward forces on the tapered portions of the hull, having the same resultant as the forces represented by the second term, but of opposite sign, and also acting through the center of buoyancy.

The derivation of equation (22) is far too complicated to be described here, and reference is made to Technical Note No. 184 of the National Advisory Committee for Aeronautics, for the full mathematical derivation.

The force F on the tail surfaces acts inwardly toward the center of the turn, and since the theoretical forces on the bare hull have no resultant force, and have only the resultant moment represented by the integration of the first term (on right hand side) of equation (22), F is equal in magnitude to the centrifugal force of the ship, and in order that it may offset the moment of the force on the bare hull, the radius R of the turning circle must be related to the angle of yaw at the center of buoyancy in accordance with the following equation:

$$\sin 2\,\theta = \frac{2a}{R\,(k_2 - k_1)}$$

If the distance a be measured from the center of volume of the hull to the center of area of the tail surfaces, it is found that the angle of yaw for a given turning radius is about 20% greater than has been determined experimentally for British rigid airships. It is concluded, therefore, that a more accurate calculation of the forces in a steady turn is obtained by assuming a to be about 80% of the distance from the center of volume to the center of area of the tail surfaces, and the force F is of course assumed to act at the distance a aft of the center of volume.

The forces on the hull represented by the first term on the right hand side of equation (22) in conjunction with the force F near the stern, and the centrifugal force of the mass of the hull produce bending moments tending to bend the hull in the direction the ship is turning; the magnitudes of the resulting bending moments are in the same ratios to the centrifugal force as the bending moments at a fixed angle of pitch are to the dynamic lift at a fixed angle. But the forces represented by the last two terms of equation (22) bend the hull in the opposite direction, with the result that the total bending moment in a steady turn is only about one-fifth as great as at a fixed angle of pitch, assuming equal resultant weight or inertia forces in the two cases.

Upson's correction to Munk's transverse force distribution. R. H. Upson has shown that the transverse force per unit length may be more accurately stated if the term k_2-k_1 in expression (19) is replaced by cos α, where α is the angle between the tangent to the meridian and the longitudinal axis of the hull. The corrected expression for transverse force is:

$$\frac{dF}{dx} = \frac{dS}{dx}\frac{\rho v^2}{2}\cos^2\alpha \sin 2\theta \qquad (23)$$

Normal Pressure Variation

The pressure on the extreme bow of an airship in excess of the static atmospheric pressure at the same height is equal to the aerodynamic head, $\rho v^2/2$, usually denoted by q. The quantities ρ and v must be expressed in absolute units.

Example. Find the pressure q on the extreme bow of an airship moving at a speed of 80 ft./sec. through air weighing .076 lb./ft.³

$$v = 80 \text{ ft./sec.}$$
$$\rho = .076/32.2 = .00236 \text{ lb. sec.}^2/\text{ft.}^4$$
$$q = \frac{\rho v^2}{2} = \frac{.00236 \times 80 \times 80}{2} = 7.55 \text{ lb./ft.}^2$$

It may be noted that $\rho v^2/2$ is the kinetic energy per unit volume of air moving at velocity v, and q is the potential or pressure energy per unit volume after the velocity is wholly converted into pressure.

It is customary to specify the aerodynamic pressure on all parts of an airship in terms of q. Proceeding from the bow, the pressure falls off rapidly, and is usually zero at only about 3% to 5% of the length aft of the bow. The pressure is negative along the hull until within about 10% of the length from the stern. Around the stern there is an area of positive pressure. Figure 26 shows the pressure distribution found by the National Physical Laboratory (Teddington, England) on a model of the rigid airship *R*-33 at various angles of yaw. Figure 27 shows the distribution of transverse force obtained by integrating the pressure around the cross-sections.

Theoretical transverse force curves in accordance with the three terms on the right hand side of expression (22) are shown

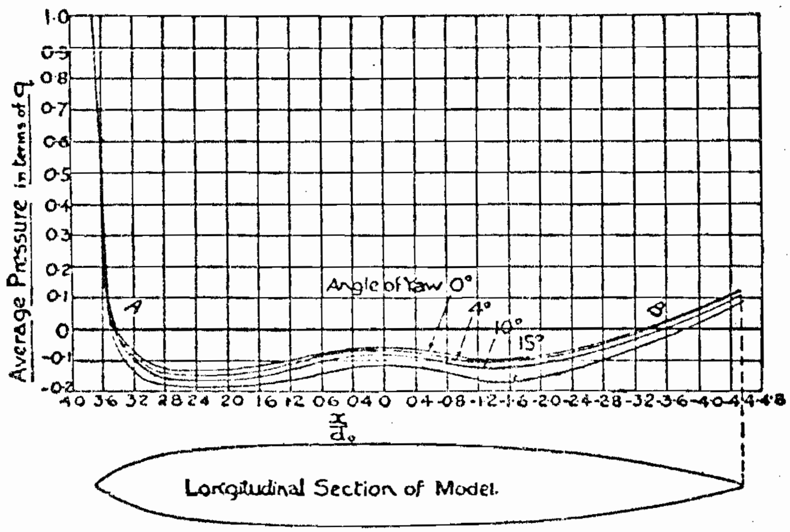


Figure 26. Pressure Distribution on the Hull of Airship *R-33*

in Figure 28. Only curve I in Figure 28 applies to a fixed angle of pitch or yaw. Curves II and III represent forces due to turning; and it may be seen that they largely neutralize the forces shown by curve I.

Additional Mass Coefficients

Table 12 of additional mass coefficients is taken from National Advisory Committee for Aeronautics Technical Note No. 184.

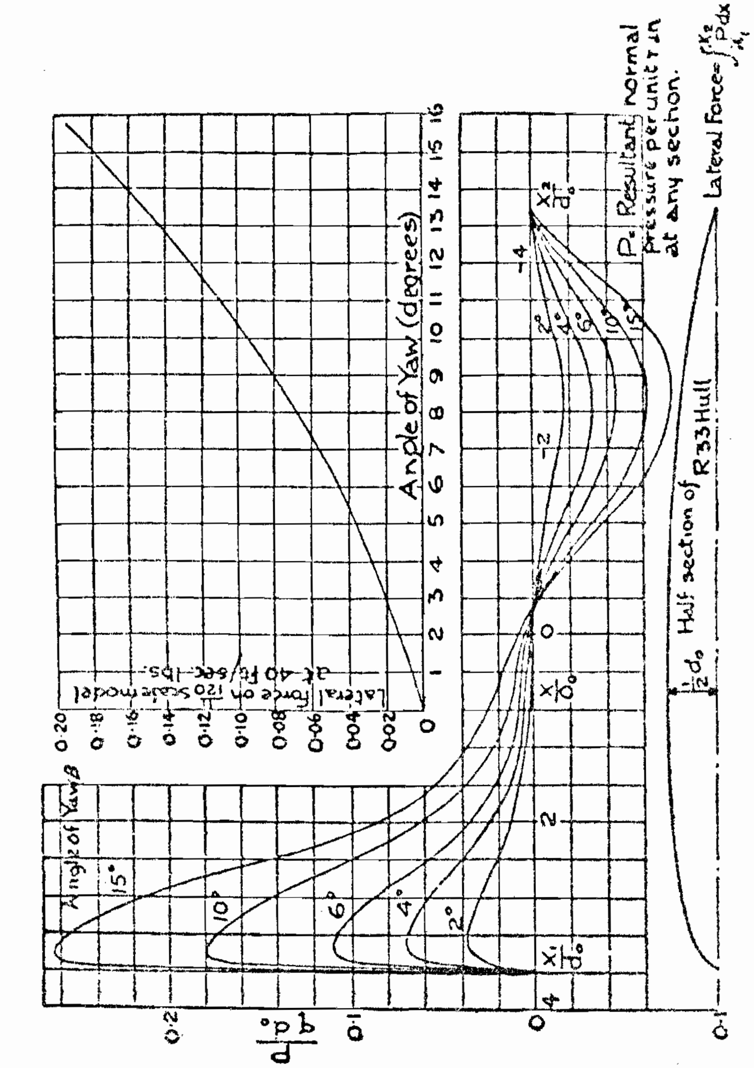


Figure 27. Distribution of Transverse Force or Resultant Pressures Over Hull of the *R-33*

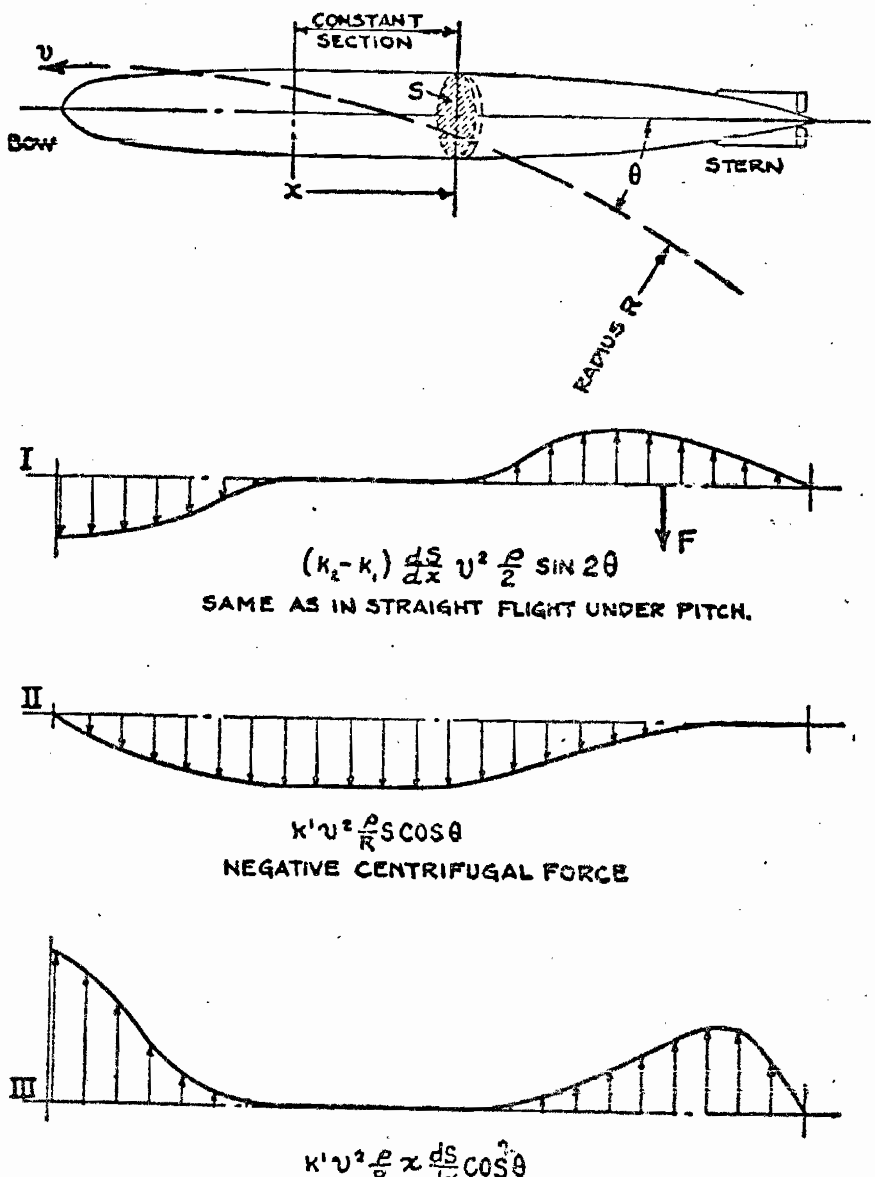


Figure 28. Munk's Theoretical Distribution of Transverse Forces

The coefficients, k_1, k_2 and k' are, respectively, the coefficients of the additional mass longitudinally, transversely, and rotationally. Their use in calculations of aerodynamic forces has been explained. It should be noted, however, that the length/diameter ratio in the table is not that of the actual airship, but of the equivalent ellipsoid, and the ratio for the ellipsoid is obtained from the equation:

$$\frac{L}{D}\text{ (ellipsoid)} = \sqrt{\frac{\pi}{6}\frac{L^3}{V}}\text{ (ship)}$$

Forces in Gusts

Measurements with strain gages on the longitudinal girders of the airships *Shenandoah* and *Los Angeles* have shown that the stresses in normal flight through rough air exceed any which can be produced by maneuvers in still air. There is great difference of opinion in regard to the maximum forces to which an airship may be subjected in heavy squalls; and this is not surprising in view of our meager knowledge of the wind structure at such times. Comdr. J. C. Hunsaker has summed up the situation as follows:

"The existence of veritable fountains of upward rushing air whose sides at times and places are sharply separated from the surrounding atmosphere must be taken into account in the design of airships. The most violent of such currents, the tornado, combines vertical velocity with rotation, but fortunately can be seen from a great distance, and can and must be avoided. The thunderstorm with large fully developed cumulus tops is also conspicuous and avoidable. It would appear to be folly to enter such a cloud and subject the ship to the unknown dangers of wind, rain, hail, and lightning. Barring such spectacular hazards, there remain convection currents which the ship may run into at full speed. There is ample evidence that upward velocities as high as 10 feet per second may be encountered. This vertical air velocity u, combined with the relative horizontal speed v of the airship, will give the effect of a change of pitch of $\tan^{-1} u/v$."

Table (14) shows the calculation of the forces and the resulting shear and bending in the U.S.S. *Shenandoah* when running suddenly at a speed of 85 ft./sec. into a gust having a vertically upward velocity of 10 ft./sec.

Aerodynamic Stability

The fundamental equations for airship stability have been derived independently by several investigators. The following derivation is due to Dr. A. F. Zahm. It has been published in detail by the National Advisory Committee for Aeronautics as Report No. 212, "Stability Equations for Airship Hulls."

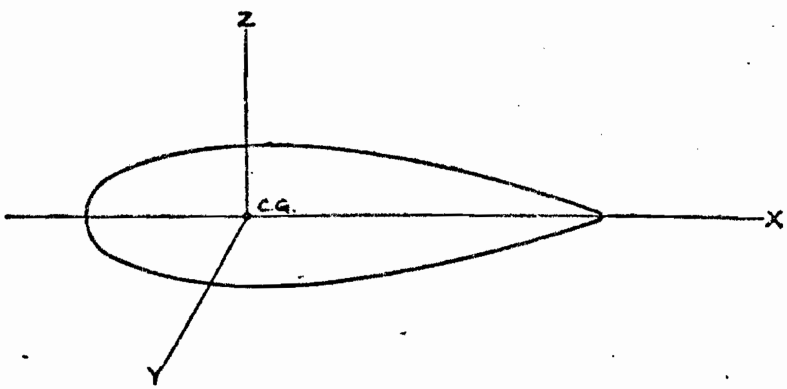


Figure 29. Stability Notation

Zahm's criterion for stability in pitch. To simplify the treatment, assume the hull symmetrical about its longitudinal axis, the centers of mass and volume coincident, the weight balanced by buoyancy, the controls neutral, the motion head-on through still air.

Starting from the centroid in Figure 29, the axes are x running aft, y aport, and z upward. The increments of velocity along and about these axes are designated, respectively, u, v, w, p, q, r. The increments of the air force and moment referred to x, y, z, fixed in the hull and moving with it, are X, Y, Z, L, M, N. Initially, all the components of velocity and moment are zero, except the forward speed U, and the resistance X_0, which latter has a negligible effect on the motion other than straightforward.

Motion in simple pitch. If now the ship with steady speed U is given a slight pitch θ, the entire system of external forces other than X_0 is equivalent to the normal air force Z at the centroid, and the pitching moment M about it. Hence, by d'Alembert's principle, the conditions for kinetic equilibrium are

$$Z = m\dot{w}_1 = m(\dot{w} - U^2\dot{q}) \quad (24)$$

$$M = B\dot{q} \quad (25)$$

where m is the effective mass of the ship; B its moment of inertia about y; while the space acceleration of the centroid is $\dot{w}$, if referred to the z axis as instantaneously fixed in space direction, but $\dot{w}$-Uq if referred to z moving with the ship.[2] If m' is the actual mass of the ship, the effective mass m, that is, the ship's actual mass plus its virtual mass due to accelerating the fluid, can be taken as about 1.5 m' for transverse accelerated motion whether normal or lateral.

Since both static and damping forces, due to θ and q, are present, it is advantageous to place in evidence the components of Z and M. The part of Z due to θ is θZ_θ, where Z_θ denotes $\delta Z/\delta\theta$; the part due to q is qZ_q. They are the usual lift and the damping force, and compose practically the whole of Z. Similarly the static and damping parts of M are θM_θ; qM_q. Substituting these four components in (24) and (25),

$$\theta Z_\theta + qZ_q = m(\dot{w} - qU) \quad (26)$$

$$\theta M_\theta + qM_q = B\dot{q} \quad (27)$$

Conditions for stability in pitch. The angular velocity is constant for $\dot{q} = 0$, and declines on addition of some restoring

[2] Books on mechanics prove $\dot{w}_1 = \dot{w} - Uq$. See Wilson's Aeronautics, §48.

moment, due, for example, to the lowering of the centroid. Making $\dot{q} = 0$ reduces (27) to

$$\theta M_\theta + qM_q = 0 \tag{28}$$

Also for such speeds that $\dot{w}$ is small compared with qU, one can write (26) more simply:

$$\theta Z_\theta + q(Z_q + mU) = 0 \tag{29}$$

These two equations are simultaneous, and taken together represent approximately the conditions essential for dynamically stable motion, i.e., for unamplifying pitch, expressed in familiar aerodynamic quantities.

By (28) the damping moment qM_q rotationally balances the disturbing moment θM_θ; by (29) the disturbing force θZ_θ; the damping force qZ_q, and the inertia force qmU, are in translational balance.

Criterion of pitch stability. In (28) and (29) the variables θ and q are the angular displacement and velocity of the small stable oscillation; the other six quantities are specified constants. Eliminating θ, q, gives the stability condition, viz., the relation between these constants which is necessary to satisfy (28) and (29) for all small values of θ and q

$$\frac{Z_\theta M_q}{M_\theta (Z_q + mU)} = 1 \tag{30}$$

This criterion can also be written in terms of model data. If s is the scale ratio, u the model speed, and if primes mark the model symbols, the values in (30) are related to the model values thus:

$$\left.\begin{aligned} Z_\theta/M_\theta &= \frac{1}{s} Z'_\theta/M'_\theta \\ M_q/U &= s^4\mu/u \end{aligned}\right\} \tag{31}$$

where μ is the coefficient of damping moment in pitch for the model with head-on speed u. One may write $Z_q + mU = nmU$, where n is to be found experimentally, since Z_q is proportional to

U. Putting these new values in (30) the working criterion for pitch becomes:

$$a \frac{\mu}{u} \frac{Z'_\theta}{M'_\theta} = 1 \tag{32}$$

where $a = s^3/mn$. The stability criterion (32) is now in a form convenient to use with familiar wind tunnel data. The damping coefficient μ is determined experimentally in the wind tunnel.

The weak point in this treatment is the difficulty of determining Z_q experimentally. It was formerly assumed that this term is negligible in comparison with mU, so that $n = 1$. The wind tunnel criteria based on this assumption showed great instability, even for ships which proved amply stable in actual flight. Hence it must be inferred that Z_q is by no means negligible.

Jones' criterion of stability. R. Jones, of the National Physical Laboratory, Teddington, England, found that in British rigid airships, $R\theta = .9\,l$, where R is the radius of turning circle, θ is the angle of yaw at the C. B. in radians, and l is the distance from the C. B. to the center of pressure of the tail surfaces. Using Zahm's symbols, Jones' criterion is based on the condition that:

$$\theta M_\theta + (U/R)M_q = 0 \tag{33}$$

which is the same as (28) above, since $q = U/R$, and the observational fact that:

$$R\theta = .9l \tag{34}$$

Combining (33) and (34),

$$\frac{M_q U}{M_\theta} = .9l$$

In terms of wind tunnel data, satisfactory stability may be expected when

$$\frac{s\,(\mu/u)\,u^2}{M'_\theta} > .9l \tag{35}$$

Example. A model of a rigid airship constructed to the scale 1/120 is tested in the wind tunnel. It is found that $\mu/u = .0052$, and $M'_\theta = .125$ ft. lb. per degree at $u = 58.6$ ft./sec. The length l from the C. B. to the center of area of the tail surfaces is 320 ft., full scale, whence:

$$\frac{s\,(\mu/u)\,u^2}{M'_\theta} = \frac{120 \times .0052 \times 58.6^2}{.125 \times 57.3} = 300 \text{ ft.}$$

and
$$.9l = .9 \times 320 = 288 \text{ ft.}$$

The ship should be stable by a small margin.

The limitation of this method is that the coefficient .9 is based on observation of conventional rigid airships, and might not apply to ships of widely different form or slenderness ratio.

Dynamic Lift

Moderate inequalities of the static forces of weight and buoyancy may be compensated by aerodynamic forces imposed upon the hull and horizontal tail surfaces through controlled operation of the elevators. This dynamic lift varies as the square of the speed, and increases with the angle of attack up to large angles, provided the speed is constant; but the increase of drag with the angle of attack reduces the speed so rapidly that at any given horsepower, the maximum dynamic lift is obtained at about 8° pitch. Figure 30 shows curves of the speed and dynamic lift of the U.S.S. *Los Angeles.*

It should be noted that the dynamic lift, like other aerodynamic forces, varies as the square of the linear dimensions, whereas weight and buoyancy forces vary as the cube of these dimensions. It follows that with increasing size of airships it becomes increasingly difficult to use dynamic lift to compensate for disturbances of the static equilibrium, unless the speed increases as rapidly as the square root of the linear dimensions. This relation of speed to linear dimensions would require the horsepower to be proportional to $L^{3.5}$ or to $V^{7/6}$, where L and V are length and volume, respectively.

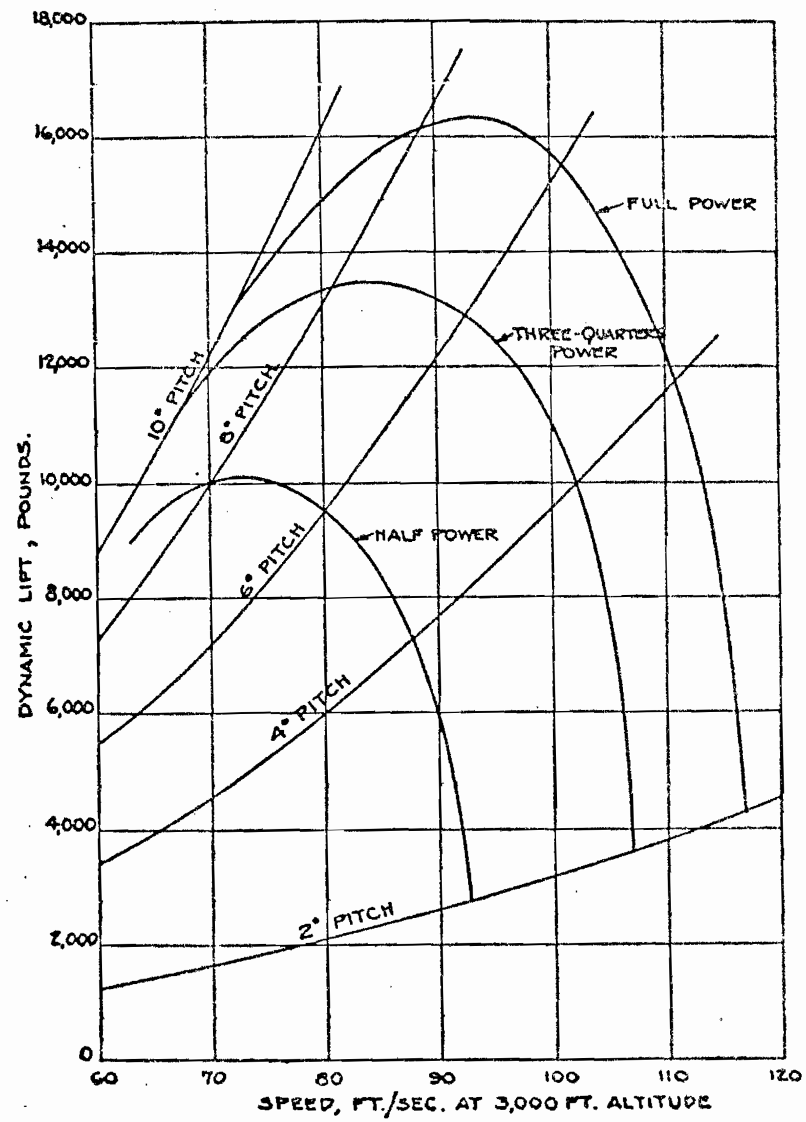


Figure 30. Dynamic Lift Curves of U.S.S. *Los Angeles*

Empirical Formulas for Maximum Aerodynamic Bending Moments and Forces on the Tail Surfaces

A very convenient formula for the maximum aerodynamic bending moment is given by Dr. H. Naatz of the Parseval Airship Company,[3] as follows:

$$M_b = .005\,\rho v^2\,V^{2/3}L \qquad (36)$$

where M_b = maximum bending moment
V = air volume of the ship
ρ = density of the air
v = air speed of the ship
L = length of the ship

All units are absolute, and .005 is a nondimensional coefficient.

Naatz derived this expression from theoretical considerations, and checked it by the internal pressure required to prevent buckling of the envelope of the large Parseval nonrigid airship *PL-27* during the course of two years' service in the stormy area of the Baltic. It has also been found to agree well with the maximum aerodynamic bending moment calculated for the U.S.S. *Shenandoah* from strains indicated by strain gages on the longitudinals, when flying in disturbed air over mountainous country in a high wind.

For design purposes, some convenient coefficients to determine approximately the maximum forces on the tail surfaces are highly desirable. Such coefficients are most suitably given in the nondimensional form:

$$C = \frac{2p}{\rho v^2}$$

for local loads where p = the load per unit area, or:

$$C = \frac{2P}{A\rho v^2}$$

where P is the total load upon a surface of area A.

[3] National Advisory Committee for Aeronautics Technical Memoranda Nos. 275 and 276, "Recent Research in Airship Construction."

The magnitudes of p can only be found from pressure distribution experiments on airships in flight or on wind tunnel models. The total load P on the surfaces can be investigated much more readily than p, either by experimental determination of the moments produced by the tail surfaces with various rudder settings on a wind tunnel model, or by observation of the angular acceleration produced by rudder action on the airship in flight. P may also be calculated by integration of model or full scale pressure plotting results.

Extensive pressure distribution experiments have been carried out under the direction of the National Advisory Committee for Aeronautics upon the tail surfaces of the U. S. Navy Airship *C-7*.[4] The following outstanding pressures were observed:

The maximum total load on a complete surface was 352 lbs., or 1.3 lbs./ft.² on the bottom fin and rudder during a reversal of the rudder from 24° left to 18° right at 40.5 m.p.h. (= 59.5 ft./sec.), and ρ = .00242 slugs/ft.³, whence:

$$C = \frac{2 \times 1.3}{.00242 \times 59.5^2} = .302$$

The maximum load on a fin without rudder was 311 lbs., or 1.7 lbs./ft.² on the top fin during a steady circle with 44° right rudder at 35 m.p.h. (= 51.4 ft./sec.), and ρ = .00242 slugs/ft.³, whence:

$$C = \frac{2 \times 1.7}{.00242 \times 51.4^2} = .532$$

The maximum total load on a movable surface equalled 2.9 lbs./ft.² with 44° rudder at the start of a turn at 45.5 m.p.h. (67 ft./sec.), and ρ = .00241 slugs/ft.³, whence:

$$C = \frac{2 \times 2.9}{.00242 \times 67^2} = .536$$

The largest local load was 7.3 lbs./ft.² near the leading edge

[4] N. A. C. A. Report No. 223.

of the top fin in a steady turn at 45 m.p.h. with 44° right rudder, and $\rho = .00233$ slugs/ft.[3], whence:

$$C = \frac{2 \times 7.3}{.00233 \times 66^2} = 1.44$$

The maximum coefficient of .302 found for a complete surface of the *C*-7 is approximately equal to the coefficient of tail surface force found on the wind tunnel model of the U.S.S. *Shenandoah* at 5° yaw with 20° contrary rudder.

It seems reasonable to design conventional types of tail surfaces for a mean pressure corresponding to $C = .40$. The leading edge of the fins or overhung balancing surfaces should be designed for loads equal to about four times the mean load. Assuming that $C = .40$, and the area of the surfaces is given by $A = .13V^{2/3}$ (see page 57), the total transverse force P on either vertical or horizontal surface is given by $P = .026\ V^{2/3}\ \rho v^2$.

CHAPTER VI

STRENGTH OF NONRIGID AIRSHIPS

In the year 1911 the Siemens-Shuckert Co., Berlin, constructed a large experimental nonrigid airship, 395 ft. in length by 46.6 ft. diameter. There were three cars suspended from a curious external nonrigid keel of fabric. As might be expected from the dimensions, the airship was weak and the shape difficult to preserve. This weakness turned out to be of great benefit to the science of airship construction, for it led to an exhaustive investigation into the theory of strength of nonrigid airships by two German engineers, Rudolf Haas and Alexander Dietzius, undertaken with great care and thoroughness, and published in 1912. An English translation of this report by Prof. K. K. Darrow was published in The Third Annual Report (1917) of the National Advisory Committee for Aeronautics. Although purely nonrigid airships may appear to be a vanishing type, except for small sizes, a thorough knowledge of the stresses in nonrigid airship envelopes is of great importance, and later in dealing with the strength of rigid airships frequent reference will be made to the principles established in this chapter.[1]

Bending of the envelope. The ordinary formula, $f = My/I$ for dealing with the bending of a homogeneous straight rod or beam, is derived from two simple assumptions; (a) that transverse sections that are plane before bending remain plane after bending, (b) that the stress in the material due to the bending is directly proportional to the extension or contraction produced by the bending. The first of these relations is known as the Navier hypothesis, and the second as Hooke's law.

In order to determine the applicability of the ordinary

[1] See also "Pressure Airships," Blakemore and Pagon, Ronald Aeronautic Library.

formula to the envelopes of nonrigid airships, experiments were undertaken by Haas and Dietzius to ascertain how far the bending of a model of a nonrigid airship filled with water conforms to Navier's hypothesis, and also how far the stretching of balloon fabric conforms to Hooke's law.

The model was constructed of three-ply rubberized balloon fabric, having two parallel plies or layers of cloth (i.e., with threads running longitudinally and transversely) and one bias ply (i.e., with threads inclined at an angle of 45° to the threads of the parallel plies), and thin layers of rubber to bind the plies together and to make the fabric gas-tight. While the model was inflated with air and undeformed, transverse circles were drawn upon its surface indicating the cross-sections. The model was then filled with water and suspended by rigging extending for only a short distance along the middle, so that a very considerable bending was produced.

The model remained in the loaded state for three weeks in order to allow for the gradual stretch of fabric under stress, so that the deformation reached a permanent value. The shapes of the cross-sections were then determined by photographs combined stereoscopically, and the plates were magnified to permit of examination with greater accuracy. No deviation of any cross-sections from a true plane could be observed, and it was concluded that the Navier hypothesis may be assumed valid for the bending of nonrigid airship envelopes.

Experiments were made to determine the validity of Hooke's law by means of diagrams showing the relation of stress to strain in balloon fabrics, and it was found that this relation depends in an irregular manner on the stress normal to the direction under investigation, but when the normal stress is constant, the stress-strain diagram is approximately in accordance with Hooke's law. It was therefore concluded that without appreciable error Hooke's law may be assumed to hold over any cross-section of a nonrigid airship envelope.

Since Navier's hypothesis and Hooke's law are valid, the

ordinary formula for stress due to bending is also valid. It should be noted, however, that in dealing with the fiber stress and moment of inertia of an airship envelope it is customary to consider the cross-section of the envelope as an infinitely thin ring, so that the stress is expressed in lbs./in., instead of in

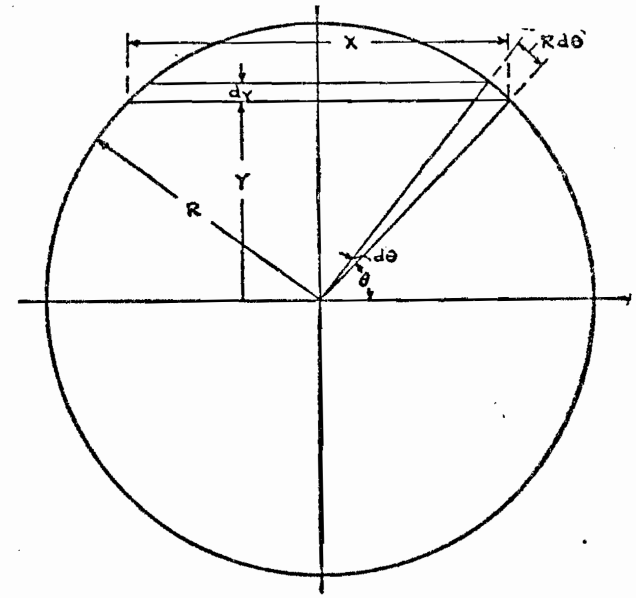


Figure 31. Moment of Inertia of Circular Ring

lbs./in.², and the dimensions of the moment of inertia are L^3 instead of L^4 as in ordinary mechanics.

In a circular cross-section (Figure 31), considered as an infinitely thin ring, the moment of inertia is given by:

$$I = 4\int_0^{\frac{\pi}{2}} y^2 R \, d\theta$$

and $y = R \sin \theta$

$$\text{Therefore } I = 4R^3 \int_0^{\frac{\pi}{2}} \sin^2 \theta \, d\theta$$

$$I = \pi R^3 \qquad (37)$$

Calculation of the bending moment. The static bending moment in a nonrigid airship is composed of three parts, as follows:

(1) The bending moment due to the vertical forces of the weight and buoyancy distributed unequally along the envelope.

(2) The moment of suspension due to the car suspension cables which are usually attached to the envelope some distance below the level of the longitudinal axis, and have longitudinal components which produce bending moments comparable to that caused by an offset load upon a column.

(3) The bending moment due to the increase of gas pressure upwards causing the longitudinal force to be greater above the axis than below.

When the bending moment is such that the ends of the envelope tend to bend upward, the bending moment is said to be positive, or "sagging;" and when the ends tend to droop the bending moment is said to be negative or "hogging." The terms "sagging" and "hogging" are borrowed from naval architecture.

It may be noted that the moment of suspension and the gas pressure moment are negative, while the moment due to the weight and buoyancy forces is usually positive. When the distribution of weight and buoyancy in the envelope and the tensions in the car suspension cables are known, the bending moment due to the vertical forces may be calculated by the methods described in Chapter IV.

The suspension diagram. About two-thirds of the gross-weight of nonrigid airships is usually concentrated in a single car, less than one-fifth as long as the hull or envelope. If this weight were suspended vertically from the envelope it would produce an excessive sagging bending moment. This is obviated by splaying the suspension cables forward and aft from the car to the envelope so as to distribute the car load along the envelope.

The difference between the gross buoyancy and the weight of the envelope is called the net lift of the envelope; and for

static equilibrium the weight of the car and its load must equal the net lift, and for level trim the center of gravity of the car must be vertically below the center of buoyancy of net lift. These centers are calculated in the usual manner by taking moments.

When the weight and position of the car are determined in accordance with the net lift, the suspension system must be designed. An infinite variety of arrangements of cables and tensions is possible, provided three conditions are fulfilled, as follows:

(1) The sum of the vertical components of the tensions in all the cables must equal the net lift or the weight of the car and its load.

(2) The sum of the longitudinal components of the tensions in all the cables must equal zero.

(3) The sum of the moments of the tensions in all the cables about any point in the vertical line through the center of gravity of the car must equal zero.

To ensure that conditions (1) and (2) are fulfilled, a suspension diagram (Figure 32) is constructed. Lines are drawn, end to end, each parallel to the projection of a suspension cable upon the longitudinal vertical plane, and of a length representing to scale the tension in the cable to which it is parallel. When lines representing all the suspension cables are drawn in this manner, the straight line joining the beginning of the first line to the end of the last line is the resultant of the tensions in all the cables. If the length of the resultant line equals the weight of the car when measured to the scale used for the lengths of the component lines, condition (1) is fulfilled; and if the resultant line is vertical, condition (2) is also fulfilled. To find the moments for condition (3), it is most convenient to prepare a table of moments, in which the tension of each cable is obtained from the suspension diagram and is multiplied by the perpendicular distance of the cable from a fixed point in the vertical line through the center of gravity of the car.

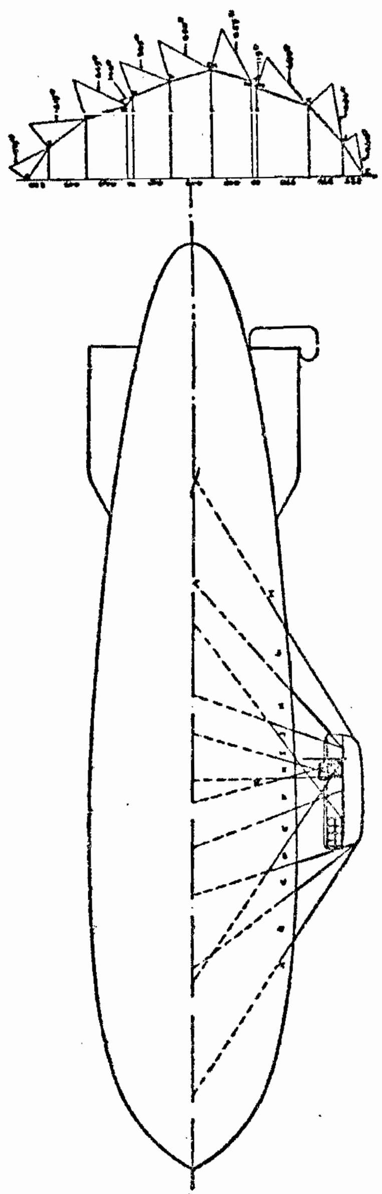


Figure 32. Suspension Diagram of Nonrigid Airship

It is a simple matter to prepare a suspension diagram to fulfill conditions (1) and (2), but it will generally be found that some juggling by trial and error is necessary to obtain a layout and choice of tensions in the cables to link up condition (3) with (1) and (2). Besides these three conditions which must be fulfilled, there are others at which the designer should aim. The arrangement of the car suspension cables should be designed with the object of producing the smallest practicable final bending moment in the envelope consistent with the requirement that the weight of the car be taken by cables as nearly vertical as possible in order to avoid compressive forces in the envelope. Another objection to putting much tension in inclined wires is that these wires take heavy loads when the airship is inclined longitudinally, even if their tensions are quite small when in horizontal trim. A further desirable condition, but usually of minor importance, is that the bending moment in the car should be small. With the short cars of modern airships this condition is easily realized.

Although there are no hard-and-fast rules for the design of the car suspension by which the necessary or desirable conditions may be fulfilled, a convenient method is to prolong the suspension lines (Figure 32), neglecting the bridles usually provided at their upper ends, until they intersect the longitudinal or neutral axis of the envelope. If the suspension cables are assumed to be attached to the envelope where their projection intersects the longitudinal axis, the moment of suspension vanishes, and the moment due to the vertical forces becomes very small by assigning to each cable a tension such that its vertical component is approximately equal to the net lift over a length of the envelope equal to the mean distances between the intersections of consecutive suspension lines with the longitudinal axis of the envelope.

In semirigid airships it is customary to choose the tensions in the suspension cables so that the vertical component of the tension in a cable where it is fastened to a joint in the keel equals the net lift of the envelope transmitted to that joint. By this arrangement the keel receives directly all the longitudinal force of the suspension.

Effect of inclination of the airship, and of propeller thrust. A large angle of inclination of the airship longitudinally throws heavy loads upon some of the inclined suspension cables, and it is necessary to provide "martingales," which are suspension cables at as flat an angle as practicable, leading aft from near the bow of the car, and forward from near the stern, to prevent the car from tilting with respect to the envelope. The effect

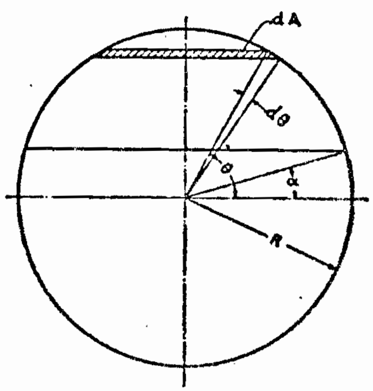


Figure 33. Gas Pressure Bending Moment and Longitudinal Force

of inclination of the airship is to cause a reduction of the force between the car and the envelope in the direction perpendicular to the longitudinal axis of the airship and to introduce a force parallel to the axis. The effect of the propeller thrust is very similar to the effect of longitudinal inclination, except that there is no change in the total force perpendicular to the longitudinal axis, although there will be in most cases a redistribution of the tensions, and hence of the vertical components, among all the suspension cables.

Accepted practice is to design the suspension cables for a factor of safety of 5 or 6 in the normal condition, with little or

no load in the martingales. The latter are designed to take the total end force due to propeller thrust or inclination of the ship, with a factor of safety of about 3.

Longitudinal force and bending moment from gas pressure. The gas pressure produces a longitudinal force and a bending moment at any cross-section of the gas container. When the cross-section is circular, the force F, and the bending moment M may be calculated as follows:

Figure 33 represents a circular cross-section of radius R inflated with gas of unit lift k down to the level given by $R \sin \alpha$ from the horizontal diameter.

$$dA = 2R^2 \cos^2\theta \, d\theta$$
$$dM = p\, R \sin\theta dA$$
$$p = k\, R(\sin\theta - \sin\alpha), \text{ assuming no superpressure}$$

Whence:

$$dM = 2\, k\, R^4 \sin\theta \cos^2\theta \,(\sin\theta - \sin\alpha)\, d\theta$$

$$M = 2kR^4 \left\{ \int_\alpha^{\pi/2} \sin^2\theta \cos^2\theta \, d\theta - \int_\alpha^{\pi/2} \sin\alpha \sin\theta \cos^2\theta \, d\theta \right\}$$

$$= 2kR^4 \left[-\frac{1}{8}\left(\frac{1}{4}\sin 4\theta - \theta\right) - \sin\alpha \left(-\frac{\cos^3\theta}{3}\right) \right]_\alpha^{\pi/2}$$

$$= \frac{kR^4}{2}\left(\frac{\pi}{4} - \frac{\alpha}{2} - \sin^3\alpha \cos\alpha + \frac{1}{2}\sin\alpha\cos\alpha - \frac{4}{3}\sin\alpha\cos^3\alpha\right)$$

$$M = \frac{kR^4}{24}\left[3\pi - 6\alpha + (2\sin^2\alpha - 5)\sin 2\alpha\right] \qquad (38)$$

And

$$dF = p\, dA$$
$$= 2\, k\, R^3 \cos^2\theta \,(\sin\theta - \sin\alpha)\, d\theta$$

$$F = 2kR^3 \left\{ \int_\alpha^{\pi/2} \sin\theta \cos^2\theta \, d\theta - \int_\alpha^{\pi/2} \sin\alpha \cos^2\theta \, d\theta \right\}$$

$$= 2kR^3 \left[-\frac{\cos^3\theta}{3} - \sin\alpha \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) \right]_\alpha^{\pi/2}$$

$$F = \frac{kR^3}{6}\left[4\cos^3\alpha - (3\pi - 6\alpha - 3\sin 2\alpha)\sin\alpha\right]$$

(When there is no superpressure.)

Superpressure p_0 (i.e., pressure at the bottom of the gas space) increases the pressure uniformly all over the cross-section, and has no effect upon M, but it increases F by the area of the section multiplied by the unit superpressure, or by $\pi R^2 p_0$. The general expression for F is therefore:

$$F = \pi R^2 p_0 + \frac{kR^3}{6}\left[4\cos^3\alpha - (3\pi - 6\alpha - 3\sin 2\alpha)\sin\alpha\right] \quad (39)$$

An important special case is when the airship is completely inflated. In that case, $\alpha = -\pi/2$, and

$$M = \frac{\pi}{4}\,k\,R^4 \quad (40)$$

$$F = \pi R^2 p_0 + \pi k R^3 \quad (41)$$

Example. Find the longitudinal force and bending moment due to the gas pressure at a circular cross-section 80 ft. diameter in an airship inflated with gas lifting .064 lb./ft.³ down to a level 15 ft. below the horizontal diameter.

$$\sin\alpha = -15/40 = -.375$$
$$\sin^2\alpha = .1407$$
$$\alpha = -22°1' = -.352 \text{ radians}$$
$$\cos\alpha = .9271$$
$$\sin 2\alpha = -.6952$$

By equation (38),

$$M = \frac{.064 \times 40^4}{24}\left[9.425 + 2.112 + (-4.7186 \times -.6952)\right]$$
$$= 101{,}000 \text{ ft. lbs.}$$

By equation (39),

$$F = \frac{.064 \times 40^3}{6}\left[3.19 - (9.425 + 2.112 + 2.085) \times -.375\right]$$
$$= 5{,}670 \text{ lbs.}$$

The area of the cross-section of the inflated portion of the

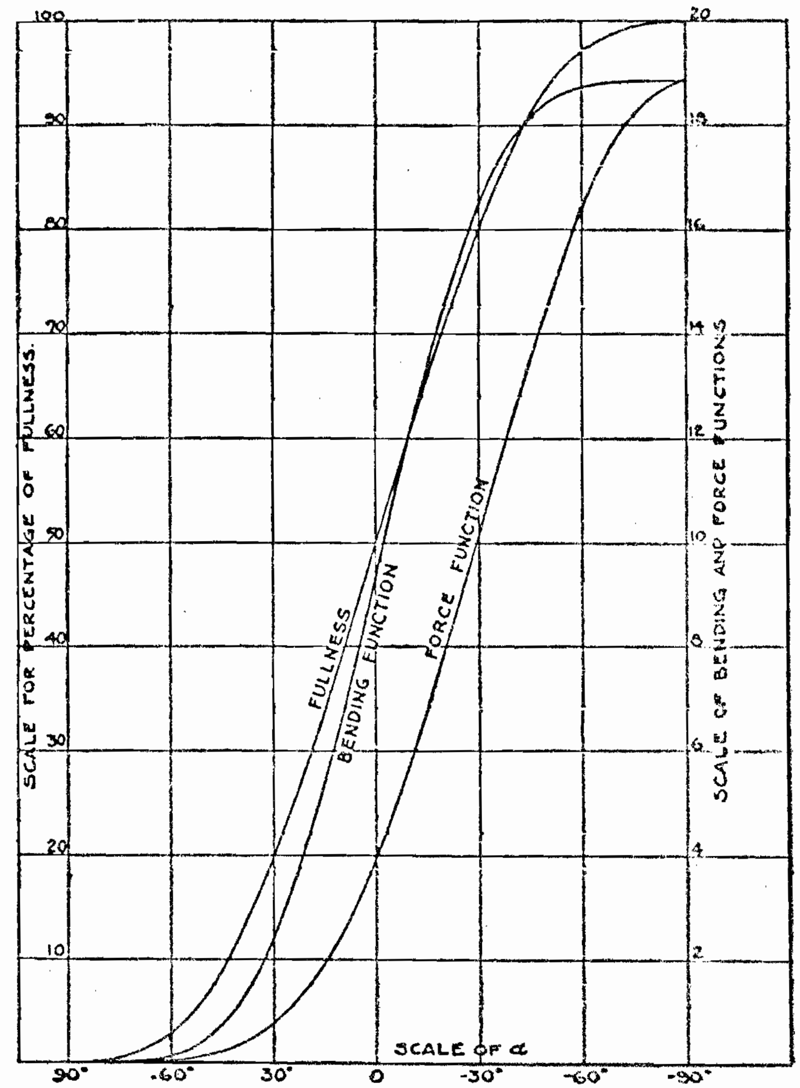


Figure 34. Gas Pressure Bending Moment and Force Functions

cell or envelope is $(\pi R^2/2) - R^2$ (sin α cos $\alpha + \alpha$), and the percentage fullness is

$$100 \times \left[1/2 - \left(\frac{\sin\alpha\cos\alpha + \alpha}{\pi}\right)\right]$$

The longitudinal force function $4\cos^3\alpha - (3\pi - 6\alpha - 3\sin 2\alpha)\sin\alpha$, the bending moment function $3\pi - 6\alpha + (2\sin^2\alpha - 5)\sin 2\alpha$, and the percentage fullness are all plotted against α in Figure 34.

Stress in the Fabric

A fundamental formula may be derived to express the relations between the tensions in an element of fabric and the pressure upon it.

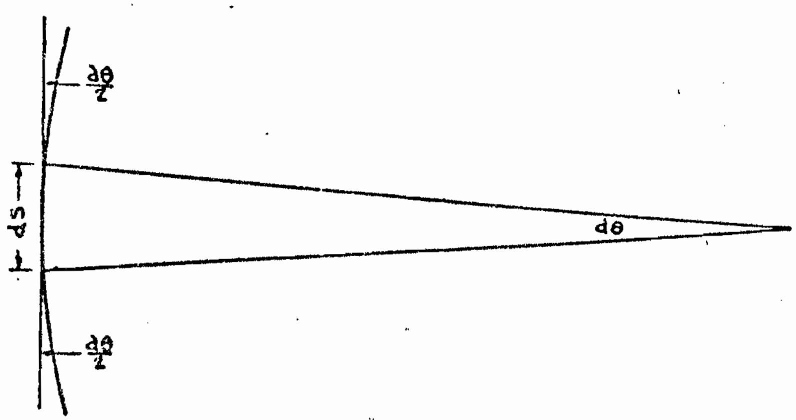


Figure 35. Tension in a Wire

In Figure 35 let element ds of a fine wire subtend an angle $d\theta$ at its center of curvature. Since the element is short, its radius of curvature may be assumed constant irrespective of variations in the curvature of any considerable length of the wire. By geometry the tangents to the wire at the ends of the element make an angle equal to $d\theta/2$ with the straight line through the ends of the element. If the wire sustains a transverse load p per unit of length, equilibrium requires that this transverse force be opposed by the tension T in the wire, so that:

$$p\,ds = 2T\sin\frac{d\theta}{2}$$

Since the sine of a small angle equals the angle,

$$p\,ds = T\,d\theta \qquad (42)$$

and $$d\theta = \frac{ds}{R}$$ where R is the radius of curvature of the element ds.

Therefore $$p = \frac{T}{R}$$

or $$T = Rp \qquad (43)$$

In a similar manner, the condition for equilibrium in the element of area of a fabric curved in two directions and with a transverse load of intensity p per unit of area, as shown in Figure 36, is that:

$$p\,ds_1\,ds_2 = 2T_1\sin\frac{d\theta_1}{2}ds_2 + 2T_2\sin\frac{d\theta_2}{2}ds_1$$

where T_1 and T_2 are the tensions in the two directions per unit width of fabric. As in the case of a transversely loaded wire this equation may be reduced to:

$$p = \frac{T_1}{R_1} + \frac{T_2}{R_2}$$

Or in the envelope of an airship:

$$p = \frac{T_t}{R_t} + \frac{T_l}{R_l} \qquad (44)$$

where the subscripts t and l indicate that the tension or the radius of curvature is transversely or longitudinally as the case may be.

The equation (44) is in general indeterminate since it contains two unknowns. It becomes determinate when R_l is infinite because the last term is then zero, or when T_l is known.

T_l may be calculated from the bending moment, the longitudinal forces, and the geometrical properties of the cross-section of the envelope at the point where the tensions in the fabric are desired. The longitudinal tension T_l may be regarded as consisting of two parts superimposed upon each other. The first

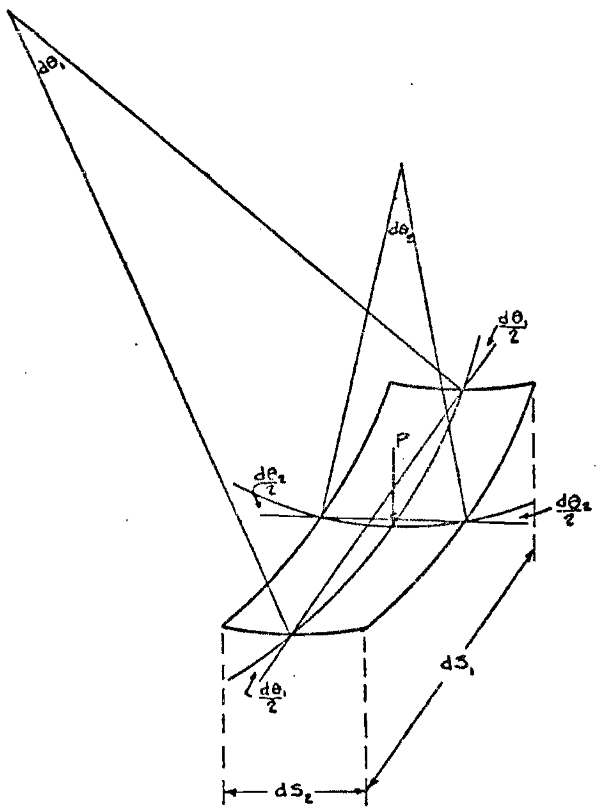


Figure 36. Tension in Fabric

is due to the total longitudinal force upon the envelope at the cross-section considered, and the second part is due to the bending moment at that section. The first part of T_l is equal to the total longitudinal force upon the envelope at the cross-section, divided by the perimeter of the cross-section, and

the second part is obtained from the usual formula for the stress due to bending.

A correction of about 5% to the stresses in the fabric may be made in most cases to allow for the increased strength derived from the overlapping of the seams. Corrections to allow for deformation of the envelope will be considered later.

Where the longitudinal tangent to the envelope is inclined at an angle α to the longitudinal axis, a correction must be made for the difference between the actual tension in the fabric and the component of the tension parallel to the longitudinal axis. This component is found by neglecting α, and T_l is equal to the component divided by cos α.

The total longitudinal force upon a cross-section of the envelope is made up of the internal gas pressure, producing tension, and the longitudinal components of the tensions in the suspension cables producing compression. The total gas pressure force is equal to the area of the cross-section of the envelope multiplied by the mean gas pressure, i.e., the pressure at mid-height of the envelope. The compressive force due to the suspension cables may be found from the suspension diagram. The fact that the resultant of the longitudinal forces does not lie along the longitudinal axis of the envelope need not be considered, because the effect of the offset position of the resultant is included in the calculation of the bending moment.

In most problems dealing with the strength of fabric required in an airship envelope, R_l is so much greater than R_t that the last term in equation (44) may be neglected, and the formula for transverse tension in a cylinder or cone may be used:

$$T_t = pR_t \qquad (45)$$

If R_l is neglected, the tapered portion of an envelope should be considered as consisting of a series of truncated cones. In that case, the transverse radius, R_t, at the point P, Figure 37, is the length of the line PQ drawn perpendicularly to the longitudinal tangent at P and intersecting the longitudinal axis at

Q, and not the line PO which is the radius of the transverse section at P. The reason for this depends upon the theory of conic sections and need not be discussed here, except to note that R_t must obviously be determined in a plane which is perpendicular and not oblique to the surface at P.

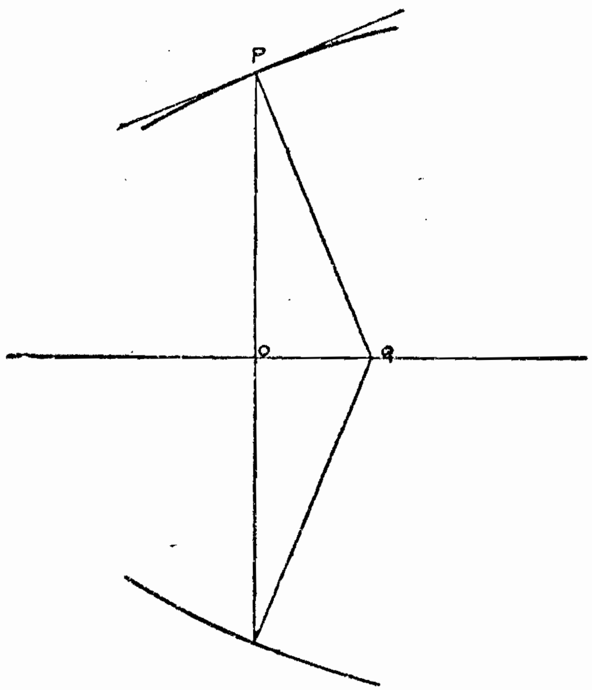


Figure 37. Transverse Radius of Curvature in Conical Portion of an Airship

Calculation of shearing stresses. In Figure 38 the small quadrilateral *abcd* having sides of the length dx and dy is subject to a shearing force of intensity f per unit of length along the sides *ad* and *bc*. These forces produce a couple $f\ dy\ dx$ tending to produce clockwise rotation of the quadrilateral. Equilibrium requires that this couple be opposed by an equal and opposite one. If the shearing force on the sides *ab* and *cd* is also of in-

tensity f, the couple on the quadrilateral will be $f\ dx\ dy$ tending to produce anti-clockwise rotation. This is the couple required for equilibrium, and it follows that for equilibrium conditions, the intensity of the shearing stress at any point must be the same longitudinally as transversely. It is therefore permissible to investigate the intensity of the shearing stress in an airship envelope by considering the shear either longitudinally or transversely.

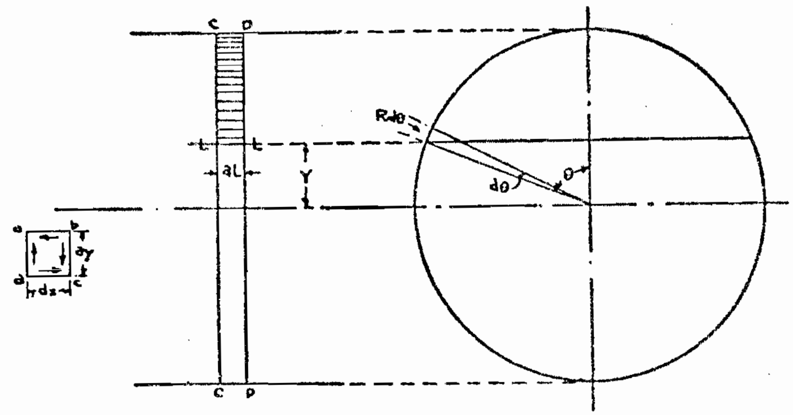


Figure 38. Longitudinal Shear in Nonrigid Airship

Shear longitudinally. Let the total transverse shear force at the section CC in Figure 38 be F, and let the shear per unit of length of the fabric be f. It is required to find the value of f at the intersection of the cross-section CC with the longitudinal-section LL.

The total longitudinal stress in the fabric at CC between LL and the top of the cross-section due to the bending moment is in accordance with the theory of bending equal to:

$$\frac{M}{I}\int_0^\theta yR\ d\theta$$

But $y = R\cos\theta$

$$\text{Therefore the integral is } \frac{MR^2}{I}\int_0^\theta \cos\theta \, d\theta$$

$$= \frac{MR^2}{I}\sin\theta$$

Similarly the total longitudinal stress in the fabric between LL and the top of the cross-section DD at an indefinitely small distance dl from CC is equal to:

$$\frac{(M - dM)R^2}{I}\sin\theta$$

where dM is the change of the bending moment between CC and DD. By the theory of shearing forces and bending moments, the change of bending moment between two cross-sections equals the integration of the shear between those sections; therefore, $dM = F\,dl$.

The difference of the total longitudinal forces at CC and DD between LL and the top of the envelope is equal to:

$$\frac{R^2 \sin\theta \, dM}{I} = \frac{R^2 \sin\theta}{I} F\,dl$$

This difference of the longitudinal forces at CC and DD must obviously equal the total shear along LL between CC and DD. It follows that:

$$f\,dl = \frac{R^2}{I}\sin\theta \, F\,dl$$

In a circular cross-section: $I = \pi R^3$

Therefore
$$f = \frac{F \sin\theta}{\pi R} \tag{46}$$

Shear transversely. Equation (46) may also be obtained by considering the shear transversely in Figure 39. Suppose that the section DD moves downward relatively to CC a small distance dy. This movement has a tangential component $dy \sin\theta$ at LL, and a radial component $dy \cos\theta$. The radial component has no effect upon the stresses in the fabric, but the tangential component produces shearing strains and stresses.

The shearing stress is proportional to $dy \sin\theta$, and it follows that if f_m is the maximum intensity of the shearing stress on the cross-section CC, the intensity of shear at any point on CC is given by:

$$f = f_m \sin\theta$$

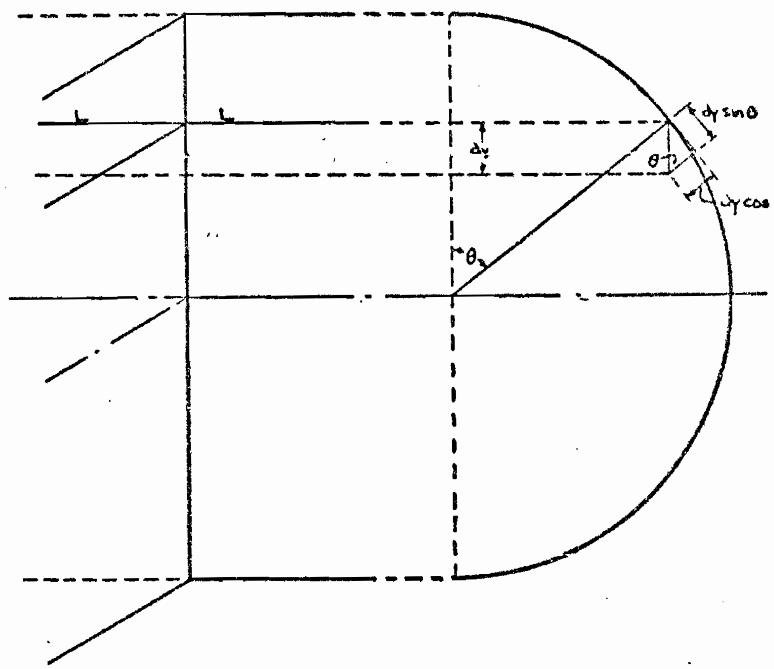


Figure 39. Transverse Shear in a Nonrigid Airship

The vertical component of the shearing stress is $f \sin\theta$, and the total vertical shearing force must equal the integration of the vertical components over the whole cross-section. Therefore:

$$F = \int_0^{2\pi} f \sin\theta \, R \, d\theta$$

$$= R f_m \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= R f_m \pi$$

or $$f_m = \frac{F}{\pi R} \qquad (47)$$

and $$f = \frac{F \sin \theta}{\pi R} \qquad (46a)$$

This is the same result as was obtained by considering the shear longitudinally; but this agreement is found to exist only when the cross-section is circular, which indicates that the ordinary bending moment formula is strictly applicable only for circular cross-sections. That is to say, when the cross-section is other than circular, bending produces a distortion of the cross-section so that the Navier hypothesis is not precisely fulfilled.

If a cross-section on a conical portion of the envelope is considered, the tangential component of the shear still equals $dy \sin \theta$, and the integration of the shearing stresses also remains unchanged, so that the value of f is the same as in a cylinder. The only difference between the cone and the cylinder is that to a distance dl parallel to the longitudinal axis of the envelope corresponds a breadth of fabric equal to $dl/\cos\alpha$, instead of equal to dl, where α is the inclination of LL to the longitudinal axis. A given shearing force produces, therefore, a greater value of the displacement dy in the ratio of $1 : \cos\alpha$ for the cone as compared with the cylinder.

Ellipse of stress. The methods of finding the intensity of the tension and shear in the fabric longitudinally and transversely at any point in the envelope have been investigated in the preceding sections. The maximum intensity of the stress may not, however, lie in either of these directions, and it is often desirable to know the direction and magnitude of the maximum and minimum tensions. This problem is especially liable to occur in the design of reinforcements for the envelope, such as trajectory bands. The magnitude of the tension and shear in any direction may be determined by a geometrical analysis known as "the ellipse of stress."

In Figure 40 let OX and OY be two perpendicular coordinate axes. In an elementary rectangular area of fabric, $ABCD$, having sides parallel to these axes, let the tensions parallel to OX and OY per unit width of fabric be T_x and T_y, respectively. Let the shear be zero in the directions of OX and OY. In the

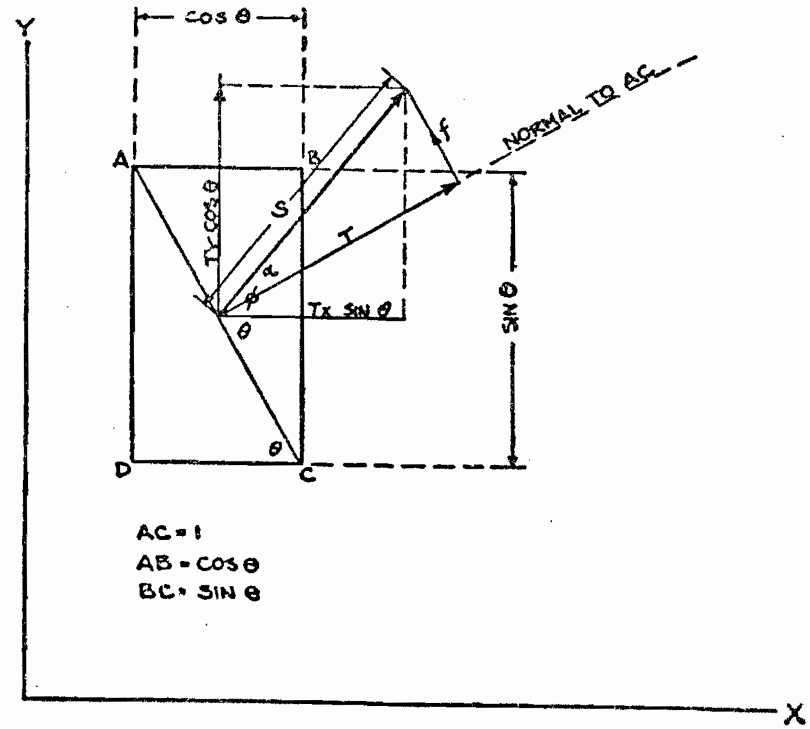


Figure 40. Principal Stresses in a Nonrigid Airship Envelope

elementary rectangle $ABCD$, let the diagonal AC be inclined to OX at an angle θ, and let the length of AC be unity. Then the forces upon AC due to the unit tensions T_x and T_y are $T_x \sin\theta$ and $T_y \cos\theta$, respectively. Let the resultant of these two forces be S, inclined to OX at the angle ϕ. Then from the resolution of forces, Figure 40,

$$S \cos\phi = T_x \sin\theta$$

and $$S \sin\phi = T_y \cos\theta$$

Whence $$\frac{S^2 \cos^2\phi}{T_x^2} = \sin^2\theta$$

and $$\frac{S^2 \sin^2\phi}{T_y^2} = \cos^2\theta$$

But $$\sin^2\theta + \cos^2\theta = 1$$

Therefore $$\frac{S^2 \cos^2\phi}{T_x^2} + \frac{S^2 \sin^2\phi}{T_y^2} = 1$$

This is the equation of an ellipse in which S is the radius vector, and T_x and T_y are the axes.

The resultant stress S upon any line of unit length may be resolved into two components, a tension T normal to the line, and a shear f parallel to the line. In Figure 40 let α be the angle between the normal to AC and the direction of S. Then:

$$\alpha = \theta + \phi - 90^\circ$$

Also $$T = S \cos\alpha$$

and $$f = S \sin\alpha$$

From Figure 40, $$\tan\phi = \frac{T_y \cos\theta}{T_x \sin\theta}$$

Example. Given $T_x = 7$ lbs./in., $T_y = 11$ lbs./in., find S, T, and f upon a line inclined at 25° to OX.

$$\theta = 25^\circ,\ \sin\theta = .4226,\ \cos\theta = .9063$$

$$\tan\phi = \frac{11 \times .9063}{7 \times .4226} = 3.37$$

$$\phi = 73^\circ 28',\ \cos\phi = .2845$$

$$S = \frac{T_x \sin\theta}{\cos\phi} = \frac{7 \times .4226}{.2845} = 10.4 \text{ lbs./in.}$$

$$\alpha = \theta + \phi - 90^\circ = 25^\circ + 73^\circ 28' - 90^\circ = 8^\circ 28'$$

$$T = S \cos\alpha = 10.4 \times .989 = 10.28 \text{ lbs./in.}$$

$$f = S \sin\alpha = 10.4 \times .1472 = 1.53 \text{ lbs./in.}$$

Only in the directions of the axes of the ellipse of stress is the resultant stress a pure tension without shear, for $\alpha = 0$ only when the stress is considered upon a line parallel to one of these axes.

In the preceding investigation it was assumed that the directions of the axes of the ellipse were known; but in actual problems dealing with airship envelopes, the more general case is when the longitudinal and transverse tension and shear

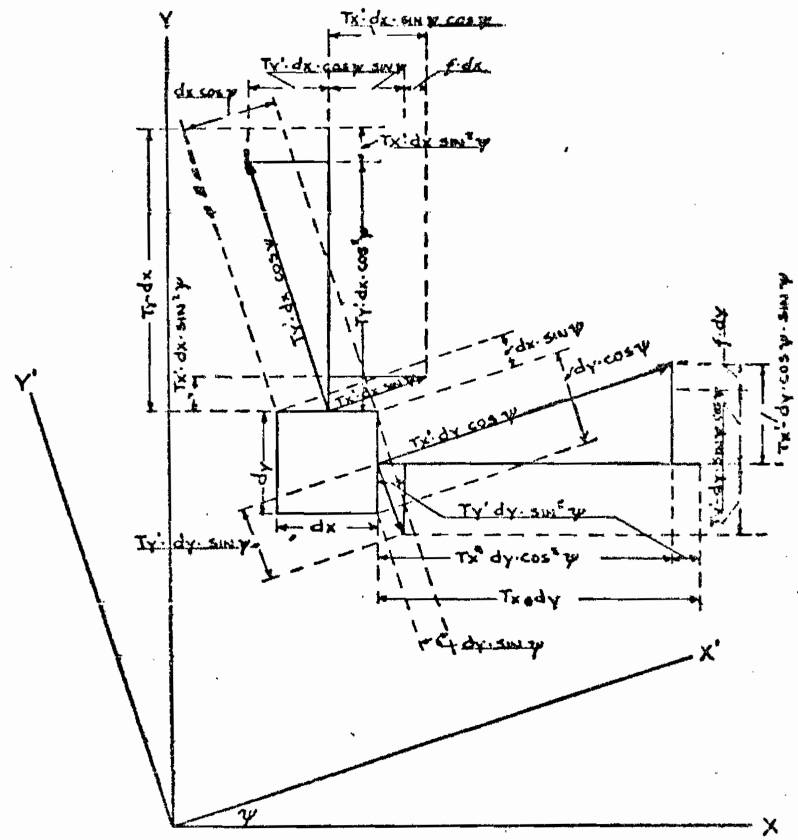


Figure 41. Tension and Shear in Any Direction

stresses are known, but not the directions of the axes of the ellipse of stress. The effect of the shear in the longitudinal and transverse directions is to rotate the axes of the ellipse to the directions in which there is no shear. The new axes give the direction and magnitude of the maximum and minimum tensions in the fabric at the point considered.

Suppose that the unit tensions T_x and T_y and the unit shear stress f parallel to the coordinate axes OX and OY in Figure 41 are known. Let the axes of the ellipse of stress be OX' and OY' inclined at the angle ψ to OX and OY, respectively. The tensions parallel to OX' and OY' have components in the directions of OX and OY equal to the known tensions and the shear in those directions. Let the tensions parallel to OX' and OY' be T_x' and T_y', respectively. By equating the components of T_x' and T_y' in the directions of OX and OY to the values of T_x and T_y, the relations are obtained (Figure 41):

$$T_x\,dy = T_x'\,dy\cos^2\psi + T_y'\,dy\sin^2\psi$$

and

$$T_y\,dx = T_y'\,dx\cos^2\psi + T_x'\,dx\sin^2\psi$$

These equations reduce to:

$$T_x = T_x'\cos^2\psi + T_y'\sin^2\psi \tag{48}$$

and

$$T_y = T_y'\cos^2\psi + T_x'\sin^2\psi \tag{49}$$

The shearing force is connected to the tensions by the equation:

$$f\,dy = -T_x'dy\cos\psi\sin\psi + T_y'\,dy\sin\psi\cos\psi$$

or

$$f = (T_y' - T_x')\sin\psi\cos\psi \tag{50}$$

The unknowns, T_x', T_y', and ψ may be determined from equations (48), (49), and (50).

By substituting $1 - \sin^2\psi$ for $\cos^2\psi$ in equation (48):

$$\sin^2\psi = \frac{T_x - T_x'}{T_y' - T_x'} \tag{51}$$

By the same substitution in (49):

$$\sin^2\psi = \frac{T_y' - T_y}{T_y' - T_x'} \tag{52}$$

From (51) and (52):

$$T_x + T_y = T_x' + T_y' \tag{53}$$

From (50) and (51):

$$f^2 = (T_y' - T_x')^2\left[\frac{T_x - T_x'}{T_y' - T_x'} - \left(\frac{T_x - T_x'}{T_y' - T_x'}\right)^2\right] = (T_x - T_x')(T_y' - T_x)$$

Substituting the value of T_y' from (53):

$$f^2 = T_x'^2 - (T_x + T_y)\,T_x' + T_x\,T_y$$

or substituting for T_x':

$$f^2 = T_y'^2 - (T_x + T_y)\,T_y' + T_x\,T_y$$

From these two equations:

$$T_x' \text{ or } T_y' = \frac{T_x + T_y \pm \sqrt{(T_x + T_y)^2 - 4\,T_x\,T_y + 4f^2}}{2} \tag{54}$$

The two roots of this quadratic are the values of T_x' and T_y'.

Example. Given $T_x = 7$ lbs./in., $T_y = 11$ lbs./in., and $f = 4$ lbs./in., find the directions and magnitudes of the maximum and minimum tensions in the fabric. (Note that from the preceding analysis these are the directions of no shear.)

The lengths of the axes of the ellipse of stress are from equation (54) equal to

$$\frac{7 + 11 \pm \sqrt{(7 + 11)^2 - (4 \times 7 \times 11) + 4 \times (4)^2}}{2} = 9 \pm \frac{\sqrt{80}}{2} = 13.47 \text{ or } 4.53 \text{ lbs./in.}$$

These are the maximum and minimum tensions at the point considered. It is immaterial which of the new axes is denoted by X' and which by Y'.

Let the minor axis be denoted by X'. Then:

$$\sin^2\psi = \frac{7 - 4.53}{7 + 11 - 9.06} = .276$$

$$\sin\psi = .525, \text{ and } \psi = 31°42'$$

That is, the directions of T_x' and T_y' are rotated 31°42′ anti-clockwise from the directions of T_x and T_y, respectively.

Critical shear. Since fabric is incapable of sustaining compression, folds will begin to appear in the envelope whenever the tension in any direction falls below zero. It may be seen from the last example that when a shearing stress is added to known values of T_x and T_y, the effect is to rotate the axes of the ellipse of stress and to change their relative lengths. Clearly, if folds in the fabric are to be avoided, a limiting value of the shear stress is when the length of one axis is zero. This will occur when the quantities outside of and under the square root sign in equation (54) are equal, i.e., when

$$T_x + T_y = \sqrt{(T_x + T_y)^2 - 4\, T_x\, T_y + 4f^2}$$

Solving this equation:

$$f = \sqrt{T_x\, T_y} \tag{55}$$

which is the limiting or critical value of f.

Deformation of the Envelope

The envelope of a nonrigid airship is subject to two different kinds of distortion. Stress produces stretch or strain in balloon fabric just as in any other elastic material, and the envelope is

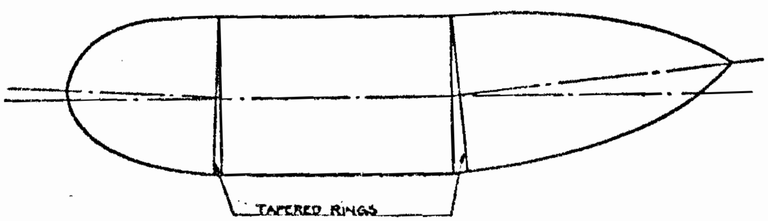


Figure 42. Tailoring of Nonrigid Airship Envelope

inevitably more or less deformed by this stretch. Independently of the stretch of the fabric, the cross-sections are distorted from true circles by the suspended loads and the variation of pressure from top to bottom of the cross-section.

Because of the great number of variables which determine the relation of stress to strain in balloon fabric, the calculation of the deformation due to stretch is difficult and of uncertain accuracy. In practice, the probable change of length, the bending of the envelope, and the increase of the perimeters of the cross-sections by permanent stretch of the fabric, are best estimated by experience with similar airships or by experiment with water models. Bending by permanent stretch may be

corrected by "tailoring" which is the insertion of tapered rings of fabric, shown in an exaggerated manner in Figure 42, to give an initial set to the envelope opposite to the anticipated bending.

Experience has shown that the permanent bending may be much reduced by care of the airship when in the shed. For example, the most serious bending of modern nonrigid airships is the so-called "tail-droop" due to the weight of the fins and control surfaces near the stern where the buoyancy is small. It has been found that if the weight of the stern is supported while in the shed, the tail-droop is much less serious than if this precaution is not taken.

The transverse deformation due to the suspended loads and the variation of gas pressure is more susceptible of calculation and is usually more important than the deformation due to stretch.

Water models. For an experimental investigation into the stresses and deformations of nonrigid and semirigid airship envelopes, a device called the water model may be used. The idea of the water model appears to have occurred independently and almost simultaneously in the year 1911 to Captain Crocco in Italy, Upson in America, and Booth in England. A scale model of the envelope of the airship, including the car suspension, is constructed of the same fabric as the actual envelope. Since $T \sim pR$, it is clear that the model will be subjected to the same intensity of stress as the airship by making p inversely proportional to R.

Suppose that the model is inflated with air and submerged in a tank of water by forces applied to the lines representing the car suspension. In the submerged model, the contained air and the surrounding water are, respectively, analogous to the contained hydrogen and the surrounding atmosphere of the actual airship. In order that the model may be comparable to the airship, it is required that the fabric tensions at homologous points of the airship and model be equal to each other.

It is necessary to find a scale ratio n between model and airship such that this condition will be fulfilled.

Let q_1 and q_2 be points on the airship at a vertical distance h apart.

Let q_1' and q_2' be points on the model, homologous to q_1 and q_2, respectively.

Let p_1 and p_2 be the pressures per unit area at q_1 and q_2, respectively.

Let p_1' and p_2' be the pressures at q_1' and q_2', respectively.

Let k be the difference between the weights of unit volumes of the gas in the airship and of the surrounding atmosphere, i.e., the lift of unit volume of the gas.

Let k' be the lift of unit volume of air submerged in water.

Then by the laws of fluid pressure:

$$p_1' = p_2' + k'h' \qquad (56)$$

and

$$p_1 = p_2 + kh \qquad (57)$$

But $h = nh'$, and it is desired that n be chosen such that:

$$p_1' = np_1 \text{ and } p_2' = np_2$$

Dividing equation (56) by equation (57) and substituting the values of the primes:

$$\frac{np_1}{p_1} = \frac{np_2 + k'\frac{h}{n}}{p_2 + kh}$$

$$\text{Whence } n = \sqrt{\frac{k'}{k}} \qquad (58)$$

$k' = 62.4$ lbs./ft.^{3}, for fresh water, and if it is assumed that $k = .068$ lb./ft.^{3},

$$n = \sqrt{\frac{62.4}{.068}} = \sqrt{918} = 30.3$$

For convenience n is usually taken as 30.0, which corresponds to $k = .0694$ lb./ft.^{3} when $k' = 62.4$ lbs./ft.^{3}

By constructing a model to a scale of 1/30th full size, and with a pressure 30 times greater than in the airship, the fabric tensions due to pressure are duplicated exactly, and therefore the deformations due to pressure are duplicated to scale. But the total deformation of the envelope is the result of the combined effects of the internal pressure and the distribution of

the load, and the water model as described does not duplicate to scale the conditions of the full size airship as regards the load. We have the following relations of model to airship:

Scale:	$1/n$
Volume:	$1/n^3$
Unit lift:	$k'/k = n^2$
Gross lift:	volume × unit lift = $1/n$
Weight of fabric = $1/n^2$	

It will be seen that the ratio weight of fabric to gross lift is n times greater in the airship than in the model, and it follows that the load taken by the suspension will be relatively greater in the model than in the airship. This may be compensated approximately by adding weights to the model equal to $(1/n - 1/n^2)$ times the weight of its fabric. Similarly, weights equal to $1/n$ times the weight of the tail surfaces and other fittings upon the envelope should be suspended from the model in order to duplicate as nearly as possible the conditions of the actual airship.

It is usually inconvenient to submerge the model in water and the same distribution of pressures may be had by suspending the model in the air, inverted and filled with water. Upward forces must be applied to the inverted model equal to the weight of the envelope of the model plus $1/n$ times the weight of the envelope and envelope fittings in the actual airship. The upward forces are usually applied to a longitudinal equatorial band around the model. An alternative method is to place an air-sack inside the model, but this is usually more complex and less satisfactory.

Water models do not reproduce the local strains at the suspension patches of an actual airship envelope. If the patches in the model were scale reproductions of the full size patches they would have an area $1/n^2$ times full size, but would be subject to a load $1/n$ times that of the actual patches. The shearing stress between envelope and patches would then be n times greater in the model than in the airship, and the patches would not hold to the model; to overcome this difficulty the patches are relatively much larger, incidentally giving the model a considerable amount of local reinforcement not present in the actual envelope.

Calculation of the deformation due to stretch of the fabric. The deformation of structures or machinery under load by stretching of the materials may be calculated provided the moduli of elasticity of the materials are known, and provided the materials are not strained beyond their elastic limits.

For homogeneous materials, such as the metals, only one characteristic, E, known as Young's modulus of (or measure of) elasticity, is required for the solution of ordinary problems in elastic stretch or deformation. E is the intensity of stress in any material required to double the length of a uniform bar of the material, assuming it to remain perfectly elastic, i.e., assuming the validity of Hooke's law, that the ratio stress/strain is constant.

In some problems, the deformation due to shear is required, and for such calculations, the modulus of elasticity in shear, E_S, is used. E_S is defined as the ratio of shearing stress per unit area to the angular distortion.

For every metal, E is a very definite quantity, so that the deformations of metal structures under loads may usually be calculated to a high order of accuracy. For wood, the values of E are less definite, and are found to depend somewhat upon time, so that a wooden structure under load deforms or sags to an extent which increases perceptibly with time. A familiar example of this is afforded by wooden ships, which as they grow old, usually become "hogged," i.e., the bow and stern droop because of the negative bending moment in the hull.

For balloon fabrics, even more than for wood, the deformation under load is of the nature of a gradual flow, continuing for days, but approaching asymptotically to a limit, which may be

regarded as permanent for loads which are more or less constant during the service of the envelope.

When balloon fabrics are subject to simple tension, it is found that the ratio stress/strain varies with the intensity of the stress and the direction in which the tension is applied in relation to the directions of the warp and filler threads of the fabric. If there is tension in the fabric in two directions at right angles to each other, the ratio stress/strain in either direction is found to depend on the stress in the other direction.

Professor Darrow, the translator of Haas and Dietzius, used the term "normal characteristic" to designate the ratio stress/strain for balloon fabrics in tension, and owing to the many variables involved, this seems a more fitting term than modulus of elasticity. Similarly, the ratio (shearing stress)/(angular distortion) is termed the "shear characteristic."

In ordinary problems upon the deformation of an airship envelope, the normal characteristic of the fabric in the two principal directions, longitudinally and transversely, i.e., parallel to the warp and filler threads of the straight plies of cloth, are required. Because of the dependence of the normal characteristic in one direction upon the tension at right angles, it follows that the value of the characteristic in each direction is represented by a family of curves. The shape of these curves for any particular type of fabric must be determined experimentally, and since the only cases of much practical value are where the fabric is given time to approach its asymptotic limit of stretch for the applied loads, it is obvious that the plotting of the curves is a long and tedious process.

The shear characteristic is even more complex than the normal characteristic. It is found that the distortion due to shear varies irregularly with the intensity of the shear and the tensions in the two principal directions. If the shear characteristic depended only upon the intensity of the shear and one of the principal tensions, the shear characteristic would be a family of curves; but since it depends upon both principal tensions, it must be represented by a family of surfaces.

The report by Haas and Dietzius contains detailed descriptions of experimental methods which they employed to determine the normal and shear characteristics of airship fabrics.

In determining the normal characteristic, the fabric is loaded in such a manner as to apply tensions parallel to the two principal

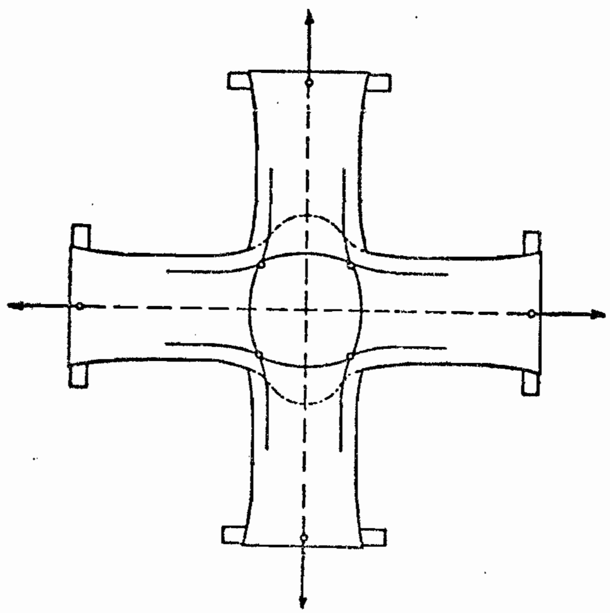


Figure 43. Method of Testing Normal Characteristics of Fabric

directions, and the variations in length in these directions are measured. The most simple procedure is to apply weights to a flat cruciform piece of fabric as shown in Figure 43. Another method is to apply hydrostatic pressure to the inside of a fabric cylinder combined with an axially applied load by means of weights in order to vary the ratio of longitudinal to transverse stress. The fabric cross method has the disadvantage that the tension is not uniform owing to distortion of the fabric in the

manner shown in exaggerated form in Figure 43. Haas and Dietzius used auxiliary clamps to pull out the corners, but even so, the distortion of the fabric and inequality of tension could not be entirely eliminated. Experimental determination of the normal characteristic by means of hydrostatic pressure in a fabric cylinder is probably more accurate than the fabric cruciform method, but the apparatus is more complex, and troubles are apt to be experienced with twisting of bias fabric; and finally, the calculations are somewhat laborious because the transverse tension cannot be assumed to vary directly as the hydrostatic pressure owing to the fact that the diameter is also found to vary with both the pressure and longitudinal tension.

Since the normal characteristic of a fabric consists of a family of curves for each of the two principal directions of stress, a great number of points must be plotted. Each observed stress/strain relation gives a point for each of the two families. The time factor must be eliminated so far as possible by giving the fabric several days to stretch under the load. Haas and Dietzius found the most satisfactory procedure was to determine each point from a separate cross of fabric, rather than all the points from one cross, or each of the curves from one cross.

The shear characteristic was determined from torsion tests on fabric cylinders subject to hydrostatic pressure.

Given the normal and shear characteristics, the calculations of the distortion of the envelope by bending and shear are in principle just as in loaded beams. The curvature of the longitudinal axis by the bending moments is given by the usual equation for the curvature of elastic beams:

$$\frac{dy^2}{dx^2} = \frac{M}{EI} \qquad (59)$$

The mathematical solution of the curvature is extremely laborious because M, E, and I are all variable along the envelope. E is the normal characteristic of the fabric, determined by experiment. The calculation is further complicated by the fact that I is increased beyond that of the designed circular cross-sections by both the stretch of the fabric and the distortion of the shape of the cross-sections. Given the longitudinal and transverse tensions, the increase in the perimeter of the cross-sections can be determined from the normal characteristic. The distortion of their shape will be considered presently.

The envelope will also change somewhat in length by stretching. Haas and Dietzius found that the length decreased owing to the transverse tension being approximately twice as great as the longitudinal tension. American nonrigid airships have been found to increase in length.

The distortion of a beam due to shear is usually neglected; but in an airship the distortion by shearing is of the same order of magnitude as that by bending. The curvature of the longitudinal axis by shear is given by the equation:

$$\frac{dy}{dx} = \gamma \tag{60}$$

where γ = the angular distortion of the axis at any point. The shear characteristic, E_S, is defined as the ratio of the shearing stress per unit length to the angular distortion, and it may be seen from this definition that:

$$E_s = \frac{f_m}{\gamma}$$

Substituting the value of f_m given by equation (47):

$$\gamma = \frac{F}{\pi R E_s} \tag{61}$$

The total deformation of the longitudinal axis of the envelope is the sum of the deformations due to bending and shear. Because of the difficulty and uncertainty in determining the normal and shear characteristics of airship fabrics, and the great labor involved in the calculation of the curvature of the axis even when the characteristics are known, the practical determination of the longitudinal distortion to be expected in

any new design of airship is almost invariably done by measurements on a water model.

Calculation of the deformation of the cross-section. In considering the transverse deformation of a nonrigid envelope, the case of a cylinder or a cone only will be investigated. The problem is thereby very much simplified, and the results attained are sufficiently accurate, because the longitudinal radius of

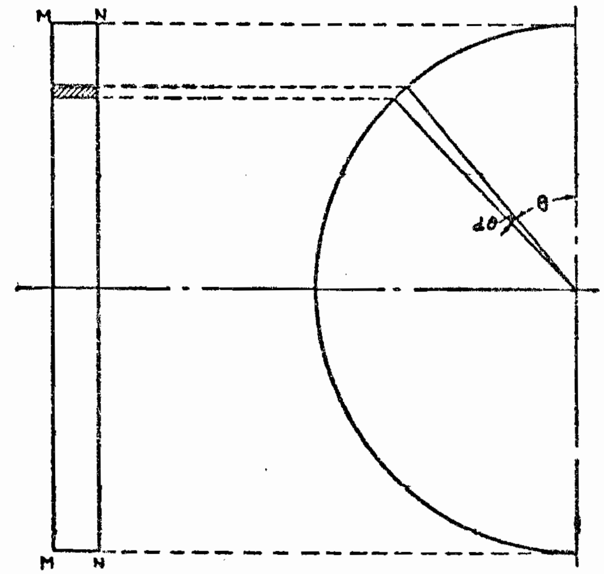


Figure 44. Effect of Shear upon the Shape of the Cross-Section

curvature is always so much greater than the transverse radius in that portion of the envelope where the deformation is appreciable, that the transverse tension in the fabric is practically independent of the longitudinal tension and curvature. The transverse forces acting upon the cross-section are the gas pressure, the weight of the fabric, the shear, and the component of the suspension in the transverse plane.

In Figure 44 let MM and NN be two cross-sections at unit distance apart. Let the shaded strip of fabric subtend the angle $d\theta$ transversely at the longitudinal axis.

Let R = the transverse radius

p = intensity of gas pressure per unit area upon the shaded part

w = weight of fabric per unit area

k = lift of the gas per unit volume

W = suspended load between MM and NN

F_1 and F_2 = total transverse shear forces at MM and NN, respectively

T_1 and T_2 = transverse stresses in the fabric per unit width, respectively above and below the shaded portion

$S = F_2 - F_1$

$dT = T_2 - T_1$

The area of the shaded strip is $R\,d\theta$ and the weight of fabric in it is $w\,R\,d\theta$. This weight has tangential and radial components, $w\,R\,\sin\theta\,d\theta$, and $w\,R\,\cos\theta\,d\theta$, respectively. The radial component acts in the direction of the gas pressure in the lower half of the envelope, and against the gas pressure in the upper half.

From equation (45) when the longitudinal curvature is neglected, the transverse tension in the fabric is given by $p = T/R$. If account is taken of the weight of the fabric, another term must be added to allow for the effect of the radial component of the weight acting with or against p. From the expression just derived for this component, and modifying equation (42), we have:

$$p\,R\,d\theta \pm w\,R\cos\theta\,d\theta = T\,d\theta$$

Whence

$$p \pm w\cos\theta = \frac{T}{R} \qquad (62)$$

The shaded strip (Figure 44) must be in equilibrium with the tangential forces acting upon it. These forces are the difference in the transverse tension above and below it, the tangential component of the weight of the fabric, and the difference between the shearing forces along its edges on MM and NN. The difference in the transverse tensions is dT; the tan-

gential component of the weight of the fabric is $w\,R\,\sin\theta\,d\theta$. The difference in shearing forces may be calculated from the total shearing forces on MM and NN obtained from equation (46). Let s = change in shear per unit width of fabric from MM to NN. From equation (46):

$$s = \frac{(F_2 - F_1)\sin\theta}{\pi R} = \frac{S\sin\theta}{\pi R}$$

The tangential component of the differences in the shearing forces upon the two ends of the shaded area is, therefore, $s\,R\,d\theta$, and the equation of equilibrium of the tangential forces upon this area is:

$$\begin{aligned} d\,T &= s\,R\,d\,\theta + w\,R\sin\theta\,d\,\theta \\ &= \frac{S\sin\theta\,d\theta}{\pi} + w\,R\sin\theta\,d\theta \\ &= \left(\frac{S}{\pi} + w\,R\right)\sin\theta\,d\theta \end{aligned}$$

If T_t = the transverse tension at the top of the section, the transverse tension at a point along the envelope a distance $R\,\theta$ from the top is given by:

$$\begin{aligned} T &= T_t - \int_0^\theta \left(\frac{S}{\pi} + w\,R\right)\sin\theta\,d\theta \\ &= T_t - \left(\frac{S}{\pi} + w\,R\right)(1 - \cos\theta) \end{aligned} \qquad (63)$$

If p_t is the gas pressure at the top of the section, the pressure at a point on the envelope a distance $R\theta$ from the top is given by:

$$p = p_t - k\,R\,(1 - \cos\theta)$$

Consider the case when $W = O$, then $S + 2\,w\,\pi\,R = k\,\pi\,R^2$; i.e., the gross lift of the volume of gas between MM and NN is balanced by the shearing force and the weight of the fabric. Let R_t = the transverse radius at the top of the section. Let it be assumed that the cross-section is circular, and upon this assumption calculate the values of the pressure and tension at any point, and from these values calculate R; if $R = R_t$ the assumption of the circular cross-section is justified. From (62) and (63) the given equations are:

$$R_t = \frac{T_t}{p_t - w\cos\theta} = \frac{T_t}{p_t - w}$$

or

$$p_t = \frac{T_t}{R_t} + w$$

$$R = \frac{T}{p - w\cos\theta}$$

$$T = T_t - \left(\frac{S}{\pi} + w\,R_t\right)(1 - \cos\theta)$$

$$p = p_t - k\,R_t\,(1 - \cos\theta)$$

$$S + 2\,w\,\pi\,R = k\,\pi\,R^2, \text{ or } \frac{S}{\pi} + w\,R_t = k\,R^2_t - w\,R_t$$

From these equations it follows that:

$$R = \frac{T_t - \left(\frac{S}{\pi} + w\,R_t\right)(1 - \cos\theta)}{p_t - k\,R_t\,(1 - \cos\theta) - w\cos\theta}$$

$$= \frac{T_t - (k\,R^2_t - w\,R_t)\,(1 - \cos\theta)}{\frac{T_t}{R_t} + w - k\,R_t\,(1 - \cos\theta) - w\cos\theta}$$

$$= \frac{R_t[T_t - (k\,R^2_t - w\,R_t)\,(1 - \cos\theta)]}{T_t + (w\,R_t - k\,R^2_t)\,(1 - \cos\theta)}$$

$$= R_t$$

This proves that when there are no concentrated weights or suspension forces upon a length of the envelope and the gross lift is absorbed by weights distributed uniformly around the cross-section, and by shear, the cross-section remains circular.

Another special case occurs when the transverse tension in the fabric is constant all the way around the cross-section, except for changes at the points of attachment of the suspension cables. This case occurs when the gross lift equals the suspended load. The weight of the fabric is then taken by shear,

and equilibrium of the shaded area in Figure 44 requires that T be the same above and below the area. This special case is most likely to occur in semirigid airships, particularly if the weight of the fabric is neglected, because the tensions in the suspension cables are adjusted to act with the keel to distribute the loads along the length of the envelope in proportion to the lift. The product, $R\,p = T$, is then constant over the cross-section above the points of attachment of the suspension cables to the envelope, or of the envelope to the keel in semirigid airships, and since p is proportional to the height, the curve of the cross-section is the lythenary:

$$R = \frac{C}{y} \quad \text{or } R = \frac{T}{p}$$

The value of the constant T may be found by trial and error, plotting curves with assumed values of T and p_t until a curve is found having the known perimeter and fulfilling the necessary conditions. In a nonrigid airship without internal rigging these conditions are that the ends of the curve be normal to the vertical center line; and that at the point of attachment of the car suspension lines to the envelope the value of T changes by the amount necessary for equilibrium with the suspension force. In a semirigid airship the position at which the lower end of the curve terminates on the keel is a known condition. The mathematical solution of this case has been obtained by W. Watters Pagon, and is fully described in "Pressure Airships" by Blakemore and Pagon (Ronald Aeronautic Library). Pagon has greatly simplified the task of plotting the shape of the cross-section, by means of an ingenious series of curves which can be used to solve almost any case for semirigid airships.

As an alternative to the mathematical solution of the problem, the mechanical method proposed by the Italian designer, Colonel Crocco, may be used. This is based on the principle that the radius of curvature of a batten or spline of uniform cross-section made of homogeneous material, and free from initial bends, is inversely proportional to the bending moment in the batten, in accordance with the fundamental equation:

$$\frac{M}{EI} = \frac{1}{R}$$

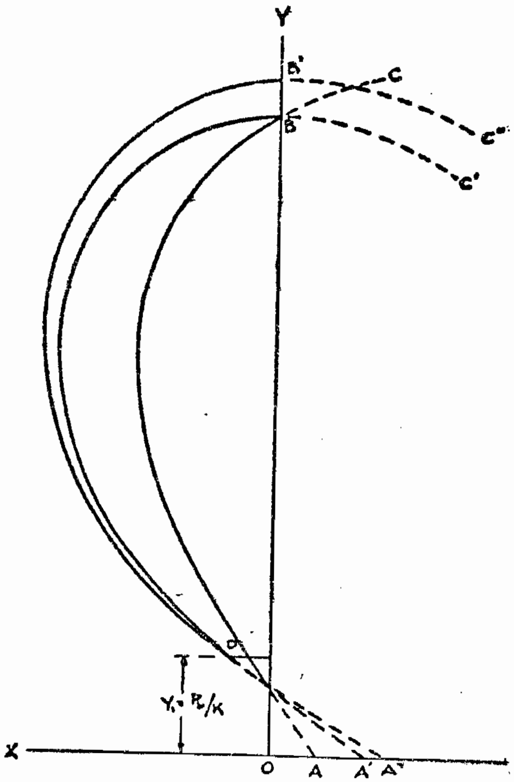


Figure 45. Mechanical Method of Drawing Cross-Sections distorted by Gas Pressure

If in Figure 45 such a batten is held by pins at points A, B, and C, of which A lies on the horizontal axis OX, and B on the vertical axis OY, the bending moment at points on the batten between A and B are directly proportional to the vertical distance from B, and the radius of curvature of the batten is in-

versely proportional to that distance. At A the bending moment is zero, and the radius of curvature is infinity. The curve of the batten between A and B conforms therefore to the equation, $R = C/y$. Suppose that it is desired to draw the cross-section of the envelope of a semirigid airship having no internal rigging. It is assumed that the perimeter of the cross-section and the position of the point D where the envelope intersects the keel, and also the gas pressure at the bottom of the envelope are known. The axis OX is chosen at a distance below the envelope representing the line of zero pressure, i.e., if an appendix opening into the envelope were extended downward, the internal and external pressures would be equal at OX. If p_b is the pressure at the bottom of the envelope, and y_1 the distance from OX to the bottom of the envelope, $p_b = ky_1$, or $y_1 = p_b/k$. If the envelope extended down to OX, the radius of curvature at OX would be infinite, just as it is in the batten. The curve of the cross-section passes through D, and the top is normal to OY; the position of the top is not yet known, but it is assumed to be at B (Figure 45). The pin at A is then moved along OX to some point A', and the pin at C to some point C' such that the batten passes through D and is normal to OX at B. The curve of the batten then necessarily fulfills all the conditions except the perimeter. The perimeter is measured, and if found to be correct, the curve is the one desired. If the perimeter is too small or too great, the point B is moved up or down along OY to B', and the points A and C readjusted to maintain the required conditions at D and B', and the new perimeter measured. By successive approximations in this way, the true curve of the cross-section is found.

Two important special cases have now been investigated. Particular attention should be given to them because the mistake is often made of thinking that the second case, when Rp = constant, is the general case; and the cross-section obtained from that formula is sometimes called the "natural section," and it has even been proposed to build rigid airships with the cross-section curved in that manner. In reality, the first special case, when the lift is taken by shear and loads distributed around the circumference, is most likely to apply to rigid airships, except at the transverse frames.

The General Case of the Shape of the Cross-Section

In the general case when the conditions of neither of the two special cases are fulfilled, the second special case may be made to apply by assuming an equivalent lift of the gas, determined in the following manner:

It is assumed that the known quantities are w, p_t, and the perimeters of the section above and below the attachment of the suspension cables, and the force and direction of the pull of the suspension cables per unit length of the envelope at the cross-section under consideration. In accordance with the principle established by the first special case, the shear and the weight of the envelope produce no distortion of the cross-section. The equivalent unit lift of the gas is equal to the vertical component of the suspension force per unit length of the envelope divided by the area of the cross-section. The shape of the cross-section may then be treated similarly to the second special case, using the equivalent lift instead of the actual lift of the gas. As the suspension force approaches zero, so does the equivalent lift, and consequently the level of zero equivalent gas pressure approaches an infinite distance below the envelope (since p_t is assumed as a given quantity); but in practice, when this level is too far below the envelope to be conveniently handled, the curves of the cross-section are so nearly circular arcs, that they may be drawn as arcs without serious error.

In these methods of finding the shape of the cross-section, either in the general or special cases, each cross-section is considered independently of its neighbors, so that discontinuous longitudinal distribution of the load would theoretically cause abrupt changes in the shapes of successive cross-sections. This

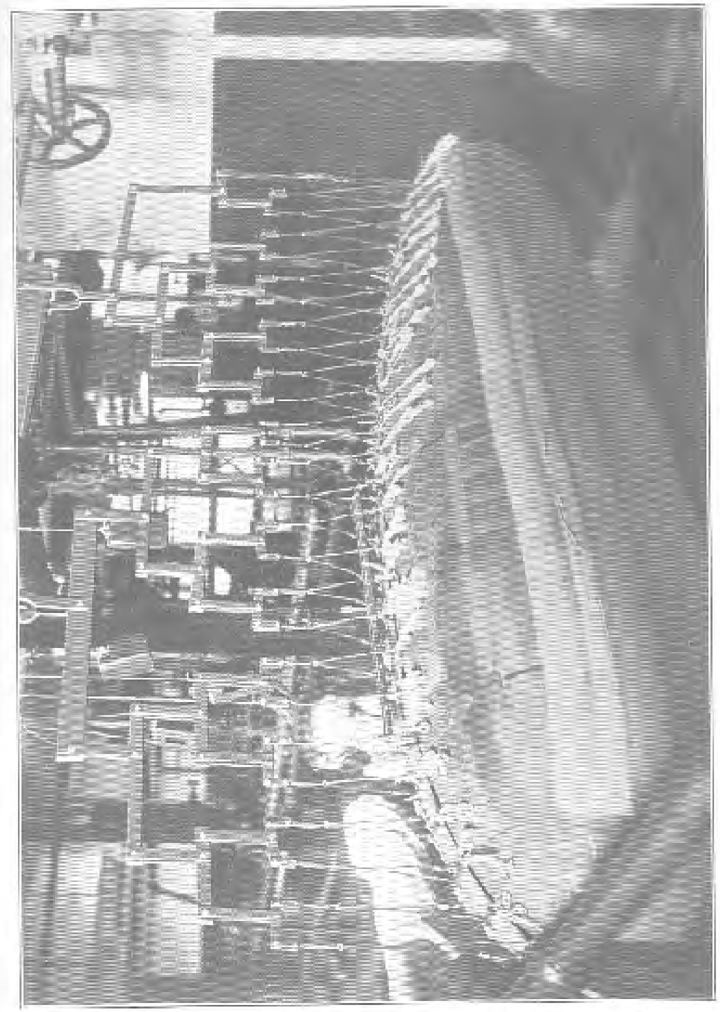


Figure 46. Water Model of Semirigid Airship SS1

is obviously impossible because of the longitudinal stresses which would be produced. The difficulty may be overcome in practical computations by assuming the suspension force per unit length of the envelope is the mean value of the force for a considerable distance forward and aft of the cross-section under consideration.

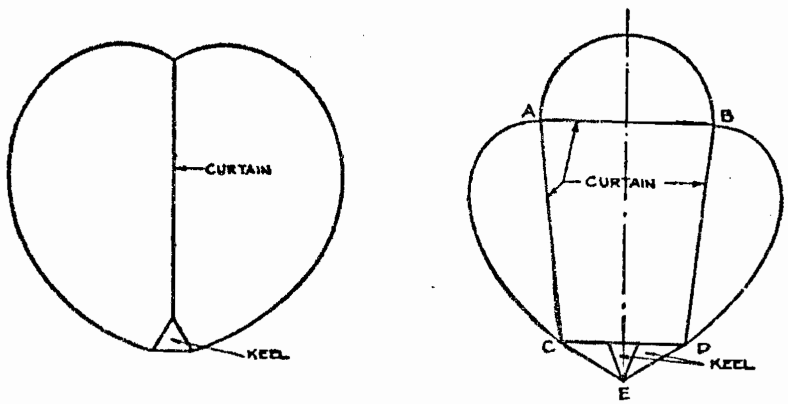
In practice the shape of the cross-section is hardly ever computed elsewhere than at the maximum diameter, unless one of the simple special cases is known to apply.

Breathing Stresses

The greater the internal pressure, the more nearly circular are the cross-sections; and it follows that the shapes of the cross-sections and of the vertical meridian plane through the envelope of a nonrigid or semirigid airship will vary with the gas pressure. The lower the pressure, the higher and narrower are the cross-sections, and the more deeply curved are the vertical meridians. In nonrigid airships, this varying shape introduces no particular stresses; but in a semirigid airship in which a fabric envelope is attached to a rigid keel, the shape of the bottom of the envelope is constrained, and its tendency to change its longitudinal outline with varying pressure produces important stresses in the keel. An investigation with strain gages in the semirigid airship *RS*-1 showed greater stresses were produced in the keel by a half-inch variation in the gas pressure than by the aerodynamic forces in very rough air.

This type of stress was first discovered by W. Watters Pagon, who gave it the name "breathing stress." The subject is fully discussed in "Pressure Airships" by Blakemore and Pagon, (Ronald Aeronautic Library). The mathematical determination of the magnitude of the breathing stress is so difficult that water-model study is almost a necessity in the design of a semirigid airship. Figure 46 shows a water model of the *RS*-1. The method of suspending the model, and the instrument for measuring the curvature of the keel may be noted.

The bi-lobe shape of the *RS*-1 (Figure 47) induces large breathing stresses. A tri-lobe shape which should almost entirely eliminate these stresses is also shown in Figure 47. The



Midship section of *RS*-1 Proposed section for semirigid airship

Figure 47. Cross-Sections of Semirigid Airships

vertical curtains from the lobe-ridges *A* and *B*, and the horizontal curtain between them, fix the positions of these points with respect to the keel, so that breathing stresses are induced only by the comparatively minor variations in the curve of the sides of the cross-section *AC* and *BD*. The rigid keel is represented by the triangle *CDE*, which is external to the envelope as in Italian practice, instead of internal as in the *RS*-1.

CHAPTER VII

LONGITUDINAL STRENGTH OF RIGID AIRSHIPS

Calculation methods. The calculation of the stresses resulting from given primary shearing forces and bending moments has been the subject of more literature and discussion than any other problem connected with the design of rigid airships. Exact computations of these stresses are desirable, but owing to the extraordinarily high degree of redundancy of the hull structure, such computations must be based on the principle of least work, or on some equivalent method of deflections involving the elastic properties of the entire structure. The application of such methods to the hull as a whole has been found impracticable owing to the great number of unknown terms and equations involved. Even the exact calculation for the simple case of a hexagonal braced structure, five frame spaces in length, and with symmetrical loading, requires the solution of ten simultaneous equations. Furthermore, the solution involves differences between quantities very nearly equal, so that unless the numerical work is correct to six or seven significant figures, very considerable errors are inevitable in the results. An exact computation of the stresses in an airship structure would be vastly more difficult than in the comparatively simple case of the hexagonal braced tube, even if the modifications introduced by the rigidity of the joints are neglected. Since exact computation appears to be impracticable, an approximate solution and a measure of the degree of precision of the approximation must be sought.

The primary structural members of the hull of a rigid airship are ordinarily the longitudinal and transverse girders, and the diagonal wires, called shear wires. The first Schütte-Lanz airship had girders running spirally around the hull in place of the usual longitudinal girders and shear wires. Owing to the great complexity of the spiral construction, it has never been repeated and it will be neglected in our study. In general, the longitudinal forces due to the bending moments in the hull are opposed chiefly by the longitudinal girders (or more simply,

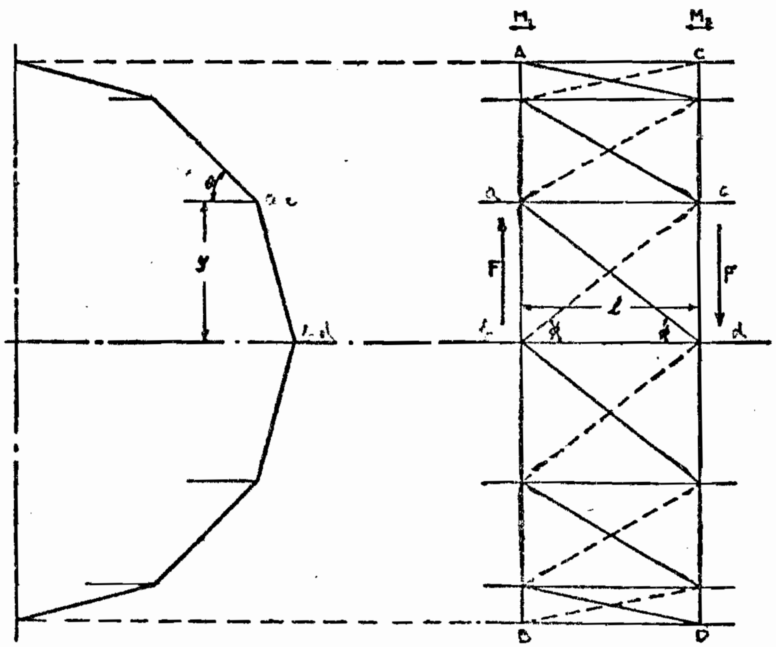


Figure 48. Illustrating Shear and Bending Stresses in Rigid Airships

the "longitudinals"); and the shearing forces are opposed by the shear wires. But the shear wires also play a considerable part in the longitudinal forces; and the longitudinals if carried continuously across the transverse frames offer an appreciable resistance to the shear.

The usual methods of calculating the stresses in the longitudinals and shear wires are based upon the only approximately true assumptions, that plane transverse sections remain plane,

that the longitudinals take all the longitudinal forces, and the shear wires take all the shear. Two methods of computation are fairly well known and have been described by Lewitt and other writers. These may be called the methods by bending moments and by transverse shears. They are analogous to the two methods of calculating the shearing stresses in a nonrigid airship, described in Chapter VI. In the calculation by the bending moment method, the fiber stress and the total forces in the longitudinals are obtained from the bending formula:

$$\frac{M}{I} = \frac{f}{y}$$

And the forces in the shear wires are obtained from the longitudinal shear between the longitudinals.

By the method of shear, the forces in the shear wires due to the transverse shearing forces in the hull are first calculated, and the forces in the longitudinals are obtained by summation of the forces applied by the shear wires at the joints.

The procedure by the two methods will now be described. In Figure 48, let AB and CD be two consecutive main transverse frames in the hull of a rigid airship.

Let L = length of the shear wires
ϕ = inclination of the shear wires to the longitudinals
l = distance between the main transverse frames
β = inclination of the longitudinals between AB and CD to the longitudinal axis of the hull
A = cross-sectional area of a longitudinal girder
M_1 and M_2 = bending moments in the hull at frames AB and CD, respectively
F = transverse shear, assumed constant between frames AB and CD
I_1 and I_2 = moments of inertia of the cross-sections of the hull at frames AB and CD, respectively
y_1 and y_2 = distances of the center of gravity of any longitudinal from the transverse neutral axis of the hull at frames AB and CD, respectively
T = tension in a shear wire

From the bending moment theory, the forces in any longitudinal at frames AB and CD are, respectively: M_1Ay_1/I_1 and M_2Ay_2/I_2. In Figure 48 the longitudinal forces above the panel $abdc$ at these frames are, respectively:

$$\frac{M_1\Sigma Ay_1}{I_1} \text{ and } \frac{M_2\Sigma Ay_2}{I_2}$$

where the summations are taken over all the longitudinals above the panel *abdc*. The difference between these two sets of forces is the longitudinal shear taken by the shear wire *ad* and the corresponding wire on the opposite side of the hull. The longitudinal component of the tension in the shear wire is $T \cos\phi \cos\beta$, and this is equated to the longitudinal shear, giving:

$$2T\cos\phi\cos\beta = \frac{M_1\Sigma Ay_1}{I_1} - \frac{M_2\Sigma Ay_2}{I_2} \quad (64)$$

where, as before, the summations are taken over all longitudinals above the panel *abdc;* and if A is constant between transverse frames:

$$\frac{I_2}{I_1} = \frac{y^2_2}{y^2_1} \text{ and } \frac{\Sigma Ay_2}{\Sigma Ay_1} = \frac{y_2}{y_1}$$

It follows that:

$$\frac{M_1\Sigma Ay_1}{I_1} - \frac{M_2\Sigma Ay_2}{I_2} = \left(M_1 - M_2\frac{y_1}{y_2}\right)\frac{\Sigma Ay_1}{I_1}$$

and

$$T = \frac{\left(M_1 - M_2\frac{y_1}{y_2}\right)\Sigma Ay_1}{2I_1\cos\phi\cos\beta} \quad (65)$$

By the theory of bending moments the integration of the transverse shear along the hull gives the magnitude of the bending moment, so that $M_1 - M_2 = Fl$. In the parallel portion of the hull $y_1/y_2 = 1$, and $\cos\beta = 1$, so that equation (65) may be reduced to:

$$T = \frac{Fl\Sigma Ay}{2I\cos\phi} \quad (66)$$

This is the expression for the tensions in the shear wires by the bending moment method.

For the method of transverse shear, the following notation is added to that given above for Figure 48:

Let Δy = small displacement of frame CD parallel to frame AB due to the action of the shearing force F

ΔL = increase of L due to the small movement Δy

θ = transverse inclination of a panel to the horizontal

a = cross-sectional area of a shear wire

E = modulus of elasticity of the material of the shear wires

The displacement Δy has a component $\Delta y \cos\theta$ normal to the panel, and a component $\Delta y \sin\theta$ parallel to the panel. Only the component $\Delta y \sin\theta$ throws tension into the shear wire. From the elastic properties of materials:

$$T = \frac{E\,a\,\Delta L}{L}$$

And by geometry:

$$\Delta L = \Delta y \sin\theta \sin\phi \quad (67)$$

Hence:

$$T = \frac{E\,a\,\Delta y \sin\theta\sin\phi}{L} \quad (68)$$

It is assumed that the movement Δy is wholly due to the yielding of the shear wires, and hence the work done by the shearing force may be equated to the work done in the wires; i.e.,

$$F\,\Delta y = \Sigma T\,\Delta L \quad (69)$$

where the summation is taken over all the shear wires between the frames or transverses AB and CD

From equations (67), (68), and (69):

$$F = E\,\Delta y\,\Sigma\frac{a}{L}\sin^2\theta\sin^2\phi$$

or

$$\Delta y = \frac{F}{E\Sigma\frac{a}{L}\sin^2\theta\sin^2\phi} \quad (70)$$

From equations (68) and (70):

$$T = \frac{F\,a\sin\theta\sin\phi}{L\Sigma\frac{a}{L}\sin^2\theta\sin^2\phi} \quad (71)$$

If L and ϕ are the same in all the panels between the frames considered, equation (71) may be reduced to:

$$T = \frac{F\,a \sin\theta}{\sin\phi \Sigma a \sin^2\theta} \qquad (72)$$

And if a is also constant, the expression may be still further reduced to:

$$T = \frac{F \sin\theta}{\sin\phi \Sigma \sin^2\theta} \qquad (73)$$

This is the method of transverse shears for calculating the tensions in the shear wires. To find the stresses in the longitudinals it is necessary to go through the tedious process of summing the forces applied by the shear wires to the longitudinals at the joints with the transverses.

Equations (66) and (73) for the tensions in the shear wires depend upon different sets of variables and do not, in general, give the same values of T.

Both the shearing and bending methods have had their advocates. Colonel Crocco of the Italian Central Aeronautic Institute adopted the bending method. E. J. Temple, formerly of Vickers, Ltd., favored the shearing method. The late C. I. R. Campbell, designer of the *R*-38, adopted the compromise procedure of applying the bending theory for the stresses in the longitudinals, and the shear theory for the stresses in the shear wires. It is believed that E. H. Lewitt published the first discussion of both these methods of calculating the longitudinal strength of rigid airships. His articles on the subject appeared in the British aviation magazine, *Aeronautics*, during 1919 and 1920.

Modification of the bending theory. The U. S. Navy Department retained Professor Wm. Hovgaard, of the Massachusetts Institute of Technology, in an attempt to place the theory of the longitudinal strength of rigid airships on a firmer basis than was evinced by the discordant theories of shear and bending. Professor Hovgaard declared himself in favour of the bending theory; but he pointed out that there was a fundamental error in the assumption that the longitudinals take all

the bending forces; and he showed that the lengthening or shortening of the panels, due to the bending of the hull, throws forces into the shear wires, and the longitudinal components of these forces contribute to the strength of the hull against bending. Professor Hovgaard suggested that the assistance given to the longitudinals by the shear wires be calculated by assuming fictitious bars coincident with the longitudinals, and of the same material, and to be included in the calculation of the moment of inertia of the cross-section of the hull. He showed that the areas of the fictitious bars could be computed by the formula:

$$A_f = qa \cos^3\phi \qquad (74)$$

where

A_f = the cross-sectional area of a fictitious bar

q = the ratio of the modulus of elasticity of the material of the wires to that of the girders

a = the cross-sectional area of the wire or wires on one side of a joint

ϕ = the inclination of the wires to the longitudinals

Professor Hovgaard has published his views in a paper, "The Longitudinal Strength of Rigid Airships," read before the American Society of Naval Architects and Marine Engineers in November, 1922.

It will now be shown that if allowance be made for the bending forces taken by the wires, the actual stresses in both the longitudinals and the wires will lie between the limits determined by the methods of shear and bending.

Since the forces in the longitudinals are due to the forces applied by the shear wires at the joints, it is possible to choose shear wires proportioned to any system of longitudinals in such a manner that the tensions in the wires will so load the longitudinals that the frames remain plane.

When there is no distortion of the frames, the conditions are satisfied for the stresses in the longitudinals (and in the wires due to the bending) to be correctly given by the bending theory, and for the stresses in the wires due to the transverse shear to be correctly given by the shear theory. Since the forces at the joints must be in equilibrium, the stresses in the wires in this case are also in accordance with the longitudinal shear as determined by the bending theory; and the stresses in the girders are in accordance with the forces applied by the wires to the joints as determined by the shear theory. It follows that when the sizes of girders and wires are so proportioned to each other that the frames remain plane, the stresses are correctly given by either the shear or bending theory.

Suppose now that a longitudinal be replaced by a new one of different cross-sectional area without other change in the structure. The new longitudinal will yield either more or less than the old one, under the forces applied by the shear wires, according to whether it is smaller or larger than the old one. If it is larger and yields less, the unit stress and total force will be less than by the bending theory; and conversely if it is smaller. At the same time the smaller yielding of the larger longitudinal will cause an increase of the forces applied to it by the shear wires, so that the total force in the longitudinal will be greater than by the shear theory.

It is thus evident that if the new longitudinal is larger than the old one without other change in the structure, the force in it will be less than by the bending theory and more than by the shear theory; and conversely if it is smaller.

By a similar argument, it may be shown that the tensions in the wires lie between the limits determined by the shear and bending theories. No precise rule is known for determining where the stresses will lie between these limits; but, in general, large bending and shearing forces are not found in the same frame space, and where the bending moment is relatively more important it may be expected that the stresses in the longitudinals will be more nearly in accordance with the bending than the shear theory. Where the shearing forces are relatively more important it may be expected that the tensions in the shear

wires will approximate more closely to the shear theory. As we are most concerned with the girder stresses where the bending is large, and with the wire stresses where the shear is large, the practice of calculating the girder stresses by the bending theory and the wire stresses by the shear theory has much to recommend it, although it is likely to lead to an overestimate of the maximum stresses.

Table 15 shows a typical calculation of the tensions in the shear wires of a frame bay by means of the shear theory. The tensions are expressed in terms of the total transverse shear F. Tables 16 and 17 show the application of the bending theory to the calculation of the stresses in the longitudinals in terms of the bending moment M.

TABLE 15. STRESSES IN THE SHEAR WIRES IN TERMS OF THE SHEARING FORCE F, AT THE MIDSHIP SECTION OF THE U.S.S. *Shenandoah*

Panel	a in.^{2}	Θ deg.	L in.	sin Θ	sin ϕ	Q in.	T/F
	MAIN SHEAR WIRES						
AC	.00817	14.25	229	.2462	.5075	.56	.0308
CE	.00817	42.75	229	.6788	.5075	4.35	.0850
EG	.01021	71.25	229	.9469	.5075	10.32	.1482
GI	.01021	80.25	229	.9855	.5075	11.20	.1542
IK	.01021	51.75	229	.7854	.5075	7.10	.1228
KM	.00817	23.25	229	.3947	.5075	1.44	.0494
MN	.01021	57.3	362	.8415	.8480	14.05	.1354
	SECONDARY SHEAR WIRES						
AC	.00515	14.25	305	.2462	.7638	.60	.0220
CE	.00515	42.75	305	.6788	.7638	4.54	.0604
EG	.00515	71.25	305	.9469	.7638	8.83	.0842
GI	.00515	80.25	305	.9855	.7638	9.56	.0878
				$\Sigma Q =$	2 ×	72.55	= 145.1

$$Q = 1{,}000{,}000 \times (a/L) \sin^2\Theta \sin^2\phi$$

$$\frac{T}{F} = \frac{a \sin\Theta \sin\phi}{L\, \Sigma Q/1{,}000{,}000}$$

Table 16. Moment of Inertia of the Midship Section of U. S. S. *Shenandoah*

Longitudinal	Type No.	Area in.²	Height m.	H^2 m²	AH m. in.²	AH^2 m.² in.²
A	56	.166	11.82	139.71	1.96	23.19
B	21	.211	11.53	132.94	2.43	28.04
C	54	.287	10.39	107.95	2.98	30.98
D	21	.211	8.74	76.39	1.84	16.12
E	54	.287	6.44	41.47	1.85	11.90
F	21	.211	3.83	14.67	.808	3.095
G	54	.287	0.93	0.86	.267	.247
H	91	.211	− 2.02	4.08	.426	.861
I	54	.287	− 4.81	23.14	− 1.380	6.641
J	91	.211	− 7.34	53.88	− 1.55	11.37
K	54	.287	− 9.38	87.98	− 2.69	25.25
L	92	.258	− 10.83	117.29	− 2.79	30.26
M	55	.305	− 11.67	136.19	− 3.56	41.54
N	43	.161	− 8.76	76.74	− 1.41	12.36
O	31	.111	− 11.64	135.49	− 1.29	15.04
P	72	.336	− 9.66	93.32	− 3.25	31.36
		3.827			− 5.36	288.25

Neutral axis is 5.36/3.827 = 1.40 m. below the geometrical axis. Moment of inertia, I, about neutral axis is

$$I = 2\,[288.25 - (1.40^2 \times 3.827)]$$
$$= 561.50 \text{ m.}^2 \times \text{in.}^2$$

Note: The measurement of the major linear dimensions in meters, and the areas of the girders in square inches is very convenient in spite of its strange appearance. The bending moments are measured in meter-pounds, and the stresses come out in pounds per square inch.

Experimental determination of the stresses in the longitudinals. The Bureau of Aeronautics has had the stresses in a celluloid model of the *Shenandoah* determined photoelastically. In this method of investigation, polarized light is passed through the members to be studied under load; and the celluloid transforms the polarized light to colors, depending on the intensity of the stress. The technique of this method of experimentally determining stresses has been so well developed that great accuracy can be attained. The applicability of the model results to full scale in an indeterminate structure depends, of course, on securing elastic similarity by carefully proportioning the parts of the model.

Table 17. Stresses in Longitudinals at the Midship Section of U.S.S. *Shenandoah* in Terms of the Bending Moment M

Longitudinal	Area in.²	Height from Neutral Axis m	Unit Stress lb./in.²	Total Load lb.
A	.333	13.22	.0235	.00783
B	.211	12.93	.0230	.00485
C	.287	11.79	.0210	.00603
D	.211	10.14	.0181	.00382
E	.287	7.84	.0140	.00401
F	.211	5.23	.0093	.00196
G	.287	2.33	.0041	.00118
H	.211	− 0.62	−.0011	−.00023
I	.287	− 3.41	−.0061	−.00175
J	.211	− 5.94	−.0106	−.00224
K	.287	− 7.98	−.0142	−.00408
L	.258	− 9.43	−.0168	−.00434
M	.305	− 10.27	−.0183	−.00558
N	.323	− 7.36	−.0131	−.00423
O	.222	− 10.24	−.0182	−.00404
P	.336	− 8.26	−.0147	−.00494

Unit stress (*pounds/inches*²) = bending moment (*meters* × *pounds*) × height from neutral axis (*meters*) ÷ moment of inertia (*meters*² × *inches*²).

In this table, M = one meter-pound.

The celluloid model of the *Shenandoah*, as originally constructed, represented seven main frame bays of parallel middle body. It was found that the stresses in the longitudinals approximated to the shear theory. One end of the model was then completed to represent the stern of the ship, and loaded comparably to air forces on the tail surfaces. It was found that the added length, and the constraint imposed by the tapered end on the distortions of the frames caused the stresses to be more nearly in accordance with the bending theory.

Experiments with strain gages on the longitudinals of the U.S.S. *Shenandoah* and *Los Angeles* have shown that the stresses resulting from the aerodynamic forces in flight approximate to the bending theory.

Inverse ratio method. A procedure, to which the name "inverse ratio method" may be applied, takes better account of the interaction of the longitudinal members and the shear wires than any of the methods previously described. It is based on the principle that when a given external load is borne by n systems, each of which is statically determinate, these systems divide the load between them in inverse ratio to their internal work when each alone carries the load. In mathematical symbols:

$$q_1 Q_1 = q_2 Q_2 = \dots\dots\dots = q_n Q_n$$

and

$$q_1 + q_2 + \dots\dots\dots + q_n = 1$$

where Q = the internal work in any system when alone carrying the given load; and q = the fraction of the given load carried by any one of the statically determinate systems when all are acting. The subscripts are used to designate the different systems.

It is obvious that the sum of the q terms is one. A proof that $q_1 Q_1 = q_2 Q_2$ is as follows: The work in a structure not strained beyond the proportional limit varies as the square of the applied load. Therefore the work in each of the statically determinate systems when all are acting is $q^2_1 Q_1$, $q^2_2 Q_2$, etc. The work in each system is also $q_1 Fx/2$, $q_2 Fx/2$, etc., where F is the total applied load, and x is the deflection of the system at the point of application of F. Combining these two sets of expressions for the work in each system:

$$q^2_1 Q_1 = q_1 Fx/2$$
$$q^2_2 Q_2 = q_2 Fx/2$$
etc. etc.

Dividing through by the q terms, the right hand expressions all become $Fx/2$; and $q_1Q_1 = q_2Q_2$, etc., which proves the proposition.

An alternative proof that the load taken by any two statically determinate systems is divided between them in inverse ratio to their internal work when each alone takes the load, may be derived from the principle of least work, as follows:

When there are two systems sharing a given load, $q_2 = 1 - q_1$; and as before, the total work is

$$W = q^2_1 Q_1 + q^2_2 Q_2$$
$$= q^2_1 Q_1 + (1 - q_1)^2 Q_2$$

$$\frac{dW}{dq_1} = 2q_1 Q_1 + (2q_1 - 2) Q_2$$
$$= 0 \text{ for minimum strain energy,}$$

or $$q_1 = \frac{Q_2}{Q_1 + Q_2} \quad (75)$$

Similarly $$q_2 = \frac{Q_1}{Q_1 + Q_2} \quad (76)$$

Dividing (75) by (76),

$$\frac{q_1}{q_2} = \frac{Q_2}{Q_1}$$
$$q_1 Q_1 = q_2 Q_2$$

Application of the inverse ratio method to longitudinal strength calculations. The inverse ratio method is strictly applicable only when there are no members common to any two of the statically determinate systems into which the redundant structure is resolved, so that the systems do not react upon each other except by way of sharing the external load. When there are members common to two or more of the statically determinate systems, it may be assumed to a good order of approximation that the total stress in such a member is the sum of the stresses to which it is subject as a member of each of the systems to which it belongs.

In applying the method to the longitudinal strength calculations of rigid airships the structure may be divided into systems, each consisting of a pair of longitudinal girders extending over one or more main frame spaces with the connecting transverse and diagonal members necessary to make a statically determinate system.

In a typical case it was found that if a load was applied at one main frame space away from a cross-section under investigation, 78% of the resulting internal work was in the shear wires, 19% in the longitudinal girders, and 3% in the transverse; but with a load applied at four main frame spaces away, only 21% of the resulting internal work was in the shear wires, 78% in the longitudinals, and 1% in the transverses. From these figures it is evident that the sizes of the shear wires are of much greater importance than the sizes of the longitudinals in determining the distribution of the load among the panels at a short distance from the point of application of a force, and hence a high order of accuracy may be expected from the shear theory for this condition; but as the distance from the point of application of the force increases, the sizes of the longitudinals become more important, and are finally the dominant factor, and the stresses are then more in accordance with the bending theory.

Computation of the longitudinal strength by the inverse ratio method is too long to be illustrated by an example here. The application of the method is shown in Chapter VIII, where it is used to calculate the distribution of the load between two statically determinate systems in a main transverse frame.

Use and computation of the quantity $\Sigma \sin^2 \theta$ in airships having regular polygonal cross-sections. In a rigid airship having longitudinals of equal sizes placed at the apexes of regular polygonal cross-sections, the moment of inertia of any cross-section is given by

$$\begin{aligned} I &= \Sigma a y^2 \\ &= a \, \Sigma y^2 \\ &= a R^2 \, \Sigma \sin^2\theta \end{aligned} \qquad (77)$$

where

I = moment of inertia

a = cross-sectional area of each longitudinal (assumed constant)

y = distance of the C. G. of any longitudinal from the diameter about which I is taken

R = radius of the cross-section of the hull through the C. G.'s of the longitudinals

θ = angle between the radial line through any longitudinal and the diameter about which I is taken

$\Sigma \sin^2\theta$ also enters into the calculations of the tensions in the shear wires due to the transverse shear in an airship having regular polygonal cross-sections with shear wires of constant size throughout any one frame space. The formula for the tensions in the shear wires in this simple case is

$$S = \frac{F \sin \theta}{\sin \phi \, \Sigma \sin^2\theta} \qquad (78)$$

where S = tension in any shear wire

F = total transverse shear

ϕ = inclination of the shear wires to the longitudinals

θ = inclination of the panel of any shear wire to the normal to the plane of shear and bending (i.e., the inclination of the panel to the horizontal in the usual case of bending in the longitudinal vertical plane)

Such simple cases of regular structures never occur in practice, and the irregularity due to the keel is especially to be considered; but for the purposes of first approximations to the stresses in the wires and longitudinals due to the primary shear and bending, these formulas for simple cases are often useful.

It will now be shown that in a regular polygonal structure of n sides

$$\Sigma \sin^2\theta = n/2$$

In such a structure when n is even, the panels or the radial planes through the longitudinals may be divided into $n/2$ pairs, each pair consisting of panels or radial planes having complementary values of θ. Or, in other words, for every panel or radial plane inclined at the angle θ to the plane of bending, there is another panel inclined at $\pi/2 - \theta$; and

$$\begin{aligned} &\sin^2\theta + \sin^2(\pi/2 - \theta) \\ &= \sin^2\theta + \cos^2\theta = 1 \end{aligned}$$

Therefore, the summation of $\sin^2\theta$ for these $n/2$ pairs of panels or radial planes equals $(n/2) \times 1$, or

$$\Sigma \sin^2\theta = n/2 \qquad (79)$$

When n is odd, the panels or radial planes cannot be divided into pairs in which the values of θ are exactly complementary; but if the odd number is 17 or greater, as it always is in practice, the error is very small.

It follows that

$$I = aR^2n/2$$
$$= AR^2/2 \qquad (80)$$

where A is the sum of the cross-sectional areas of all longitudinals.

An interesting corollary is that the moment of inertia is independent of the number of sides when the total cross-sectional area of the longitudinals is constant.

Assuming that the end loads in the longitudinals due to the primary bending are distributed in accordance with the simple bending theory, the fiber stress in any longitudinal of the simple structure under consideration is given by:

$$f = \frac{My}{I} = \frac{2M \sin \theta}{a R n}$$
$$= \frac{2M \sin \theta}{AR} \qquad (81)$$

In the most severely loaded longitudinal, $\sin \theta = 1$, and

$$f = \frac{2M}{a R n} = \frac{2M}{AR}$$

The total end load in any longitudinal is fa, and is given by:

$$fa = \frac{2 M \sin \theta}{Rn}$$
$$= \frac{2 M}{Rn} \text{ (in the most severely loaded longitudinal)}$$

Similarly, the tension in any shear wire is given by:

$$S = \frac{2 F \sin \theta}{n \sin \phi} \qquad (82)$$

These equations obviously furnish very rapid and simple means for a first approximation to the forces to be expected in the longitudinals and shear wires of an airship subjected to given primary shearing and bending forces.

Secondary Stresses Resulting from the Primary Shearing Forces

The longitudinal girders are continuous over transverse frames to which they are fastened by rigid joints, and therefore

must bend slightly to follow the contour of the axis of the ship when the latter bends as a whole. Under the theory of bending we assume that the main frames remain plane, but rotate under the action of the primary bending moments. The additional or secondary stress thrown into the longitudinal girders, through the rigid joints, by rotation of the main frames, could be allowed for in the calculation of the primary stresses if we used the distances of each channel element of a girder from the neutral axis of the ship, instead of taking an average distance for the girder. However, since the depth of any girder near the top or bottom of the ship is small with respect to its distance from the neutral axis, and since we are interested here only in these highly stressed girders, the refinement obtained by allowing for this secondary stress is of no importance and is neglected.

The secondary stresses due to the relative movements of the main transverses under the primary shearing forces remain to be considered. W. W. Pagon has given a very complete treatment of the secondary stresses in connection with the review of the design of the *Shenandoah* by the special committee of the National Advisory Committee for Aeronautics. He assumed that the transverse girders offer no resistance to torsion, but that they resist rotation of the joints by a kind of portal action. In order to determine the secondary stresses exactly, it would be necessary to consider the shearing movements of the transverses over the entire length of the hull, and the calculations would be very laborious. Mr. Pagon obtains an approximate solution by considering that the longitudinals are normal to the main frames next forward and aft of the one at which the secondary stresses are desired. His treatment of the problem is set forth in Technical Note No. 140 of the National Advisory Committee for Aeronautics.

The following analysis divides the secondary bending of the longitudinals into two parts; one in a radial plane of the hull, including the longitudinal axis of the ship and a longitudinal girder; and the other in a longitudinal plane, designated the tangential plane, including a longitudinal girder and normal to the radial plane. At the top and bottom of the hull (when the primary bending is in the vertical plane) the secondary bending is wholly radial, and is wholly tangential only at the mid-height of the hull. Radial secondary bending occurs, therefore, where the primary stresses are greatest and tangential bending where they are least. Moreover, the depth of the longitudinals is much greater than the breadth, so that the maximum secondary stresses in the radial plane are much larger than the maximum in the tangential plane. For these reasons the secondary stresses in the tangential plane may be neglected.

Since the transverse girders offer practically no resistance to torsion, the longitudinals may be considered as continuous in the radial plane over the joints, but not fixed. The secondary stresses will then be the same on both sides of a joint. Assuming the longitudinals to be normal to the main transverses next forward and aft of the one at which the secondary stresses are desired, it follows from the well-known relations between stress and deflection in a fixed-ended beam having the ends at different levels and carrying a central load that the secondary stress is given by

$$s = 3\,Ey\,(\Delta_1 + \Delta_2)/b^2 \qquad (83)$$

where

s = secondary stress in a channel of a girder
y = the distance of the channel from the neutral axis of the girder
b = distance between main frames
Δ_1 and Δ_2 = the radial movements of the girder at the main frames next forward and aft, respectively, relative to the frame at which s is taken

From the transverse shear theory of the primary stresses, the relative movement between consecutive main frames is given by

$$d = Sb/E\Sigma a \sin^2\phi \cos\phi \sin^2\theta$$

where d = the relative movement, and the other symbols have the same significance as in the transverse shear theory (page 157).

It follows that

$$\Delta_1 + \Delta_2 = (S_1 - S_2)\,b \cos u/E\Sigma a \sin^2\phi \cos\phi \sin^2\theta$$

where S_1 and S_2 are the transverse shearing forces forward and aft, respectively, of the frame under consideration, and u is the angle between the vertical and the radial plane through the longitudinal girder.

The secondary stress is given by

$$s = 3y(S_1 - S_2) \cos u/qb\Sigma a \sin^2\phi \cos\phi \sin^2\theta \qquad (84)$$

where q is the ratio of the modulus of elasticity of steel to that of duralumin.

As an example of the maximum secondary stress due to shear likely to occur in the *ZR*-1, the case of full load with a deflated gas cell between frames 80 and 90 may be taken. In this condition the transverse shear changes at frame 90 from 6,754 lbs. to -6,457 lbs. The given quantities are then

$$S_1 = -6{,}457 \text{ lbs.}$$
$$S_2 = 6{,}754 \text{ lbs.}$$
$$q = 2.86$$
$$b = 394 \text{ in.}$$
$$\Sigma a \sin^2\phi \cos\phi \sin^2\theta = .0571 \text{ in.}^2$$

And for the *A* longitudinal

$$y = 8.14 \text{ in. (apex channel)}$$
$$\cos u = 1.0$$

The secondary stress in the open channel of the *A* longitudinal is then given by

$$s = 3 \times 8.14\,(-6{,}457 - 6{,}754)/2.86 \times 394 \times .0571$$
$$= 5{,}000 \text{ lbs./in.}^2$$

Spacing of the Longitudinal Girders and the Transverse Frames

The spacing of the main transverse frames of a rigid airship is determined largely by the degree of gas space subdivision required. The greater the subdivision, the greater the safety, but the greater the weight of the structure and gas cells. Modern practice shows from 12 to 20 main frames and gas cells. It is possible that with rigid airships of lower slenderness ratios than are now common, subdivision of the gas space will be obtained partly by longitudinal diaphragms within the gas cells in addition to the separation of the cells by the main frames, thus permitting some reduction in the number of the latter.

The spacings of the longitudinals and the intermediate transverses are interrelated by the fact that the more numerous the longitudinals, the lighter and more slender they are individually, and hence the closer should be the supporting frames. It is also desirable to have the spacings such that the shear wires can be inclined as nearly as possible at 45° to the longitudinals and transverses, this being their most efficient angle, as is shown on page 177. The best slope of the shear wires may be obtained by having equal spacings of longitudinals and transverses, or by having the longitudinals half as far apart as the transverses, with the shear wires crossing the longitudinals halfway between frames.

The least expensive and lightest structure for its strength is obtained by having a small number of stout longitudinals, with complete elimination of the intermediate frames. On the other hand, the outer cover is best supported, and the maximum gas volume is obtained by having a reasonably large number of not too deep longitudinals. The table on page 43 of Chapter III shows the ratios of the areas of regular polygons of various numbers of sides to the area of the circumscribed circle. It may be noted that very little area is gained by having more than about 21 sides; but on the other hand, with less than 13 sides, the loss of area begins to be serious; and the deeper longitudinals also encroach on the gas space.

Examples of airships with close and open spacings of the longitudinal and transverse girders are afforded by the U. S. Navy's *Los Angeles* and the new British 5,000,000 cu. ft. airships. In the former, which was designed and built by the Zeppelin Company, there are 24 lines of longitudinals at 11.9 ft. apart amidships, with main frames at 49.2 ft., and intermediate

frames at 16.4 ft. apart. According to the published descriptions of the British airships, there are 17 lines of longitudinals at about 24 ft. apart, with main frames every 60 ft., and no intermediate frames. Moreover, the longitudinals of the *Los Angeles* are only about 10 in. deep, and the gas cell netting is permitted to bulge to within about 4 in. of the outer cover midway between the longitudinals; whereas in the British ships, the longitudinals are about 3 ft. in depth; in order to relieve their long unsupported span from gas pressure loads, the netting is secured to the transverse frames without touching the longitudinals, and the lift of the gas is transmitted directly by the netting to the main frames. The British construction saves the weight of the intermediate frames, and gives much less total weight of the longitudinals by the elimination of gas pressure loads and stresses from the distortion of the intermediate frames.

It is believed that the saving in weight of girders effected by the British type of construction is approximately offset by the heavier netting required, and the loss of gas lift. The advantage of less cost and greater facility of construction remains with the more open spacing adopted by the British; but the scale may finally be turned in favor of the Zeppelin type by the better support it gives to the outer cover.

Main and secondary longitudinals. Some rigid airships have two types of longitudinals, designated main and secondary. This scheme was introduced by the Zeppelin Company in 1916, with the object of obtaining a 25-sided ship in which there were only 13 main longitudinals taking part in the primary strength of the structure. Midway between each main longitudinal, except the two bottom ones, there was a light secondary longitudinal to support the outer cover and assist in taking gas pressure loads. It was found impracticable to make the secondary longitudinals less than about two-thirds the weight of the main longitudinals; and it was a natural step to modify the construction by securing shear wires to the secondary longitudinals so that they assisted the main ones in the primary strength of the ship, and thus the two types of longitudinals became identical in

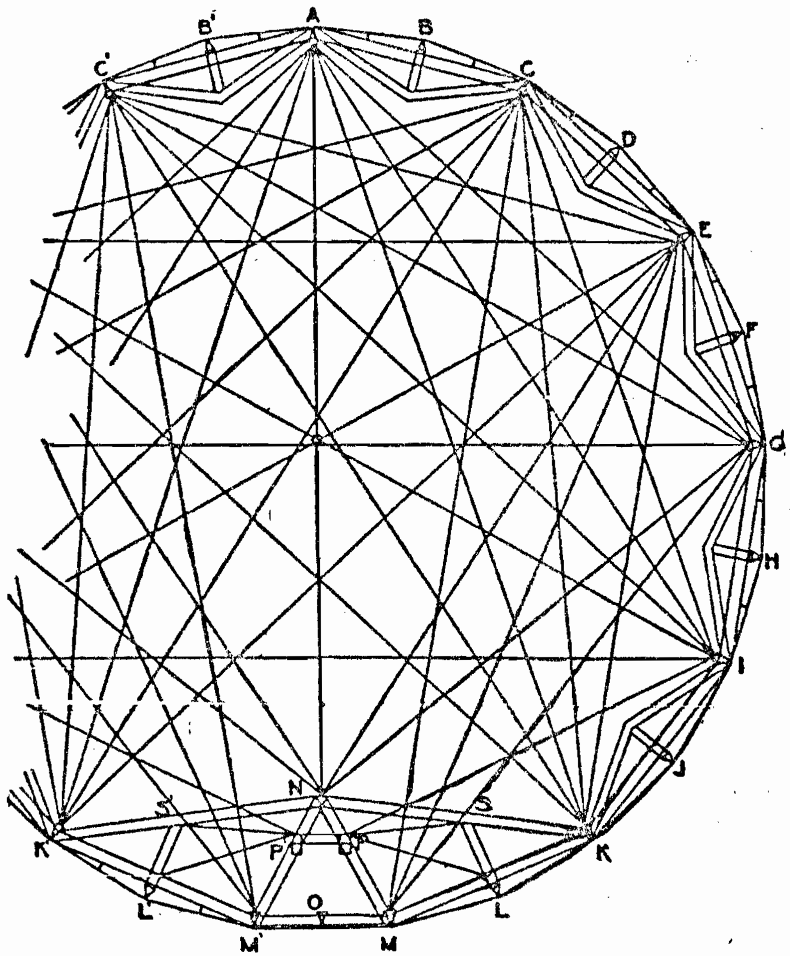


Figure 49. A Main Transverse Frame of the *ZR-1* (U.S.S. *Shenandoah*)

function although differing in size. In the *Los Angeles*, all longitudinals are equal in size, and the distinction of main and secondary longitudinals no longer remains. In Zeppelin airships

having 13 main longitudinals and 12 secondary ones, the main frames were 13-sided structures with king-posts and diagonal bracing girders as shown in Figure 49. The king-posts were carried through the main members of the transverse frames and supported the secondary longitudinals. The transverse wires were secured at the joints with the main longitudinals only. When the distinction between main and secondary longitudinals was abandoned in the *Los Angeles*, it was considered advisable to avoid the complicated and excessive redundancy resulting from a large number of sides in the main frames, so that in this airship these frames are 12-sided structures, each side or element consisting of a diamond-shaped truss as shown in Figure 1 (page 4), with longitudinal girders at the joints between the elements, and at the middle of each element where two sides of the diamond intersect. Transverse wires are secured only to the joints between the elements.

The Wiring of Rigid Airships

The main systems of wiring in rigid airships are the shear wiring in the quadrilateral panels formed by the longitudinal and transverse girders, the cross-wiring in the main transverse frames, and the gas cell netting. High tensile steel is the best material for the wiring. Its ultimate tensile strength is about 240,000 lbs./in.^{2} as compared with 55,000 lbs./in.^{2} for duralumin. The ratio of the ultimate tensile strength to the specific gravity is about 31,000 for high tensile steel as compared with 20,000 for duralumin. The smaller cross-sectional area of steel for equal weight is no disadvantage in tension members, although it is extremely serious in compression members. The temperature coefficient of expansion of duralumin is 1.78 times that of steel, and consequently the tensions of the wires, and the compressive forces they put upon the duralumin girders are influenced to some extent by the temperature. This temperature effect has been investigated by E. H. Lewitt who published his analysis in an article entitled "Temperature Stresses in the Rigid Airship" in the British aviation magazine, *Aeronautics*, for July 21, 1921. Mr. Lewitt showed that a temperature rise of 50° F. increased the tensions in the steel wires by some 2,000 to 3,000 lbs./in.² and increased compressive forces in the longitudinal girders by 310 lbs./in.² and in the main transverse girders by 770 lbs./in.². These temperature stresses are not at all serious, and are neglected in strength calculations, although it might be advisable to consider them in airships designed to operate in climates varying widely in temperature.

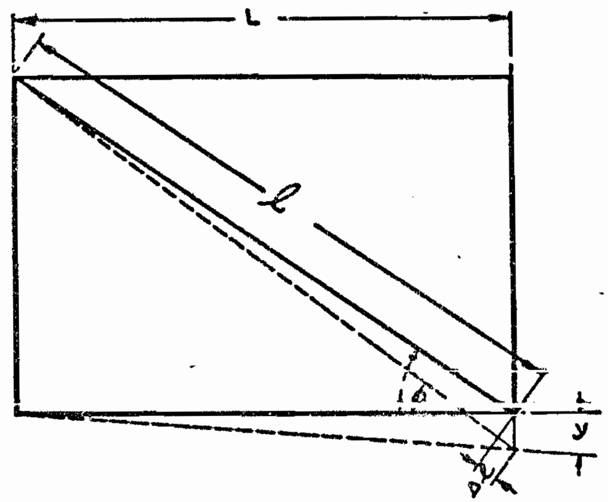


Figure 50. Illustrating Tension in Shear Wire Due to Parallel Movement of Consecutive Transverse Frames

Best slope for shear wires. Designers have exercised a wide range of choice for the inclination of the shear wires to the longitudinal and transverse girders. The following investigation shows that the greatest efficiency is obtained when the slope is 45°.

Referring to the rectangular panel of length L, shown in Figure 50, subjected to a given transverse shearing force F, the slope of the wire will be considered from the point of view of minimum shearing deflection of the panel, and also in regard to the least weight of wire.

Let T = total tension in the wire
t = tension per unit area
a = cross-sectional area
l = length of the wire
ϕ = inclination of the wire to the longitudinal
Δl = stretch of the wire
y = transverse deflection of the outboard end of the panel
E = modulus of elasticity of the material of the wire

In practice the variations in the length of the panels due to end loads are small in comparison with the transverse deflections due to shear. Neglecting changes of panel length,

$$\Delta l = y \sin \phi$$

Also

$$\Delta l = \frac{Tl}{Ea} = \frac{tl}{E}$$

Whence

$$y = \frac{tl}{E \sin \phi} = \frac{tL}{E \sin \phi \cos \phi}$$

Therefore minimum y for a given t requires that $\sin \phi \cos \phi$ be a maximum.

Let $x = \sin \phi \cos \phi$

$$\frac{dx}{d\phi} = \cos^2\phi - \sin^2\phi$$

$$= 0 \text{ when } \cos \phi = \sin \phi, \text{ i.e., when } \phi = 45°$$

Therefore 45° slope of the shear wires gives the least deflection.

Let k = specific weight of the material of the wire

The weight of the wire is kal

$$T = F/\sin \phi$$

$$a = F/t \sin \phi$$

$$kal = k\, FL/t \sin \phi \cos \phi$$

Therefore minimum weight of wire with a given t also requires that $\phi = 45°$.

Initial tension in wires. A wire with initial tension is capable of transmitting compressive forces by becoming less taut. It thus acts as a strut, and when it fails it does so without damage. It follows that by giving initial tension to the diagonal wires in the hull, the counter-shear wires act as struts, and the shearing deflections of the hull are reduced. This has the advantage of reducing the secondary stresses in the longitudinals resulting from the primary shear in the hull. The disadvantages of initial tension are that the redundancy of the structure is increased, and in many cases the total compressive forces in the longitudinal and transverse girders are also increased. Furthermore, it is difficult to obtain uniform initial tensions, as some wires and girders are likely to be overloaded. For these reasons, it is not customary to have initial tensions in excess of about 200 lbs. in the shear wires.

In the transverse wires, initial tension is undesirable because it increases the total tension resulting from a deflated gas cell. It is necessary, however, to have sufficient initial tension to prevent distortion of the frames by their own weight. When the ship is resting in its cradle without gas in the cells, the weight of the hull tends to flatten the frames and put tensions in the horizontal transverse wires. On the other hand, when the ship is inflated, the transverse frames tend to extend vertically, placing the vertical wires in tension.

The problem of obtaining sufficient initial tensions to prevent distortion of the hull structure while under construction without having more than is required, is a practical problem not very susceptible of theoretical treatment; the initial tensioning of wires is best accomplished by skilled workmen of long experience with a good sense of the "feel" of the wires.

CHAPTER VIII

GAS PRESSURE FORCES AND TRANSVERSE STRENGTH

Functions of the Gas

The gas[1] contained within the envelope or gas cells of an airship ordinarily is considered to be the source of buoyancy, and while this conception is sufficiently accurate for general purposes, a more precise understanding of the function of the gas is essential to the airship designer.

The true source of buoyancy is the surrounding air, and not the contained gas. If the gas could be removed, leaving a vacuum, without collapse of the container, the lift or buoyancy of the airship would be increased by an amount equal to the weight of gas abstracted. Although the specific weight, and hence the buoyancy of air is small, being only .07635 lbs./ft.^{3} in the standard atmosphere at sea level, its pressure is very great, being 2,120 lbs./ft.^{2} in the same conditions. No structure enclosing a vacuum and displacing a volume of air of less weight than itself could resist this pressure. The cell or container for displacing the air must therefore be filled with a gas, having an absolute pressure approximately equal to that of the surrounding air, and as light as possible, because the weight of the gas diminishes the useful lift just as much as any other equal item of weight in the ship.

Strictly speaking, the true function of the gas is to maintain the internal pressure.

Definition of gas pressure. In practice, the term "gas pressure" is used to express the difference between the absolute internal gas pressure and the external air pressure.

[1] The properties of hydrogen and helium, also methods of producing these gases, are set forth in "Balloon and Airship Gases," a volume of the Ronald Aeronautic Library.

Rate of variation of gas pressure. Because of the difference in the densities of air and gas, the pressure increases upward within a gas container. The rate of pressure increase with height is given by:

$$dp/dh = k \tag{85}$$

where

p = gas pressure per unit area
h = height
k = unit lift of the gas, or the difference between the weight of air and gas per unit volume

Within the height from the bottom to the top of an airship, k may be regarded as constant, although strictly speaking, k varies within the airship at the same rate as elsewhere.[2]

Superpressure and total pressure. The gas pressure at the lowest point within the gas container is called the "superpressure," and the pressure at any other point is given by

$$p = p_0 + kh \tag{86}$$

where p_0 is the superpressure, and h the height above the bottom of the container.

In the gas cell of a rigid airship there can be no superpressure unless the cell is completely filled with gas. In nonrigid and semirigid airships, superpressure may be obtained by forcing air into the ballonets, producing a uniform air pressure throughout the ballonet space. The superpressure in the gas is then equal to the ballonet pressure except in so far as the weight or tension of the ballonet diaphragm may reduce the superpressure slightly below the ballonet air pressure.

Magnitude of gas pressure. The gas pressure is commonly measured in inches of water; but for design calculations, the pressure in pounds per square foot is usually required.

$$\text{One inch of water} = 5.2 \text{ lbs./ft.}^2 \tag{87}$$

The pressures are very small in comparison with the unit pressures commonly encountered in other engineering calcula-

[2] See "Aerostatics," by Prof. E. P. Warner, a volume of the Ronald Aeronautic Library.

tions. A superpressure of 2-in. of water is considered large even in a nonrigid or semirigid airship; and in a rigid airship, the automatic gas relief valves are commonly set to blow at less than 0.5 in. of water superpressure.

Example. Find the pressure at 70 ft. above the lowest point in a container filled with gas lifting .068 lb./ft.^{3}, and having a superpressure of 1.5 in. of water.

From equation (87), $p_0 = 1.5 \times 5.2 = 7.8$ lbs./ft.^{2}; and from equation (86), $p = 7.8 + (70 \times .068) = 12.56$ lbs./ft.^{2}, or 2.42 in. of water.

Effect of longitudinal inclination upon gas pressure. In nonrigid and semirigid airships, the gas space is either wholly undivided by transverse bulkheads, or the bulkheads are of little effect in preventing equalization of the gas pressure on each side of them. The vertical distance between the lowest and highest points of the envelope increases with the longitudinal inclination of the airship, and since the lowest point must always have sufficient pressure to prevent collapse (usually about 1 in. of water in nonrigids), the largest gas pressures occur in the inclined position. The envelopes are usually designed for the gas pressures occurring at 30° inclination.[3]

In rigid airships, the gas cells are short in comparison with their diameters, and are separated by cross-wired transverse frames, so that inclination of the ship has very little effect upon the maximum gas pressures in the cells, although it produces differential pressures between consecutive cells, throwing loads upon the transverse bulkheads. These loads are small in comparison with those resulting from the deflation of a gas cell adjacent to a full one.

Gas pressure an asset or a liability. In pressure airships, some gas pressure is an absolute necessity to prevent collapse; and even in rigid airships, the gas pressure is often useful in assisting the hull structure to resist compressive forces. In all

[3] See "Pressure Airships," Blakemore and Pagon, a volume of the Ronald Aeronautic Library.

types of airships, excessive gas pressure must be avoided by ample provision of gas valves to liberate the gas when climbing rapidly, or when thrown violently upward by strong ascending air currents, as in a thunderstorm.

In rigid airships hitherto constructed, the gas pressure is most often a liability, throwing undesirable cross bending forces on the longitudinal girders. It seems possible by improved design to contain the gas cells within systems of wiring which will not place transverse loads of appreciable magnitude on the longitudinals, but will rather produce tensile forces that are easily taken care of, and in fact can be used as positive assets in offsetting the compressive forces which are more difficult to resist by structural members.

It has been proposed to reduce gas pressure stresses in rigid airships by having so-called "natural shape" cross-sections, i.e., the lythenary which was the second special case of cross-sectional shape considered in Chapter VI. It would be extremely difficult to provide for the variations in the cross-sections with changing gas pressure; and in fact, the natural shape between main frames is circular, for, except at those frames, the lift is opposed only by shear and uniformly distributed weight.

The longitudinal bending moment from gas pressure has been discussed in Chapter VI.

Gas pressure loads on longitudinal members. In conventional rigid airships, the stresses in the longitudinal girders due to the gas pressure loads are of the same order of magnitude as the stresses resulting from the primary bending of the hull.

The fabric of the gas cell is forced by the gas pressure against the bases of the longitudinal girders and against the netting which is secured to the girders. It is sometimes thought that the entire pressure of the gas on each panel must be taken by the longitudinals as radial loads; but in reality this would be true only if the netting bulged in a complete semi-circle (assuming that the shear wires and outer cover permitted this shape)

as shown in Figure 51. In this case the netting would impose no loads upon the longitudinals tangentially to the cross-section of the hull.

Another special case is when the netting has the same curvature as the cross-section of the hull, also shown in Figure 51. The netting is everywhere tangential to the hull, so that the tension has no component throwing radial loads upon the girders.

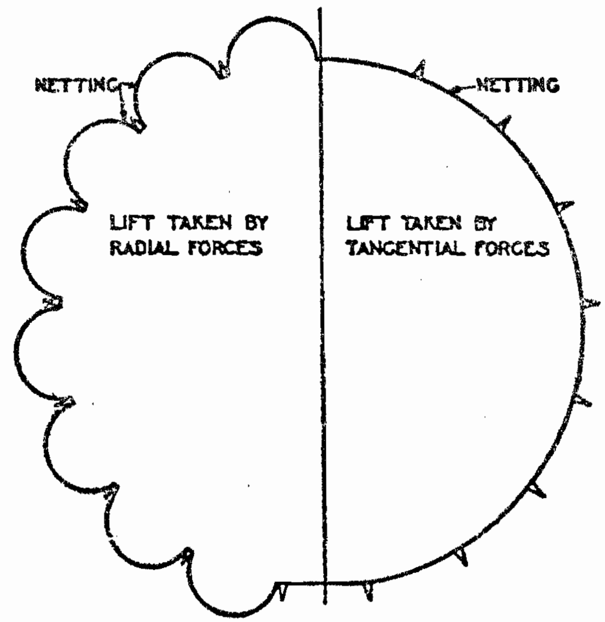


Figure 51. Illustrating Effect of Curvature of Netting on Distribution of Lift Forces

There are, however, tangential loads upon the longitudinals resulting from the increase of the gas pressure and the tensions in the nettings upward from panel to panel.

The manner in which the lift of the gas is transmitted to the hull differs very greatly in these two cases. In the first, the lift is due to the radial outward loads upon the upper longitudinals; while in the second case, the lift is the result of the tangential forces, mainly on the longitudinals at the mid-height of the hull.

The effect of the variation of curvature of the netting is of great importance; and a similar problem will occur when con-

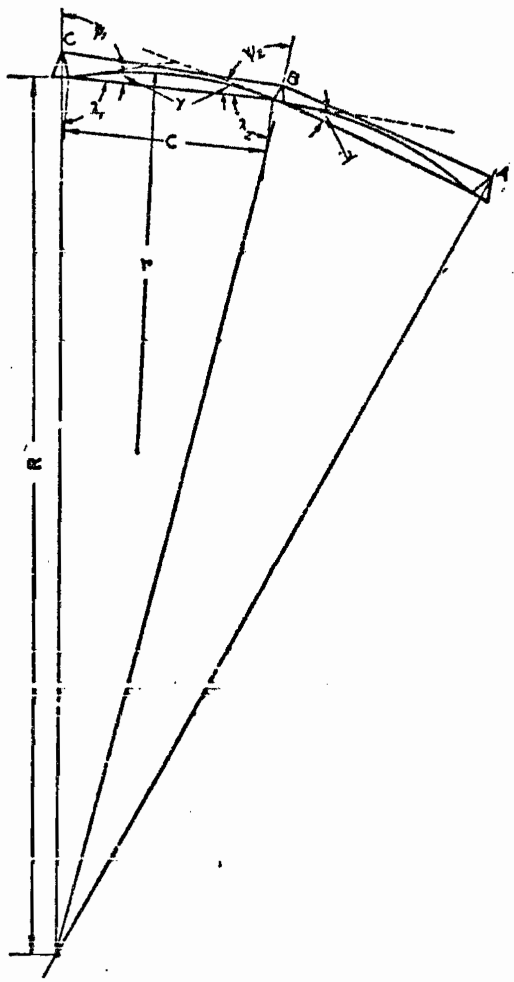


Figure 52. Illustrating Radial Load on Longitudinals from Gas Pressure on Netting

sidering the loads imposed on the longitudinals by the outer cover, resulting from differences between the external and in-

ternal pressures. The point is worth some emphasis, because it is often argued that the curvature of the netting can make no difference in the loads on the longitudinals, just as the curvature of the cables of a suspension bridge makes no difference in the loads upon the towers. The fallacy in this argument may be explained by reference to Figure 52. The radial load each panel of netting puts upon the longitudinal to which it is attached is given by

$$w = t \cos \psi \qquad (88)$$

where

w = the radial load which the netting puts upon the longitudinals per unit length

t = the tension in the netting per unit width

ψ = the angle between the tangent to the netting and the radial plane through the longitudinal

And from Figure 52,

$$\psi = \pi - \gamma - \lambda \qquad (89)$$

and

$$t = pr$$

where

p = gas pressure per unit area, assumed constant for the panel

r = transverse radius of curvature of the netting

Therefore,

$$w = pr \cos (\pi - \gamma - \lambda).$$

In the special case when $\lambda = \pi/2$

$$w = pr \cos (\pi/2 - \gamma)$$
$$= pr \sin \gamma$$

and $\sin \gamma = c/2r$, where c = the width of the panel. Therefore

$$w = pc/2$$

This case applies to the load upon the towers of a suspension bridge, but is obviously not generally applicable to the radial loads upon the longitudinals of an airship.

Another special case is when $r = R$, then $\gamma + \lambda = \pi/2 = \psi$; therefore $w = 0$.

In airships, such as the *Shenandoah*, where the main and secondary longitudinals are of different depths, λ is larger at the

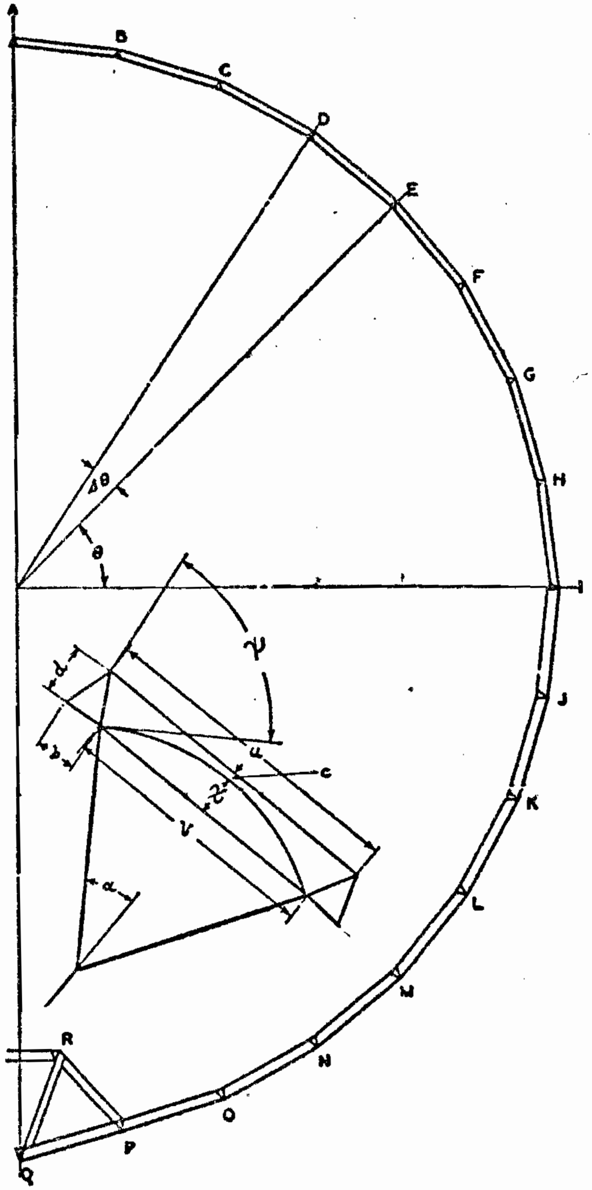


Figure 53. Cross-Section of a 32-sided Rigid Airship

main than at the secondary longitudinals, and consequently the radially outward loads are greater on the main longitudinals.

The curvature of the netting is usually such as to transmit the lift of the gas to the hull by a combination of radial and tangential loads upon the longitudinals. It is possible to brace the longitudinals much more efficiently in the tangential than in the radial direction; and in fact, the netting, when composed of diagonal wires, is in itself a tangential bracing. It is therefore desirable that the netting should bulge but little beyond the inscribed circle through the bases of the longitudinals. Unfortunately, it is not practicable to erect and maintain the netting with the exact amount of tension theoretically most desirable; and it is therefore customary to allow for the netting bulging to within about 4 to 6 in. of the outer cover. A certain amount of radially outward gas pressure load is helpful to the longitudinals against the inward loads from the outer cover. The direct gas pressure on the base of the longitudinal girders provides this load.

Sample computation of the gas pressure loads on the netting and longitudinals. In the following sample computation of the gas pressure loads in a 32-sided airship (Figure 53), it is given that:

R = radius of cross-section of hull over apexes of longitudinals
= 61.0 ft.
$\Delta\theta$ = angle subtended by a panel at the center of the cross-section
= $11°15'$
d = height of longitudinal girder
= 1.18 ft.
b = width of longitudinal girder
= .885 ft.
c = minimum clearance between netting and outer cover
= .5 ft.
r = radius of curvature of netting

u = distance between apexes of longitudinals
v = distance between bases of longitudinals
x = maximum depth of curve of netting
θ = angle between the horizontal and a radial plane through any longitudinal girder
H = height of any longitudinal above the bottom of the gas cell
k = unit lift of the gas
= .064 lb./ft.^{3}
t = transverse tension in the netting
p = unit gas pressure at any longitudinal
p_0 = unit gas pressure at bottom of cell
= 0.5 in. of water = 2.6 lbs./ft.^{2}
α = half the arc of the netting
ψ = angle between the radial plane and the tangent to the netting at any longitudinal
R_1 = the mean radius of the gas space from the center of the cross-section of the hull

From the given quantities we derive by geometry the following:

$$u = 2\,R \sin(\Delta\theta/2)$$
$$= 2 \times 61 \times .098 = 11.95 \text{ ft.}$$
$$v = u - 2d \sin(\Delta\theta/2) - b \cos(\Delta\theta/2)$$
$$= 11.95 - (2.36 \times .098) - (.885 \times .995) = 10.84 \text{ ft.}$$
$$x = d \cos(\Delta\theta/2) - \frac{b}{2} \sin(\Delta\theta/2) - c$$
$$= (1.18 \times .995) - (.442 \times .098) - .5 = .63 \text{ ft.}$$
$$r = v^2/8x$$
$$= (10.84)^2/(8 \times .63) = 23.4 \text{ ft.}$$
$$\psi = 90° + \frac{\Delta\theta}{2} - \alpha$$
$$\alpha = \sin^{-1}(v/2r) = \sin^{-1} .232 = 13°25'$$
$$\psi = 90° + 5°37\ 1/2' - 13°25' = 82°\ 12\ 1/2'$$
$$\sin\psi = .9907$$
$$\cos\psi = .1356$$
$$H = R_1 (1 + \sin\theta)$$
$$= 60.2 (1 + \sin\theta)$$

$$p = p_0 + kH$$
$$= 2.6 + (.064 \times 60.2)(1 + \sin\theta)$$
$$= 6.46 + 3.86 \sin\theta$$
$$t = rp = 151 + 90.4 \sin\theta$$
$$\text{Direct radial load} = bp$$
$$= 5.72 + 3.42 \sin\theta$$
$$\text{Radial load from netting} = 2\,t \cos\psi$$
$$= 41.08 + 24.45 \sin\theta$$
$$\text{Total radial load} = 46.8 + 27.9 \sin\theta \text{ (lbs./ft.)}$$
$$\text{Tangential load} = (\sin\psi)\,\Delta t$$
$$= (\sin\psi)\,r\Delta p$$
$$= (\sin\psi)\,rk\Delta H$$
$$\Delta H = R_1 \cos\theta \sin\Delta\theta$$
$$\text{Tangential load} = R_1\,r\,k \sin\psi \sin\Delta\theta \cos\theta$$
$$= 60.2 \times 23.4 \times .064 \times .195 \times .9907 \cos\theta$$
$$= 17.45 \cos\theta \text{ (lbs./ft.)}$$

The running radial and tangential loads upon the longitudinals, and the mean transverse tensions in the netting are

TABLE 18. COMPUTATION OF GAS PRESSURE, NETTING TENSION, AND RADIAL AND TANGENTIAL LOADS UPON THE LONGITUDINALS OF A 32-SIDED AIRSHIP, 122 FT. DIAMETER

Longl.	θ	sin θ	cos θ	p lbs./ft.^{2}	t lbs./ft.	Radial Load lbs./ft.	Tangential Load lbs./ft.
A	90° 0′	1.0	0	10.32	242	74.7	0
B	78° 45′	.9808	.1950	10.24	240	74.1	3.40
C	67° 30′	.9239	.3827	10.03	236	72.6	6.68
D	56° 15′	.8315	.5555	9.67	226	70.0	9.70
E	45° 0′	.7071	.7071	9.19	215	66.5	12.33
F	33° 45′	.5555	.8315	8.60	201	62.3	14.5
G	22° 30′	.3827	.9239	7.94	186	57.5	16.1
H	11° 15′	.1950	.9808	7.21	169	52.2	17.1
I	0	0	1.0	6.46	151	46.8	17.45
J	−11° 15′	−.1950	.9808	5.71	134	41.4	17.1
K	−22° 30′	−.3827	.9239	4.98	117	36.1	16.1
L	−33° 45′	−.5555	.8315	4.32	101	31.3	14.5
M	−45° 0′	−.7071	.7071	3.73	87	27.1	12.33
N	−56° 15′	−.8315	.5555	3.25	76	23.6	9.7
O	−67° 30′	−.9239	.3827	2.89	68	21.0	6.68
P	−78° 45′	−.9808					

Curvature of netting = r = 23.4 ft.

given in Table 18. The true netting tension in any panel may be taken as the mean of the tensions given in the table which are calculated for the positions between consecutive panels.

When the curvature of the netting or the depth of the longitudinals are variable, ψ is also variable, and the computations of the loads on the longitudinals are more complicated, but the same in principle as in the foregoing example.

In another section it will be shown how to compute the initial tension or slackness which must be given to the netting to produce the bulge or curvature desired at any given gas pressure.

The Gas-Cell Netting

The gas cells should be supported between the longitudinals by netting having a mesh not greater than about 18 in. Wire netting of this mesh must consist of very small wires, and because of the great number of wires in each panel it is difficult to obtain the uniform initial tensions which are necessary if the gas pressure loads upon the longitudinals are to be kept within the designed limits. For these reasons, it is customary to use a steel wire netting having an average mesh of about 3 to 4 ft., combined with ramie cord netting having a mesh about one-half to one-third as large. The cord netting is not considered in the calculations of the gas pressure loads, which are assumed to be taken directly by the wire netting.

There have been two principal types of wire netting, known as the diagonal mesh and circumferential systems. Zeppelin airships previous to the *Los Angeles* had simple diagonal mesh nettings. In the *R*-38, the British used an ingenious system of circumferential wires running transversely around the hull, passing through holes in the base channels of the longitudinal girders, and secured to catenary loop wires, resembling inverted suspension bridge cables, in the planes of the panels between successive joints of the main longitudinals with the main and

intermediate frames. These catenary loops received the tangential forces resulting from the changes in the tensions in the circumferential wires due to the increase of gas pressure upwards; and thus relieved the longitudinals of tangential loads.

The gas-cell netting of the *Los Angeles* is arranged as shown in Figure 54. The arrangement is a compromise between the

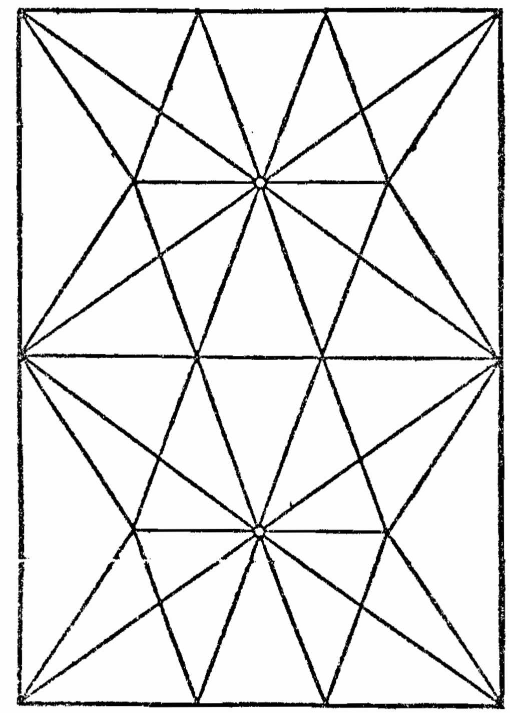


Figure 54. Two Typical Panels of Gas-Cell Netting of the U.S.S. *Los Angeles*

diagonal mesh and circumferential systems. The wires pass through holes in the base channels of the longitudinals, as in the circumferential system, so that there are no tangential loads upon the longitudinals; and several wires converge at each joint so as to transmit a large part of even the radial gas pressure forces directly to the joints, instead of to the longitudinals.

In designing the gas-cell netting, the primary objects to be sought for are:

(*a*) Small radial and tangential loads upon the longitudinals
(*b*) Largest possible gas space
(*c*) Small longitudinal components of the wire tensions
(*d*) A light and simple and easily adjustable system of wiring

These requirements are mutually conflicting to some extent, and the designer must make a compromise between them.

Tangential loads upon the longitudinals are easily avoided by allowing the netting to pass freely through holes in the base channels, and having all wire terminals at the joints with the frames. Radial loads may be eliminated by having the transverse radius of curvature of the netting equal to the radius of the cross-section of the ship through the bases of the longitudinals; but this conflicts with the requirement for maximum gas volume, since it leaves a considerable air space between the cells and the outer cover; and furthermore it is a condition which can be only approximately attained in practice, owing to the unavoidable variation of the curvature with varying gas pressure. The only way completely to eliminate radial and tangential gas pressure loads upon the longitudinals appears to be to attach the wires only to the transverse frames, drawn so tight that they will not touch the longitudinals under any conditions. Owing to the loss of gas space, and the difficulty of setting up the wires to the correct tension, it is doubtful if such an arrangement is practicable or desirable.

The longitudinal tension put upon the hull by the gas pressure may be useful in assisting the longitudinal girders against compressive forces resulting from the primary bending of the hull; but it is difficult to avoid absorbing the greater part of the longitudinal gas pressure force in the longitudinal components of the tension in the gas-cell netting. For this reason it is desirable to have the greater part of the wires running as nearly as practicable in the circumferential direction; although there

is difficulty in securing the proper tension and curvature due to the great lengths of the wires which become necessary if this principle is carried too far.

Tension due to a transverse load upon a wire initially taut. In design calculations for airships it is frequently necessary to compute the tension due to a transverse load from gas or air pressure upon a wire initially taut. This problem rarely occurs

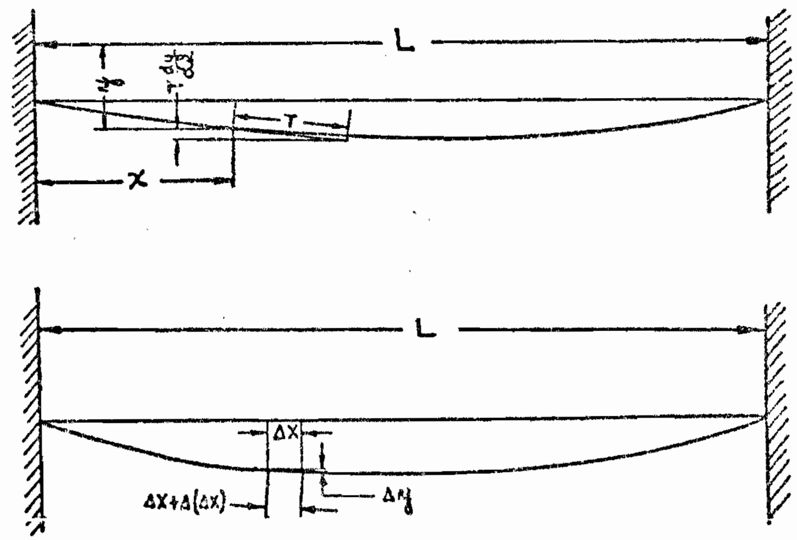


Figure 55. Tension due to Transverse Load on a Wire Initially Taut

in other engineering work, and the procedure for handling it is but little known to engineers. For these reasons, formulas and their derivations for computation of the tension in an initially taut wire subjected to transverse loads of various distributions will be given here.

Figure 55 shows a wire initially stretched taut between two fixed points of attachment, and loaded transversely.

Let L = initial length of wire
ΔL = increase in length of wire due to the transverse load
a = cross-sectional area of the wire

E = modulus of elasticity of the material of the wire
y = deflection of the wire at distance x from one end
p = transverse load per unit length of the wire
P = total transverse load upon the wire
T_o = initial tension of the wire
T = final tension of the wire
R = radius of curvature of the wire

The effect of the weight of the wire is neglected, because in airship problems this weight is small in comparison with the transverse loads to be considered. Since transverse forces resulting from fluid pressure can have no tangential component along the wire, the tension is constant throughout the wire.

The fundamental and well-known relation between R, p, and T is $Rp = T$. And since dy/dx (the inclination of the wire to its initial position) is always small, $1/R = d^2y/dx^2$, whence

$$T\frac{d^2y}{dx^2} = p \tag{90}$$

The deflection of the wire increases a short length from Δx to $\Delta x + \Delta(\Delta x)$, and from Figure 55

$$\left[\Delta x + \Delta(\Delta x)\right]^2 = (\Delta x)^2 + (\Delta y)^2$$

Whence:

$$\Delta(\Delta x) = 1/2\left(\frac{\Delta y}{\Delta x}\right)^2 \Delta x$$

And

$$\Delta L = \int_0^L 1/2\left(\frac{dy}{dx}\right)^2 dx \tag{91}$$

And by the properties of materials strained within their proportional limits:

$$\Delta L = \frac{(T - T_0)\,L}{E\,a} \tag{92}$$

By integrating equation (90) twice, T can be found in terms of p, x, and y, plus a constant of integration which can be determined from the fact that $y = 0$ when $x = L$.

From equations (90), (91), and (92), the tension and deflection can be completely determined when there is no relative displacement of the ends of the wire due to the loads. If there is a known displacement of the ends of the wire, the final tension can be calculated by assuming that the initial tension is modified by the amount which it would be if the displacement occurred before the application of the transverse load, instead of afterward.

Tension due to a uniformly distributed transverse load. By double integration of equation (90),

$$T\frac{dy}{dx} = \int_0^x p\,dx + C = px + C$$

$$Ty = \frac{px^2}{2} + Cx$$

When $x = L$, $y = 0$, whence:

$$\frac{pL^2}{2} + CL = 0$$

$$-C = \frac{pL}{2}$$

$$\frac{dy}{dx} = \frac{px}{T} - \frac{pl}{2T}$$

$$\Delta L = \frac{(T - T_0)L}{E\,a} = \int_0^L 1/2\left(\frac{dy}{dx}\right)^2 dx$$

$$= \frac{p^2}{2T^2}\int_0^L\left(x^2 - Lx + \frac{L^2}{4}\right)dx$$

$$= \frac{p^2L^3}{24T^2} = \frac{P^2L}{24T^2}$$

Whence:

$$24\,T^3 - 24\,T_0T^2 - E\,a\,P^2 = 0 \tag{93}$$

When $T_0 = 0$, T is given by

$$T^3 = \frac{EaP^2}{24}$$

Other useful relations which may be easily derived for a uniform transverse load and $T_0 = 0$ are:

$$R^3 = \frac{E\,a\,L^3}{24\,P}$$

$$y_{max} = \frac{L^2}{8R}$$

$$y^3_{max} = \frac{3L^3P}{64\,Ea}$$

Let $T/a = t$, then

$$t = E\frac{\Delta L}{L}$$

$$T^2 = \frac{EP^2}{24\,t}$$

$$y^2_{max} = \frac{3L^2t}{8\,E}$$

$$R^2 = \frac{L^2E}{24\,t}$$

Tension due to non-uniformly distributed transverse loads. For a single concentrated transverse load at the center of a wire initially just taut,

$$T^3 = \frac{E\,a\,P^2}{8}$$

A frequently occurring case is a distributed load in which $p = kx$, where k = constant. In this case

$$\frac{45T^3}{2} - \frac{45\,T_0T^2}{2} - E\,a\,P^2 = 0$$

This expression and similar ones for other types of non-uniformly distributed loads may be readily derived by procedure similar to that used above for uniform loading.

Determination of initial tension or slackness for a given final tension in a wire loaded transversely. A frequently occurring problem in airship design is to find the initial tension or slackness required for a wire which will have a specified final tension or curvature when subjected to a given transverse load.

This problem must be solved by calculating the value of the initial tension T_0, which will give the required T.

The following example illustrates various ramifications of the problem of determining tension in a transversely loaded wire.

Example. Find the initial tension and cross-sectional area required in a wire to bulge 6 in. under a uniform transverse load of 18 lbs./ft. with a tension of 50,000 lbs./in.², given that $L = 13$ ft., and $E = 30{,}000{,}000$ lbs./in². From the foregoing data,

$$P = 18 \times 13 = 234 \text{ lbs.}$$

$$R = \frac{L^2}{8y} = \frac{13 \times 13}{8 \times .5} = 42.25 \text{ ft.}$$

$$T = Rp = 42.25 \times 18 = 760 \text{ lbs.}$$

$$a = \frac{T}{t} = \frac{760}{50{,}000} = .0152 \text{ in.}^2$$

From the expression

$$24\,T^3 - 24\,T_0\,T^2 - E\,a\,P^2 = 0$$

$$T_0 = \frac{24\,T^3 - E\,a\,P^2}{24\,T^2}$$

$$= \frac{(24 \times 760^3) - (30{,}000{,}000 \times .0152 \times 234^2)}{24 \times 760^2}$$

$$= -1{,}040 \text{ lbs.}$$

A wire cannot be set up with an initial compression, and so the negative T_0 must be converted into terms of initial slack, given by

$$\Delta L = \frac{T_0L}{E\,a}$$

$$= \frac{-1{,}040 \times 156}{30{,}000{,}000 \times .0152}$$

$$= -.356 \text{ in.}$$

i.e., the wire must have an initial length .356 in. greater than the 13 ft. between its terminals.

Effect of initial netting tension on gas pressure loads on longitudinals. The following investigation into the effect of the tension in the gas cell retaining netting upon the radial gas pressure loads on the longitudinals is confined to the special case of longitudinals of equal depth at 11°15′ apart around the circumference of a cross-section of the hull of a rigid airship 135 ft. diameter.

The character of the calculations is such that a general expression for the radial loads cannot be handled, and every case must be considered by itself. Nevertheless, some deductions of general applicability are made.

The method of analysis is as follows:

Let v = clear distance between bases of consecutive longitudinals
x = maximum depth of the curve of the netting between longitudinals
r = radius of curvature of the netting
α = half of the arc subtended by the netting at its center of curvature
$\Delta\theta$ = the angle between consecutive sides of the ship
ψ = the angle between the radial line through a longitudinal and the tangent to the netting at the base of the longitudinal
p = the mean unit gas pressure on the panel of netting
t_0 = the initial tension in the netting per unit width
t = the final tension in the netting due to the pressure p
w = the running radial load put upon a longitudinal by the netting on one side of it
E = modulus of elasticity of the wires
a = cross-sectional area of the wires, assumed to run transversely at unit distance apart

The following simple relations between the above quantities are readily derived from the preceding discussion:

$$w = t\cos\psi$$
$$\psi = 90^\circ + \Delta\theta/2 - \alpha$$
$$\sin\alpha = v/2r$$

$$x = v^2/8r$$
$$r = t/p$$
$$t^3 - t_0 t^2 = Ea\,(vp)^2$$

Let it be given that v = 12.5 ft., E = 20,000,000 lbs./in.², and a = .01 in.² for transverse wires at 1 ft. apart.

The values of t, w, x, etc., are plotted in Figures 56 and 57 against pressure from 0 to 20 lbs./ft.² (3.85 in. of water). Five different values of t_0 are considered: − 200 lbs., − 100 lbs.,

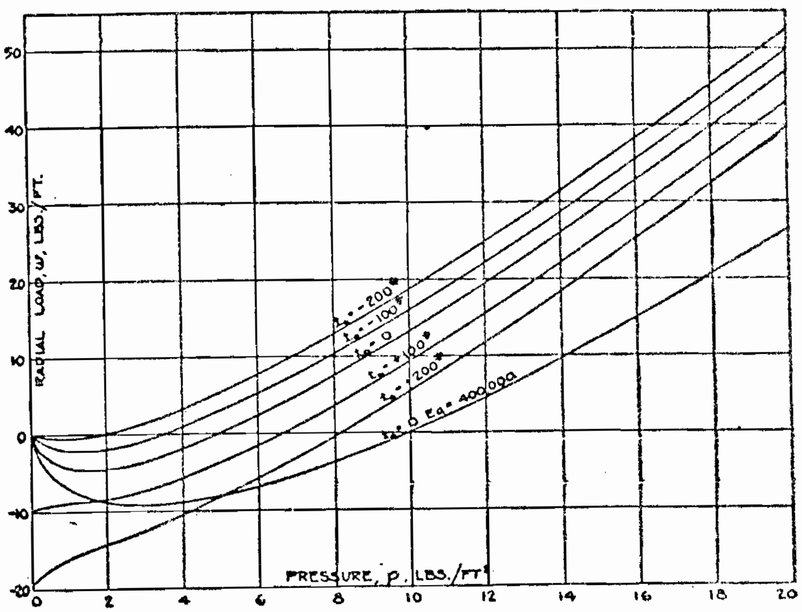


Figure 56. Radial Load on Longitudinals from Gas Cell Netting of Various Initial Tensions

0, + 100 lbs., and + 200 lbs. Negative initial tension in the usual sense of the term (i.e., compression) is, of course, impossible with wires. The term is used here to indicate that the wires are slack, having the additional amount of length, which would produce the specified negative tension if they could sustain compression without buckling. For example: v is the distance between the wire terminals, and Δv is the additional length of the wire. By the theory of elasticity, $-\Delta v = v\, t_0/E\, a$.

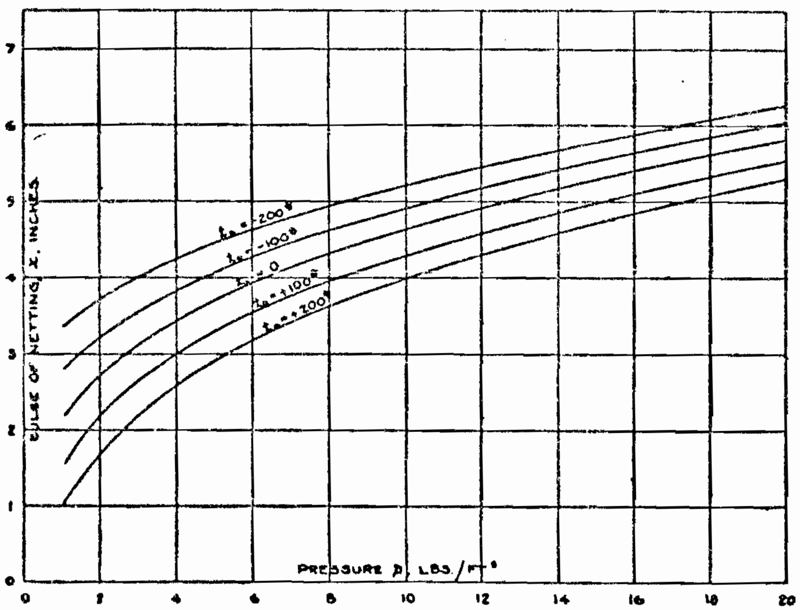


Figure 57. Bulge of Gas Cell Netting of Various Initial Tensions

The initial lateral deflection, x_0, which can be produced by plucking at the center of the wire to take up the slack without producing tension is given by:

$$x^2_0 = v\Delta v/2$$

In the case under investigation,

When $t_0 = -200$ lbs.

$$\Delta v = 150 \times 200/200{,}000 = .150 \text{ in.}$$
$$x^2_0 = 150 \times .150/2 = 11.25 \text{ in.}^2$$
$$x_0 = 3.36 \text{ in.}$$

When $t_0 = -100$ lbs.

$$\Delta v = 150 \times 100/200{,}000 = .075 \text{ in.}$$
$$x^2_0 = 150 \times .075/2 = 5.625 \text{ in.}^2$$
$$x_0 = 2.37 \text{ in.}$$

Practical application. The curves in Figure 56 show the variation of w with p for constant values of t_0; and by following along the vertical lines, the variation of w with t_0 when p is constant may be noted.

It should be the aim of the designer to keep w small, although some positive value for it is desirable to offset the inward load from the outer cover. For any particular gas pressure p, there is an initial tension t_0, which will result in $w = 0$; but p varies with the fullness of the gas cell and the superpressure; and it is not possible to adjust t_0 with any great accuracy. Probably the practical range of uncertainty of t_0 may be set at 100 lbs. for panels of the size under consideration. Figure 56 shows that within the range of $t_0 = \pm$ 50 lbs., the variation of w remains about 4 lbs./ft. for the entire range of pressure. Since netting is secured to both sides of the longitudinals, the range of uncertainty for w for any given pressure is about 8 lbs./ft.

As a typical case, we may consider a longitudinal girder 18 ft. in length between fixed ends, and having a section modulus $Z = 2.5$ in.^{3} The maximum bending moment is given by:

$$M = w\,L^2/12$$

For the uncertain load of 8 lbs./ft.,

$$M = 8 \times 18^2/12 = 216 \text{ ft. lbs.}$$
$$= 2592 \text{ in. lbs.}$$
$$f = M/Z = 2592/2.5 = 1{,}037 \text{ lbs./in.}^2$$

That is, there is an uncertainty of about 1,000 lbs./in.^{2} stress due to uncontrollable variations in t_0.

The curves in Figure 56 are carried up to a pressure of 20 lbs./ft.^{2}, but in practice it is hardly necessary to consider pressures over about 12 lbs./ft.^{2}, even at the top of an airship of this great size.

When $p = 12$ lbs./ft.^{2}, $Ea = 200{,}000$ lbs., and $t_0 = 0$; $w = 19.4$ lbs./ft., $t = 570$ lbs., and $t/a = 57{,}000$ lbs./in.^{2} The radial load w is high; and the unit stress of 57,000 lbs./in.^{2} is very moderate for high tensile steel wire. Better results are obtained with $t_0 = 200$ lbs. We then have $w = 11.8$ lbs./ft., $t = 647$ lbs., and

$t/a = 64{,}700$ lbs./in.[2] This is a considerable improvement in w over the case when $t_0 = 0$, and the unit stress is not high. The objection to such a large initial tension is the inward load put upon the longitudinals when there is no gas pressure.

It is obviously desirable to have w vary as little as possible with varying pressure. This may be accomplished by increasing the cross-sectional areas of the wires. The curve of w against p with $t_0 = 0$ and $Ea = 400{,}000$ lbs., is plotted in Figure 56, and is found to be very much flatter than the curves with $Ea = 200{,}000$ lbs. At $p = 12$ lbs./ft.[2], $w = 4.40$ lbs./ft. only, $t = 720$ lbs., and $t/a = 36{,}000$ lbs./in.[2] The more constant value of w is obtained at the cost of greater weight of the gas-cell wiring.

It should be noted that reducing w by increasing either t_0 or a decreases the gas volume through the reduced bulge of the gas cell between the longitudinals

The best compromise between maximum volume of the gas cells with light netting wires on one hand and minimum load on the longitudinals on the other hand, must be determined by investigation of each individual case. No general rule can be given.

Estimating E for steel wires at 20,000,000 lbs./in.[2], instead of 30,000,000, gives a reasonable allowance for stretch of the wire terminals.

The Main Transverse Frames

The transverse strength and rigidity of rigid airships is derived almost entirely from the cross-wired main frames. The intermediate frames are necessarily without cross-wiring because of the presence of the gas cells, and are therefore very flexible. Besides giving transverse rigidity to the airship, the main frames also act as bulkheads between adjacent gas cells, and the most severe loads which come upon these frames are the gas pressure forces resulting from the deflation of a gas cell adjacent to a full one. The only other important loads carried by the main frames

result from the concentrated loads in the keel, which are balanced by the lift forces carried to these frames through the longitudinals and the shear wires.

The redundancy of the main frame structure is so great that it is very difficult to make exact computations of the stresses in the girders and transverse wiring due to the lift and weight forces upon the frame. A good approximation may be obtained by dividing up the greater part of the wiring between three or four statically determinate systems, and apportioning the load between these systems according to the "inverse ratio method" described in the previous chapter.

The stresses resulting from the weight and buoyancy forces on the main frames are rarely more than 30% as large as those resulting from the deflation of a gas cell.

Note on the numbering of the frames. It is customary to number the transverse frames of rigid airships by their distances in meters from the sternpost, (i.e., the cruciform frame at the trailing edge of the fins). The five-meter frame spacing which has hitherto been standard practice makes this system of numbering very convenient. Frames aft of the stern-post are designated by minus signs before their numbers.

Specimen computation of stresses in a main transverse frame. A main frame 79 ft. diameter is loaded with a weight of 6,500 lbs. at the keel. This load is balanced by lift forces applied to the joints of the frame, partly as radial loads from the longitudinals, but to a much greater extent as tangential forces transmitted to the joints by the longitudinals, the netting, and most of all, the shear wires. The calculation of the stresses in the frame can be much simplified by assuming that the lift forces at the joints are entirely tangential; and the small error which results is upon the safe side, since the actual radial forces at the joints produce tensions in the girders, relieving to some extent the compressive forces produced by the other loads. It is sufficiently accurate to assume that in accordance with the

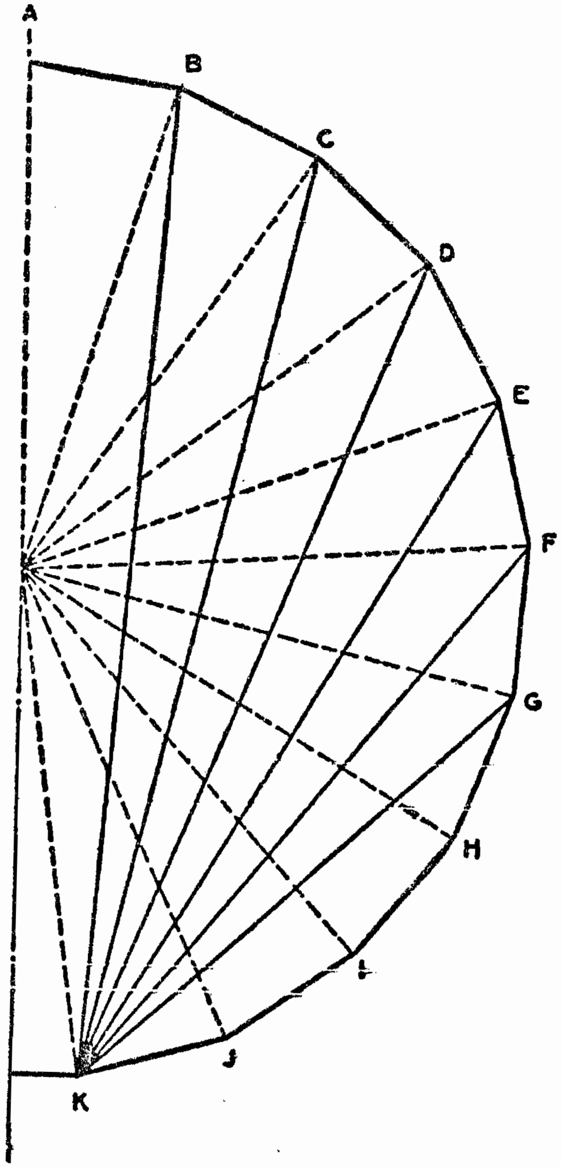


Figure 58. Simplified Main Frame

shear theory, the tangential forces at the joints are proportional to sin θ, and the vertical forces to $\sin^2\theta$. The tangential force at any joint is then 6,500 sin $\theta/\Sigma \sin^2\theta$. The tangential forces at the joints and the 6,500 lbs. vertical load at the keel are assumed to be shared between two statically determinate systems. System 1 consists of the transverse girders and the radial wires shown in Figure 58. System 2 consists of the girders and the chord wires converging at the joints at the bottom of the keel. Each system is solved by itself by ordinary analytical or graphical methods for determinate structures. The stresses S, and the internal work $S^2L/2EA$, for both systems are shown in Table 19, from which the sums of the work, Q_1 and Q_2 are obtained.

TABLE 19. COMPUTATION OF STRESSES IN A MAIN FRAME

Member	L in.	A in.²	E lbs./in.²	System 1		System 2		q_1S_1 lbs.	q_2S_2 lbs.	$S = q_1S_1 + q_2S_2$ lbs.
				S_1 lbs.	$\frac{S_1^2L}{2EA}$ in. lbs.	S_2 lbs.	$\frac{S_2^2L}{2EA}$ in. lbs.			
AC	236	.40	10,500,000	−2,944	244	−1,775	89	−1,150	−1,082	−2,232
CE	236	.40	10,500,000	−2,436	167	−1,570	69	− 950	− 958	−1,908
EG	236	.40	10,500,000	−1,545	67	−1,110	35	− 603	− 677	−1,280
GI	236	.40	10,500,000	− 485	7	− 485	7	− 189	− 296	− 485
IK	236	.40	10,500,000	+ 485	7	+ 485	7	+ 189	+ 296	+ 485
OA	475	.00817	30,000,000	1,451	4,080			566		566
CC	475	.00817	30,000,000	1,326	3,410			517		517
OE	475	.00817	30,000,000	980	1,860			382		382
OG	475	.00817	30,000,000	500	485			195		195
MA	945	.00817	30,000,000			400	618		244	244
MC	900	.00817	30,000,000			920	3,120		561	561
ME	790	.00817	30,000,000			750	1,820		458	458
MG	635	.00817	30,000,000			750	845		348	348
					10,327		6,610			

$$Q_1 = 10{,}327 \text{ in.lbs.}$$
$$Q_2 = 6{,}610 \text{ in.lbs.}$$
$$q_1 = \frac{Q_2}{Q_1 + Q_2} = .39$$
$$q_2 = 1 - q_1 = .61$$

The stresses in the members as parts of systems 1 and 2 are multiplied by q_1 and q_2, respectively, and the total stresses are assumed to be $q_1S_1 + q_2S_2$.

Stresses due to the deflation of a gas cell. The pressure of a full or partially full gas cell adjacent to an empty one, or the differential pressure between two unequally inflated cells throws transverse loads upon the wiring of the frame, producing tensions in these wires, and compression in the boundary girders of the frame. Owing to the extremely complex system of transverse wiring used in modern rigid airships (Figure 49, page 174), it is impracticable to make an exact calculation of the manner in which the gas pressure loads are distributed among the wires. A very fair approximation to the loads in the frame may be obtained by assuming that the entire system of transverse wiring is replaced by a simple system of radial wiring in which the cross-sectional areas of the wires are assumed to be twice as great as the areas of the actual radial or diametral wires of the airship. Calculations of the forces at the joints of the main frames, assuming this simplified system of wiring, have been found to agree very well with the forces computed from the observed tensions in the transverse wires in all gas-cell tests for which data are available.

The computation of the tensions in an initially taut wire to which a transverse load is applied is described on page 193; and applying the method to the tensions in a system of radial wires between an empty cell and a cell fully inflated, but without superpressure, it is found that the tension in the upper vertical wire is given by:

$$\frac{TR}{Ea} = \frac{33}{280}\,\frac{k^2 R^7 \tan^2 \alpha}{T^2} + \frac{\delta^2}{2R} \qquad (94)$$

where

T = the tension in the wire
R = the radius of the transverse frame
E = the modulus of elasticity of the material of the wire
a = the cross-sectional area of the wire
k = the unit lift of the gas
α = half the angle between successive radial wires
δ = the longitudinal movement of the center ring to which the radial wires are attached

Similar expressions may be found for the other radial wires. For the horizontal and the lower vertical wires, the coefficient 33/280 is replaced by 2/45 and 17/2,520, respectively. These expressions are derived on the assumption that there is no vertical movement of the center ring. Actually, the movement of this ring adjusts the tensions of all the radial wires to approximately the tension calculated for the horizontal wires, which are not appreciably affected by the vertical motion of the center ring.

The longitudinal movement of the center ring may be calculated from the stretch of the axial cable according to the expression:

$$\delta = \frac{TL}{Ea}$$

Where T is the tension in the axial cable, and is assumed to be one-third the total gas pressure load on the transverse frame; L is the distance from the frame to the end of the ship on the side of the frame opposite to the deflated cell. Since the axial cable is of stranded wire, E is taken as 17,100,000 lbs./in.², or 57% of the value for solid steel wire.

When there is no axial cable, the tension in the horizontal radial wires is given by

$$10\,T^3 = E\,a\,R^6 k^2 \tan^2 \alpha \qquad (95)$$

This may be taken as the approximate tension in all the wires of a simple radial or diametral system of wires without an axial cable.

The compressive force in the girders of the transverse frame due to the tension T in the equivalent radial wire is given by

$$S = \frac{T}{2 \sin \alpha}$$

Applying this method to the calculation of the stresses resulting from a deflated gas cell separated from a full one by the transverse frame in which the effect of a 6,500 lb. load in the keel was computed (page 203), we have:

$R = 38.2$ ft.
$a = 2 \times .00817 = .01634$ in.^{2}
$E = 30{,}000{,}000$ lbs./in.^{2}
$k = .068$ lb./ft.^{3}
$\alpha = 14° 15'$, $\tan \alpha = .254$

The mean pressure on the end of the cell is 2.7 lbs./ft.^{2}, and the area of the end of the cell is 4,510 ft.^{2}, whence the load on the axial cable is $2.7 \times 4{,}510/3 = 4{,}060$ lbs. The cross-sectional area of the cable is .1105 in.^{2}, and the distance of the frame from the end of the ship on the side of the full cell is 600 ft., whence $\delta = (4{,}060 \times 600)/(17{,}100{,}000 \times .1105) = 1.285$ ft., and from equation (94),

$$\frac{38.2\ T}{30{,}000{,}000 \times .01634} = \frac{2}{45}\left(\frac{.068^2 \times 38.2^7 \times .254^2}{T^2}\right) + \frac{1.285^2}{2 \times 38.2}$$

Whence $T = 2{,}820$ lbs., and the compressive force in each transverse girder is $2{,}820/(2 \sin 14°15') = 5{,}730$ lbs.

Unwired main frame. In airships of about 4,000,000 cu. ft. or over, the main frames may be constructed as deep rings without transverse wiring, as shown in Figure 59. Such a frame has only two redundant members, irrespective of the number of sides, so that it may be analyzed by the conventional method of least work more accurately than by inverse ratio, and almost as readily. There are no tight wires across the frame, but only a loose netting to take the end pressure of a gas cell in case of a differential pressure between two cells, or the deflation of one of them. This loose netting permits the end of the cell to bulge deeply, so that the stresses resulting from a deflated gas cell are much less than in cross-wired frames.

The deep ring frame is not nearly as efficient as the wired frame in resisting distorting forces, such as the lift of the gas at the joints around the circumference, opposed by the weights concentrated at the corridor. This difficulty may be avoided either by carrying the principal weights well out from the center line in the main frame itself, instead of in the corridor, or by

having two corridors placed as indicated in Figure 59. Either arrangement considerably reduces the stress in the frame; but even so, the stresses due to lift and weight are greater than those due to a deflated cell.

This type of frame has a great many girders, some of which carry relatively small loads, so that because of the minimum economical limit of girder sizes (page 245) the construction is not efficient for frames less than about 100 ft. diameter. A great advantage of it is the ready access it gives to the greater part of the ship structure in flight. The deep ring frame may also be regarded as an example of the general trend of aircraft design away from complex systems of wiring with their variable and uncertain distributions of tension.

The Intermediate Transverse Frames

The intermediate transverse frames serve as struts between the longitudinals, assisting them in connection with the shear wires to maintain the longitudinal rigidity of the ship. They also reinforce the longitudinals against the gas pressure, and loads from the outer cover.

The intermediate frames are very limber because of the absence of transverse wiring, and they are distorted by the gas pressure forces applied to them by the longitudinals and gas-cell netting. Owing to their limberness, the intermediate frames are not seriously stressed by their distortion; but the distortion is very important because of the stresses it throws back into the longitudinals. The forces at each joint of the intermediate frames may be resolved in three directions as follows:

(*a*) The resultant of the radial and tangential gas pressure loads transmitted to the joint by the longitudinal girder and the gas-cell netting.
(*b*) The end load along the transverse girder above the joint.
(*c*) The end load along the transverse girder below the joint, and in the same direction, the component in the transverse

plane of the tensions in the shear wires of the panel below the joint.

A sample computation of the gas pressure loads upon the longitudinals of a 32-sided airship, 122 ft. diameter, is given on pages 187 to 189. The intermediate frame spacing is 20 ft., and the radial and tangential gas pressure forces acting at the joints of the intermediate frame are assumed to be 20 times the running loads per foot calculated on the longitudinal girders. The tension in the intermediate transverse girder above a joint equals the radial force at the joint divided by 2 sin $\Delta\theta/2$ and diminished by the tangential load divided by 2 cos $\Delta\theta/2$.

TABLE 20. STRESSES IN THE INTERMEDIATE FRAME DUE TO THE GAS PRESSURE

Intde. Trans. Girder	Radial Load / 2 sin $\Delta\theta/2$ lbs.	Tangential Load / 2 cos $\Delta\theta/2$ lbs.	Tension in Girder lbs.
AB	7,560	34	7,526
BC	7,410	67	7,343
CD	7,150	97	7,053
DE	6,780	123	6,657
EF	6,350	145	6,205
FG	5,861	161	5,700
GH	5,326	171	5,155
HI	4,775	175	4,600
IJ	4,221	171	4,050
JK	3,681	161	3,520
KL	3,190	145	3,045
LM	2,763	123	2,640
MN	2,410	97	2,313
NO	2,140	67	2,073

$\Delta\theta = 11° 15'$
2 sin $\Delta\theta/2$ = .196
2 cos $\Delta\theta/2$ = 1.9904

Table 20 shows the computations of the tensions in the girders of an intermediate frame, based on the radial and tangential loads calculated in Table 18.

Some confusion may arise from the emphasis placed here on the distortion of the intermediate frame by gas pressure, since it was shown in Chapter VI that cross-sections of the ship to which no concentrated loads are attached tend to remain circular, in spite of the gas pressure. The explanation is that the distortion is mainly in the nature of an increase in the diameter of the original circle due to the tension in the girders, while the bottom point remains fixed relatively to the main frames; and such departures as occur from a circular shape result from variations in the sizes of the shear wires and frame girders, and irregularities in the gas pressure loads due to variable curvature in the gas cell netting. There is, of course, an absolute distortion of the intermediate frame following the slight distortion which occurs in the main frames in spite of the restraining effect of the wires, but we are concerned here only with the movements of the intermediate frames relatively to the main frames.

To calculate the movements of the joints of the intermediate frame, it is necessary to know the stresses in the girders of the frame and also the tensions in the shear wires which transmit the lift from the intermediate to the main frames.

Calculation of the tensions in the shear wires from the lift of the intermediate frames. In this calculation, we determine first the component of tension in the transverse plane. This component is $S \sin \phi$, where S is the total tension in the wire, and ϕ is the inclination of the wire to the longitudinal. Beginning at the top joint of the intermediate frame, the sum of the transverse components of the tensions in the shear wires leading downward and outward from the joint equals the tension which must be added to the tension in the uppermost girders of the intermediate frame to balance the radial load at the top joint.

Similarly, at any other joint, the component of the shear wire tension in the transverse plane equals the total tension necessary to give equilibrium with the radial and tangential forces, minus the tension in the girder. Or more simply, the component of the wire tension in the transverse plane in the panel below a joint equals the radial force at the joint divided by 2 sin $\Delta\Theta/2$, plus the tangential load at the same joint divided by 2 cos $\Delta\Theta/2$, less the tension in the transverse girder of the same panel. All these forces, including the divisions by the sine or cosine of $\Delta\Theta/2$, are calculated in the determination of the forces in the transverse girders (Table 20).

An easier method of computing the shear wire stresses in the case when the netting has the same curvature in all panels, and the longitudinals are at the apexes of a regular polygon, is to apply the shear theory. (Chapter VII.)

Let S_w = the sum of the tensions in the shear wires in any panel

ϕ = the inclination of the shear wires to the longitudinal girders

F = the total gas lift upon an intermediate frame

θ = the inclination of a panel to the horizontal

Then by the shear theory, and neglecting the weight of the intermediate frame,

$$S_w \sin\phi = \frac{F \sin\theta}{\Sigma \sin^2\theta}$$

But $\Sigma \sin^2\theta = n/2$ (see page 167), where n is the number of sides of the ship; hence

$$S_w \sin\phi = \frac{2F \sin\theta}{n}$$

When the ship is fully inflated $F = k\,\pi\,R^2_1\,l$, where l is the intermediate frame spacing. In this particular problem,

$$F = .064 \times \pi \times 60.2^2 \times 20 = 14{,}600 \text{ lbs.}$$

$$S_w \sin\phi = 912 \sin\theta$$

The numerical values of $S_w \sin\phi$ for all panels are then as shown in Table 21.

Distortion of the intermediate frame. For the calculation of the movements of the joints of the intermediate frame, the structure is assumed to be pin-jointed with inelastic longitudinals, as in the shear theory of longitudinal strength. The movement of each joint is then determined by the stresses in the structure consisting of the transverse girder above the joint,

TABLE 21. TRANSVERSE COMPONENTS OF TENSIONS IN SHEAR WIRES (S_w SIN ϕ) DUE TO LIFT OF THE INTERMEDIATE FRAME

Panel	Θ	sin Θ	$S_w \sin\phi = 912 \sin\Theta$ lbs.
AB	84° 22½′	.0980	89.5
BC	73° 7½′	.2903	265
CD	61° 52½′	.4714	431
DE	50° 37½′	.6344	579
EF	39° 22½′	.7730	705
FG	28° 7½′	.8819	805
GH	16° 52½′	.9569	874
HI	5° 37½′	.9952	908
IJ	− 5° 37½′	.9952	908
JK	− 16° 52½′	.9569	874
KL	− 28° 7½′	.8819	805
LM	− 39° 22½′	.7730	705
MN	− 50° 37½′	.6344	579
NO	− 61° 52½′	.4714	431

and the shear wires in the panels above and below the joint. Considering the radial and tangential movements separately, the former is given by

$$\Delta_r = \frac{SL}{2EA \sin\frac{\Delta\theta}{2}} \text{(girder)} + \Sigma \frac{(S \sin\phi)\, L}{2EA \sin^2\phi \sin\frac{\Delta\theta}{2}} \text{(wires)}$$

And similarly, the tangential movement is given by

$$\Delta_t = \frac{SL}{2EA \cos\frac{\Delta\theta}{2}} \text{(girder)} + \Sigma \frac{(S \sin\phi)\, L}{2EA \sin^2\phi \cos\frac{\Delta\theta}{2}} \text{(wires)}$$

These equations are a three-dimensional application of Maxwell's theorem of reciprocal displacements.

In reality, the girders are continuous, instead of pin-jointed; the gas-cell netting, the outer cover, and the weight of the structure oppose the movements of the joints, so that the actual movements are only about one-half to two-thirds as much as calculated. It is on the safe side to multiply the calculated deflection by two-thirds, or to assume E increased by 50%.

Sample calculation of the distortion of an intermediate frame. The following sample calculation of the distortion of an intermediate frame is based on the longitudinal girder loads computed in Table 18. It is given that:

L (girder) $= 135$ in.
L (wires) $= 375$ in.
E (girders) $= 1.5 \times 10{,}500{,}000 = 15{,}750{,}000$ lbs./in.^{2}
E (wires) $= 1.5 \times 30{,}000{,}000 = 45{,}000{,}000$ lbs./in.^{2}
A (girders) $= .40$ in.^{2}
A (wires) $= .032$ in.^{2}
$\sin \Delta\theta/2 = .098$
$\cos \Delta\theta/2 = .9952$
$\sin^2 \phi = .590$

The magnitudes of $S \sin \phi$ are given in Table 21. From these figures, the radial movement of any joint is equal to $.000109 \times$ (tension in girder above joint) $+ .00225 \times$ (tension in wire below joint less tension in wire above joint) $\times \sin \phi$.

And the tangential movement is $.000222 \times$ (sum of tensions in wires above and below the joint) $\times \sin \phi - .0000107 \times$ (tension in girder above joint).

TABLE 22. MOVEMENTS OF JOINTS OF INTERMEDIATE TRANSVERSE FRAME

Joint	Stress in Girder above Joint lbs.	Stress in Wire above Joint × sin φ lbs.	Stress in Wire above Joint × sin φ lbs.	Radial Motion in.	Tangential Motion in.
A	0	0	89.5	.404	0
B	7,526	89.5	265	1.214	−.002
C	7,343	265	431	1.174	.076
D	7,053	431	579	1.103	.148
E	6,657	579	705	1.009	.214
F	6,205	705	805	.901	.269
G	5,700	805	874	.776	.311
H	5,155	874	908	.638	.341
I	4,600	908	908	.501	.354
J	4,050	908	874	.364	.353
K	3,520	874	805	.229	.334
L	3,045	805	705	.107	.302
M	2,640	705	579	.004	.257
N	2,313	579	431	−.081	.199

The movement of the top joint is purely radial, and the sectional area of the shear wires is doubled to allow for the wires on both sides of the ship restraining the movement of this joint.

The convention with regard to signs is that radial movements are counted as positive when outward, and tangential movements as positive when upward.

The calculations are shown in Table 22.

Stresses in the longitudinals due to distortion of the intermediate transverses. The longitudinal girders are considered to be fixed in position and direction over the main frames, and to be continuous, but not fixed in direction, over the intermediate frames. When there is one intermediate frame the bending moment in the longitudinal girder at the main and intermediate frames is given by

$$M = \frac{24\, EI\Delta}{L^2}$$

where

M = the bending moment
E = the modulus of the girder in bending
I = the moment of inertia of the cross-section of the girder about an axis perpendicular to the plane of bending
Δ = the deflection of the joint of the longitudinal girder with the intermediate frames out of the straight line between the joints with the main frame
L = the length of the longitudinal between main frames

When there are two intermediate frames between main frames, the bending moment at the main frames due to equal deflections Δ, of the two intermediate frames is given by

$$M = \frac{36\, E\, I\, \Delta}{L^2}$$

And in this case the bending moment at the intermediate frame is given by

$$M = \frac{18\, E\, I\, \Delta}{L^2}$$

The fiber stress due to the bending moment is given by: $f = My/I$, and by substituting in the foregoing equation for M when there is one intermediate frame between main frames:

$$f = \frac{24\,E\,\Delta\,y}{L^2}$$

And when there are two intermediate frames,

$$f = \frac{36\,E\,\Delta\,y}{L^2} \quad \text{at the main frames}$$

and

$$f = \frac{18\,E\,\Delta\,y}{L^2} \quad \text{at the intermediate frames}$$

For Zeppelin type girders in bending, $E = 9{,}000{,}000$ lbs./in.², approximately.

As an example of the stresses produced in the longitudinals by the distortions of the intermediate frame, it is given that longitudinal C in the foregoing example on the calculation of the distortion of an intermediate frame is of isosceles triangular section; y from the tangential neutral axis is 8.5 in. to the apex channels, and 5.7 in. to the base channels; y from the radial neutral axis is 5.3 in. to either base channel; the main frames are 60 ft. apart with two intermediate frames between them. From Table 22, the radial and tangential movements of the intermediate frame at the C longitudinal are 1.174 in. and .076 in., respectively. The resulting compressive stress in the apex channel at the main frames is

$$\frac{36 \times 9{,}000{,}000 \times 1.174 \times 8.5}{720^2} = 6{,}250 \text{ lbs./in.}^2$$

The lower base channel receives the maximum compressive stress at and between the intermediate frames where it is in compression from the bending produced by both the radial and tangential deflections; the stress is:

$$\frac{18 \times 9{,}000{,}000}{720^2}\left[(1.174 \times 5.7) + (.076 \times 5.3)\right] = 2{,}220 \text{ lbs./in.}^2$$

It may be noted that the stress produced in the apex channel by the radial deflection is of very considerable magnitude. The tangential deflection produces stresses in the base channels only.

Since the distortion of the intermediate frame is due mainly to radial gas pressure loads on the longitudinals, and these loads result from the bulge of the gas pressure netting, the large stresses which may be produced in the longitudinals by the movements of the intermediate frame furnish an important argument for keeping this bulge as small as practicable.

CHAPTER IX

DESIGN OF GIRDERS

Materials of construction. Airship girders have been constructed of wood, aluminum alloys, and high tensile steel. Wooden girders were developed to a high degree of perfection by the Schütte-Lanz Airship Company. Until the development of the aluminum alloy known as Duralumin, wood was probably the most suitable material available. The development of duralumin gave airship designers a very perfect material for their purposes. At first it was believed that this metal, among its other excellent properties, had almost complete freedom from corrosion. In recent years it has been found that, like other materials, it must be protected against the weather by protective coatings of paint or some such oxide as the recently developed electrical anodic process. The most serious feature of duralumin corrosion is that instead of being a surface action which puts forth its own warning signals like the red rust of steel, it is mainly an intercrystalline action working from the outside toward the interior of the metal and difficult to detect.

With duralumin, as with other metals, corrosion becomes less serious as the thickness of the material increases, and it is believed that with thicknesses greater than about .06 in. the loss of strength through corrosion of reasonably well protected and cared for duralumin is not likely to become serious within the life of an airship. The subject of corrosion of duralumin and methods of preventing it is very much to the fore in aeronautical engineering, and it may be expected that great progress in our knowledge of the causes and prevention of corrosion will be made in the next few years.

One advantage of duralumin is that immediately after heat

treatment it is comparatively soft, and may be conveniently worked into any desired shape before the process of aging, which takes two or three days to develop the hardness characteristic of finished duralumin. Some investigators have believed that working after heat treatment renders the material more liable to corrosion, but the basis of this belief is not well established and may be regarded with some skepticism.

A typical chemical composition of duralumin is as follows:

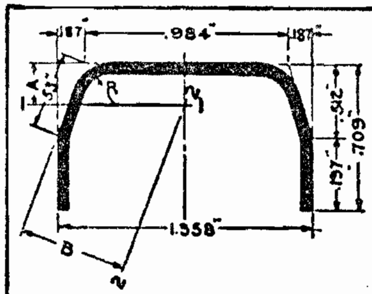
Copper	4.23%
Magnesium	.54%
Manganese	.52%
Iron	.45%
Silicon	.26%
Aluminum	94.00%

Physical properties are as follows:

Tensile strength at ultimate	55,000 lbs. per sq. in.
Tensile strength at elastic limit	30,000 lbs. per sq. in.
Elongation in 2 in. at ultimate strength	18%
Specific gravity	2.81

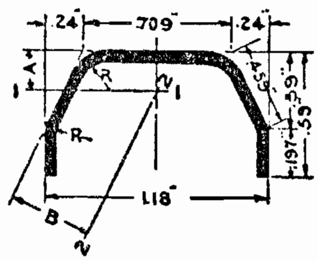
Both the yield point and ultimate tensile strength may be increased about 10,000 lbs. per sq. in. by a moderate amount of cold rolling, which reduces the elongation to about 15%.

High tensile steel. High tensile steel has certain advantages over duralumin as structural material for aircraft, provided the forces involved are sufficient to permit of a reasonable thickness of material being used. It has a somewhat greater ratio of strength to weight, and it is therefore invariably used for the wiring of airships. Loads upon present types of rigid airship girders are not sufficiently great to use steel of a practicable thickness, so that it can not be considered as a serious rival to duralumin until airships are constructed much larger than at present, or types are evolved having fewer and larger girders. The keel of a semirigid airship is a structure in which the forces in the individual members may be sufficiently great to justify the use of steel.



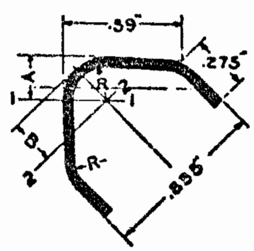
	A1	A2	A3
THICKNESS—INS.	.039	.047	.055
SECT. AREA. SQ. INS.	.09306	.1109	.12832
R. (INSIDE—RADII)	.120	.160	.160
A. (FROM TOP) INS.	.230	.231	.232
B (FROM WEB) INS.	.541	.542	.543
I. (ABOUT 1-1) INS.4	.00478	.00572	.00659
I (ABOUT 2-2) INS.4	.02064	.02433	.02763

CHANNELS A1, A2 AND A3.



	B1	B2	B3	B4	B5	B6
THICKNESS-INS	.031	.039	.047	.055	.063	.079
SECT AREA SQ INS.	.06121	.07604	.09066	.10495	.11933	.14457
R (INSIDE RADII) INS	.120	.120	.140	.160	.175	.175
A (FROM TOP) INS.	.2070	.2098	.2106	.2124	.2142	.2160
B (FROM WEB)-INS.	.4130	.4138	.4146	.4154	.4162	.4170
I. (ABOUT 1-1) INS.4	.00216	.00229	.00313	.00357	.00386	.00470
I (ABOUT 2-2)-INS 4	.00830	.0103	.01242	.01425	.01603	.01908

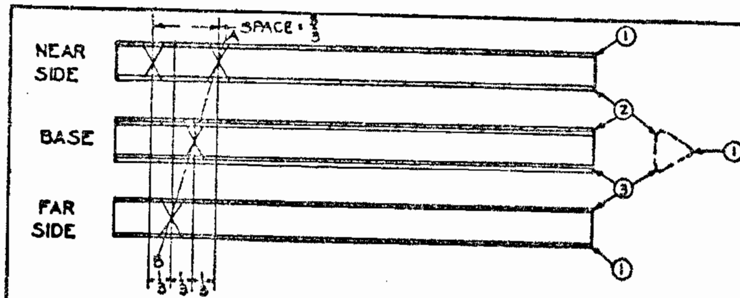
CHANNELS B1, B2, B3, B4, B5, B6.



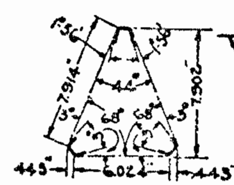
	N1	N2	N3	N4
THICKNESS-INS	.039	.047	.055	.063
SECT. AREA-SQ. INS	.063364	.07545	.08735	.09904
R (INSIDE RADII)-INS	.120	.160	.160	.175
A (FROM WEB)-INS.	.238	.242	.246	.250
B (FROM TOP)-INS.	.275	.261	.260	.256
I. (ABOUT 1-1) INS 4	.00377	.00432	.00497	.00555
I (ABOUT 2-2)-INS 4	.00214	.00230	.00257	.00290

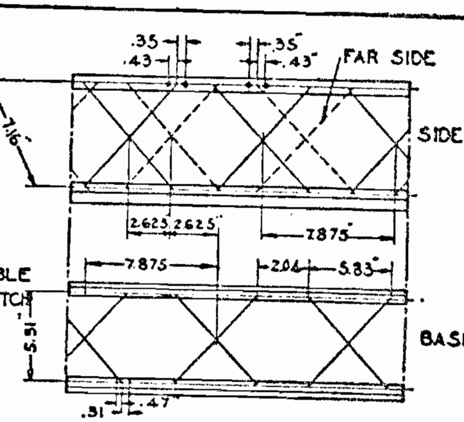
ANGLES N1, N2, N3, N4.

Figure 60. Dimensions and Properties of Channels and Angles



LINE A-B INDICATES THE DIRECTION OF STANDARD SPIRAL.
THE DIRECTION OF THE LINE OF SPIRAL IS ALWAYS N. EAST AND S. WEST.
SHOWING SPIRAL OF LATTICE CENTERS AROUND GIRDER.





GIRDER NO. 23 HAS DOUBLE LATTICE ON SIDES AT PITCH OF ½ OF 7.875"
ALL DIMENSIONS ARE IN INCHES.

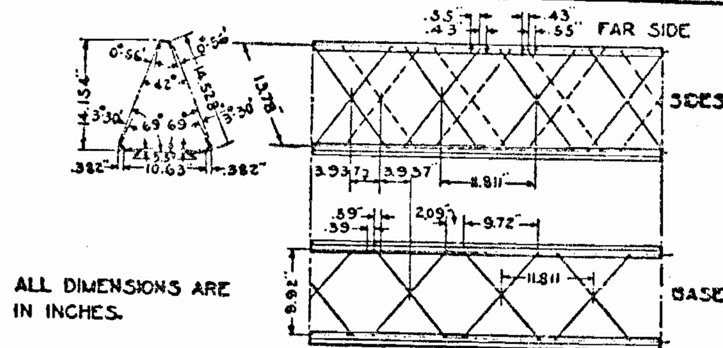
GIRDERS 21 & 23

GIRDER NO.	APEX CHAN.	THICK	BASE CHAN	THICK	THICKNESS LATTICE SIDE	THICKNESS LATTICE BASE
21	A1	.039	B1	.031	.019	.019
23	A1	.039	B2	.031	.019	.018

Figure 61. Dimensions and Properties of Zeppelin Type Girders

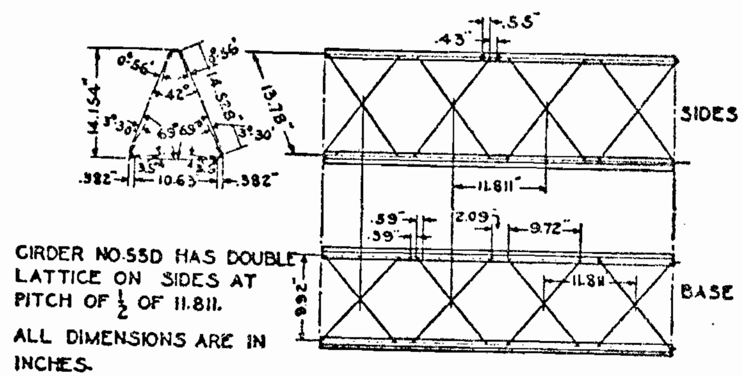
Old type of Zeppelin girder. The type of girder in most general use in rigid airships has been developed by the Zeppelin Company. It is usually of triangular section, although rectangular and "W" sections are also used for special purposes. In triangular girders there are three longitudinal members of lipped channel section, one along each apex of the triangle. In rectangular girders there are four longitudinal members of lipped angle section; in "W" girders lipped channels or angle members are used as best fits the exigencies of the case. Typical lipped angle and channel members are shown in Figures 60 and 61. The sides of the girders are laced with stamped lattices of corrugated section, and for mutual support the lattices are in pairs crossing each other at their centers and riveted together at the crossing.

A peculiarity of the Zeppelin girder which at once strikes the eye of the engineer accustomed to other types of structures, is the curious unsymmetrical placing of the lattices. One of the fundamental rules usually accepted in girder design is that the center line of lattice members should intersect on the center line of the longitudinal members. This rule is altogether violated in Zeppelin girders. The reason for this peculiarity is that the transverse shearing forces induced in the longitudinal members by the gaps between opposed lattice members produce only negligible stresses in the channels; but on the other hand, the arrangement of lattices adopted by the Zeppelin Company reduces the unsupported length of channels between bracing members with a given number of these members, and this reduction of the unsupported length greatly increases resistance of the channels to compressive loads. Girders of this type are illustrated in Figures 61 to 65. In girders designed for purely compressive loads, the cross-sections are usually equilateral triangles. For girders designed to carry beam loads or combined beam and column loads, as in the longitudinal girders, isosceles triangular sections are preferred, with the altitude considerably greater than the width of the base and in the direction of the



GIRDER NO	APEX CHAN	THICK	BASE CHAN	THICK	THICK OF LATTICE	
					SIDE	BASE
52	A1	.039	B2	.039	.019	.019
53	A2	.047	B2	.039	.019	.019
54	A2	.047	B3	.047	.019	.019
55	A3	.055	B3	.047	.023	.019
55A	A3	.055	B3	.047	.023	.019

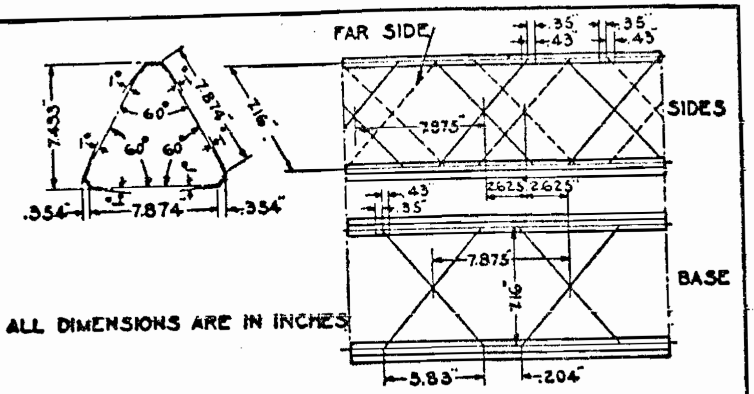
GIRDERS 52 53 54 55 55A.



GIRDER No.	APEX CHAN	THICK.	BASE CHAN.	THICK	THICK OF LATTICE	
					SIDE	BASE
55B	A3	.055	B3	.047	.023	.019
55C	A3	.055	B3	.047	.019	.019
550	A3	.055	B3	.047	.019	.019

GIRDERS 55B 55C AND 55D.

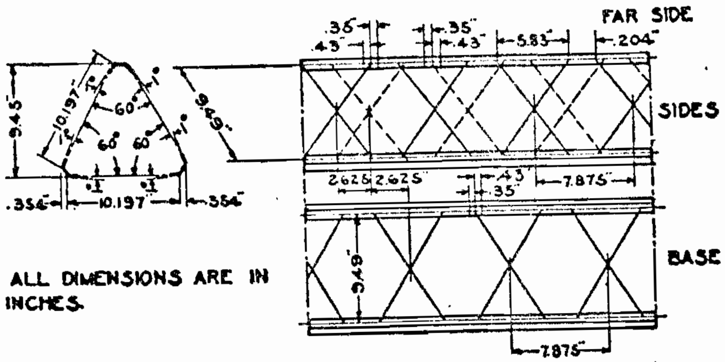
Figure 62. Dimensions and Properties of Zeppelin Type Girders



ALL DIMENSIONS ARE IN INCHES

GIRDER No.	APEX CHAN	THICK.	BASE CHAN	THICK	THICK OF LATTICE	
					SIDE	BASE
31 B	82	.039	B 2	.019	.019	.019
32 B	83	.047	82	.039	.023	.023

GIRDER 31B & 32

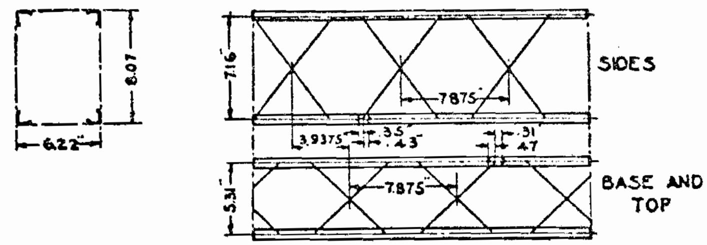


ALL DIMENSIONS ARE IN INCHES.

GIRDER No.	APEX CHAN.	THICK.	BASE CHAN.	THICK.	THICK. OF LATTICE	
					SIDE	BASE
42	83	.047	83	.047	.019	.019

GIRDER 42

Figure 63. Dimensions and Properties of Zeppelin Type Girders

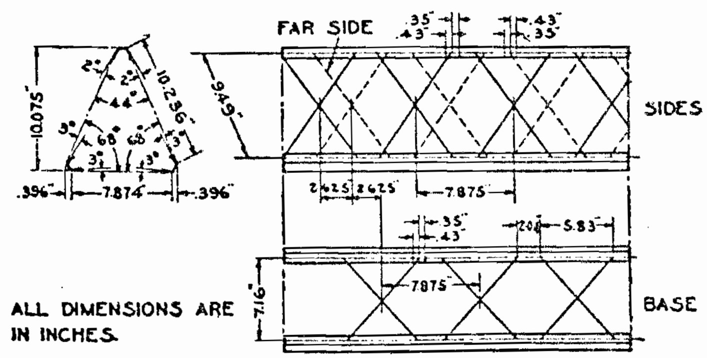


GIRDER NO. 72 HAS SPECIAL SPACING ON SIDES 5.748"

ALL DIMENSIONS ARE IN INCHES

GIRDER No.	ANGLES	THICK	THICK OF LATTICE	
			SIDE	BASE
71	N 3	.055	.019	.019
72	N 3	.055	.019	.019

GIRDERS 71 AND 72



ALL DIMENSIONS ARE IN INCHES.

GIRDER No	APEX CHAN.	THICK	BASE CHAN	THICK	THICK OF LATTICE	
					SIDE	BASE
91	A1	.039	81	.031	.019	.019
92	A2	.047	82	.039	.019	.019

GIRDERS 91 AND 92

Figure 64. Dimensions and Properties of Zeppelin Type Girders

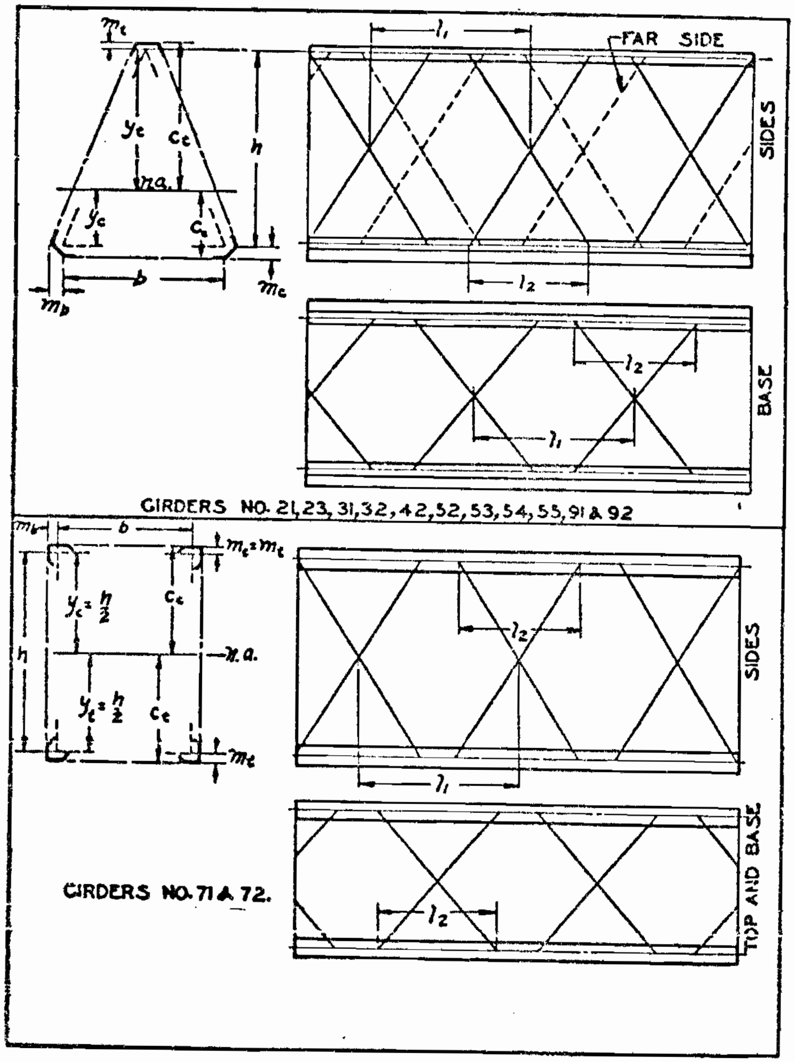


Figure 65. Dimensions of Girders

principal transverse load, i.e., radially to the cross-section of the ship.

The lattices. The lattices used in Zeppelin type girders are stampings made from duralumin sheet, usually .02 in. in thickness. This is about the minimum thickness of material which will permit of reasonable stiffness to resist damage from handling and men climbing about on the girders.

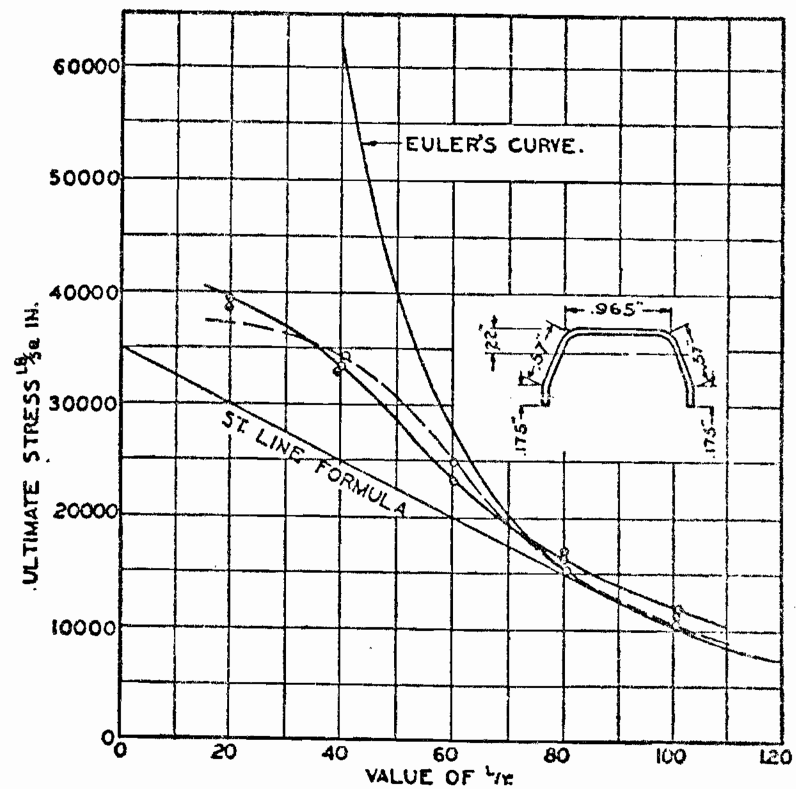
Tensionmeter readings taken at the Bureau of Standards on lattices in girders under load show that the stresses in the lattices are extremely small. Values of 2,000 to 3,000 lbs. per sq. in. are the maximum met with. The only failures encountered in the testing of 150 girders were those which occurred in the thinnest lattices and at shears in the girders far beyond the loads met with in practice. A design for a new lattice may well follow the Zeppelin prototype as to the depth of rib and bringing the edges up to the neutral axis. It should have the proper slope, according to the depth of girder, to break down the length of the longitudinal members to give the l/r ratio desired. It is believed that the high center rib should be carried out as far as possible. This adds to the stiffness of the channel when in place and assists it to carry compressive loads. It is found that the exact form of any lattice is considerably influenced by the necessity for providing certain radii and "draws" in the dies.

Compressive strength of duralumin channels and angles. Airship girders are usually designed with the ratio of the length to radius of gyration less than 50, so that primary failure by flexure under column loading hardly has to be considered; and the most important criterion of strength is the load which the longitudinal channel or angle members will sustain as columns between the lattice bracing members. This is the limiting factor for both beam and column loading. An important series of tests undertaken by the Massachusetts Institute of Technology at the instigation of the Bureau of Aeronautics is described in the National Advisory Committee for Aeronautics Note No. 208, "Tests on Duralumin Columns for Aircraft Construction," by John G. Lee, from which the following is taken:

Compression tests were made on four sets of heat-treated duralumin columns of different cross-section and varying thickness for different values of l/r. The specimens to be tested were cut to length, their ends machined square, and mounted between hemispherical bearing caps in an Olsen vertical testing machine (accurate to 20 lbs.). In mounting the specimens, a template was used to insure accurate centering. The load was applied by hand. Three individual tests were averaged for each point appearing on the plots.

The most interesting thing to be observed in the plots is the behavior of the columns at low values of l/r. While at the higher values ($l/r = 80$ and upwards) the points lie close to Euler's curve, at the lower values they do not break away in the parabolic form which we ordinarily expect. Instead, the curves break away rather earlier and in some instances remain concave upwards throughout their entire length. This tendency is most marked in thin sections with free edges far from the neutral axis, such sections being peculiarly liable to secondary failure by crinkling. Presumably, if the curves for such sections were continued below $l/r = 20$, they would continue rising more and more steeply until they reached the ultimate compressive strength (55,000 lbs./sq. in.) at $l/r = 0$. As the sections are thickened up, the points lie closer to a straight line than to a curve. It may therefore be assumed that if we continued to increase the thickness the curve will eventually become convex upward, and take the more familiar form, the maximum value being at 55,000 lbs./sq. in. at $l/r = 0$ as before. For two of the groups of sections (series A and B) the reversal in curvature actually appears.

This peculiar deviation from conventional column behavior appears to be due to local failure of the long unsupported flanges of the column. This failure is manifested in several ways. In the shorter columns it takes the form of direct local buckling.



Mark	Pt. on Curve	Thick. Stand.	Area Actual	Least Rad. Gy.
A-1	○	.035	.0816	.232
A-2	◉	.050	.1167	.230
A-3	◉	.055	.1259	.212

Figure 66. Compression Tests of Duralumin Channels and Angles

Compression tests mark "A" heat-treated duralumin columns. Columns tested as pin-ended struts. Least radius of gyration is about axis of non-symmetry.

In the longer columns both flanges occasionally bend in the same direction, which amounts to giving the center of the column an angular displacement relative to the ends, and the failure is by twisting. Again, if the flanges bend in opposite directions, distortion takes place, and not infrequently the column fails about an axis which formerly had the greatest moment of inertia. In the case of the plain angle sections, where the outstanding legs receive the least support of any, the outer portions of these legs appeared to act as separate columns and buckle separately, as could be so noticed during the test.

It is useful to adopt a formula which will approximately fit all of the various column sections with which the designer is concerned. It is bound to be over-conservative in most cases, but the designer is constantly in need of some such formula for sections on which he has no tests. The following are accordingly suggested for pin-ended columns:

From $l/r = 0$ to $l/r = 90$

$$f_c = 35{,}000 - 250\, l/r \qquad (96)$$

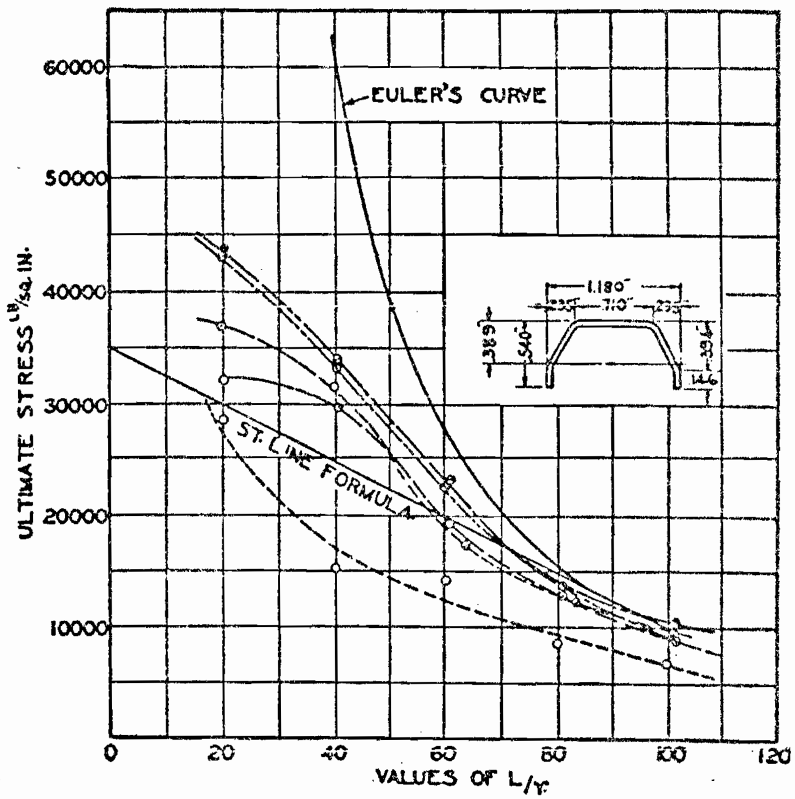
From $l/r = 90$ upwards

$$f_c = 100{,}000{,}000/(l/r)^2 \qquad (97)$$

The first of these is an entirely empirical straight-line formula which is practically tangent to the second at $l/r = 90$. The second is Euler's formula $f_c = (\pi)^2 E/(l/r)^2$, where E for duralumin has been taken as 10,130,000 lbs./sq. in., which is a very reasonable value. Another formula which is of interest is that suggested by the Army of the form

$$f_c = .8\,[47{,}000 - 400(l/r)] \qquad (98)$$

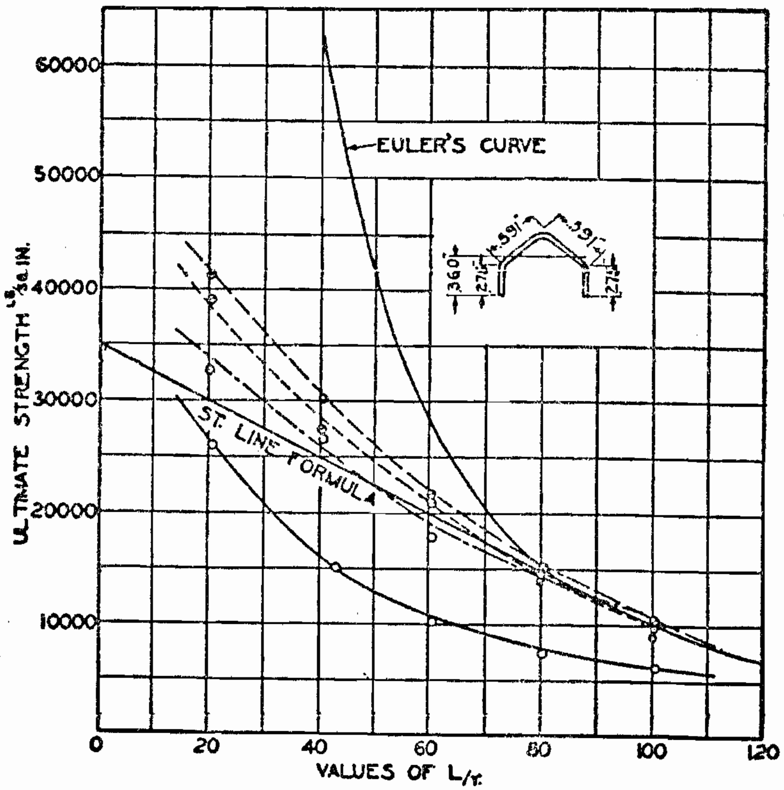
for values of l/r from 0 to 80. The .8 factor disappears for tubes and bars, but is used for all other sections. For columns fixed at the ends, the 400 becomes 280 and the formula is good up to $l/r = 110$. The principal difficulty with the Army formula is that it does not come tangent to Euler's curve by a considerable amount. It fits the points almost as well as formula (96) however.



Mark	Pt. on Curve	Thick. Stand.	Area Actual	Least Rad. Gy.
B-1	o	.0310	.0595	.194
B-2	o	.0394	.0818	.192
B-3	o	.0473	.0949	.188
B-4	o	[illegible]	[illegible]	[illegible]
B-5	o	.0630	.1282	.187

Figure 67. Compression Tests of Duralumin Channels and Angles

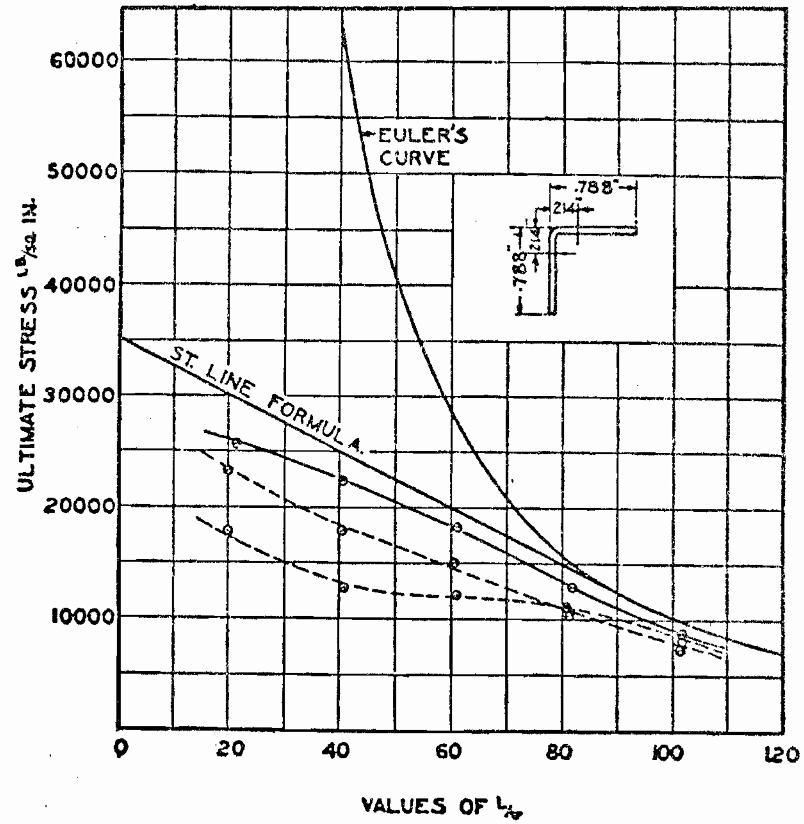
Compression tests mark "B" heat-treated duralumin columns. Columns tested as pin-ended struts. Least radius of gyration is about axis of non-symmetry.



Mark	Pt. on Curve	Thick. Stand.	Area Actual	Least Rad. Gy.
N-1	○	.0394	.068	.180
N-2	⊙	.0473	.079	.178
N-3	⊕	.0545	.084	.175
N-4	⊖	.0675	.099	.172

Figure 68. Compression Tests of Duralumin Channels and Angles

Compression tests mark "N" heat-treated duralumin columns. Columns tested as pin-ended struts. Least radius of gyration is about axis of non-symmetry.



Mark	Pt. on Curve	Thick. Stand	Area Actual	Least Rad. Gy.
S-1	⊙	.0394	.061	.157
S-2	⦶	.0473	.076	.156
S-3	⊖	.0552	.085	.156

Figure 69. Compression Tests of Duralumin Channels and Angles

Compression tests mark "S" heat-treated duralumin columns. Columns tested as pin-ended struts. Least radius of gyration is about axis of non-symmetry.

The curves determined by formulas (96) and (97) are drawn in on each of the four plots. It will be noticed in Figures 67 and 68 that the channels B-1 and N-1 fall below the curve. These two channels are so exceedingly thin for their size that they are quite uneconomical sections; their ratio of thickness to perimeter is 1/60 and 1/40, respectively. The same thing may be said of the "S" sections in Figure 69. Here we find all of the test specimens below the curve. If, however, we extrapolate from the test values, we conclude that the formula will hold for angles in which the thickness of plate is at least 1/10 of the length of a leg. Figure 69 shows very clearly the inefficiency of the plain angle section.

Tests of Airship Girders

About 150 specimen girders similar to those incorporated in the hull of the *Shenandoah* were tested to destruction at the Bureau of Standards during the construction of that airship. These tests were very fully described and discussed in a paper on "The Strength of Rigid Airships" by Burgess, Hunsaker, and Truscott, published in the Journal of the Royal Aeronautical Society, June, 1924. The principal characteristics of the different types of girders are shown in Figures 61 to 65 inclusive. The type numbers of the girders in the figures correspond to the following members in the ship:

Type Number	*Position in Ship*
21, 23	Upper secondary longitudinals
52, 53, 54, 55	Main longitudinals
71, 72	Box girders in keel
91, 92	Lower secondary longitudinals

In order to obtain design data, the tests on the girders were extended over a much greater range of l/r than occurs in the actual ship. Tests included 10-meter lengths of longitudinal girders with ball ends and no intermediate support; whereas in actual practice, the girders are held with a very considerable

degree of fixity in regard to both position and direction at 5-meter intervals.

Girders were tested as columns, as beams, and with combined column and beam loading. In the tests as beams, the girders were simply supported at the ends, and a uniformly distributed load was applied upon the base. In every case, failure occurred by compressive buckling of the base channels. The stress in these channels is given by the formula:

$$f = \frac{W\,l\,y}{8\,I}$$

where

W = the total load

l = the length of the girder

I = the moment of inertia about the minor principal axis, calculated on the assumption that the area of each channel is concentrated at its center of gravity

y = the distance from the neutral axis to the line through the center of gravity of the base channels

TABLE 23. FAILING STRESSES IN BASE CHANNELS OF LONGITUDINALS TESTED AS PIN-ENDED BEAMS

Type	Length in.	I/y in.^{3}	Average Failing Load lbs.	Fiber Stress in Base Channels lbs./in.^{2}
21	197	.837	697	20,500
23	197	.837	732	21,500
52	197	2.006	1,636	20,050
53	197	2.019	1,664	20,300
54	197	2,314	1,866	19,850
55	197	2.377	2,042	21,200
71	197	1.289	1,355	25,900
72	197	1.289	1,410	26,950
91	197	1.139	988	21,400
92	197	1.404	1,273	22,400
55c	197	2.377	1,747	18,100
55d	197	2.377	2,540	26,350
23	394	.837	349	20,550
55	394	2.377	1,024	21,250
72	394	1.289	670	25,650
92	394	1.404	622	21,800

In most cases, three specimens of each type of girder were tested.

The failing stresses in the base channels are given in Table 23. Inspection of the table shows that the regular main and secondary longitudinals (types 21, 91, and 52 to 55) failed with fair consistency at stresses of about 20,500 lbs./in.² The box girders, 71 and 72, had a failing stress about 25% higher.

The special girders 55c and 55d are interesting. It was found in the tests of the ordinary types of girder that the lattices were very lightly stressed; and the manner of failure of the channels seemed to show that the method of arranging the lattices so that those upon the two sides of the channel are not directly opposite each other, giving in effect a kind of spiral arrangement of the latticing, caused the channel to fail by twisting. It was therefore decided to make several experimental girders of type 55c, in which the lattices were .5 mm. in thickness, instead of .6 mm. as in the regular 55 type, and the lattices were riveted to the flanges of the channels at points directly opposite each other, in accordance with the more usual structural practice. The lower failing stress of type 55c shows that the gain of symmetry in the arrangement was more than offset by the increased span of the channels between supports.

Type 55d was the same as 55c, except that the lattice spacing was halved. This had the effect of increasing the failing stress in the channels by about 24% as compared with type 55, and 45% compared with 55c. The weight, exclusive of end fittings, was increased 35% over 55, and 43% over 55c. It appears that on this basis, the regular type 55 was the most efficient girder, and 55c the least efficient.

It may be noted that the apex channel is about 50% farther from the neutral axis than the base channels, so that when the base channels failed under the load, there was a tensile stress of something over 30,000 lbs./in.² in the apex channels; but this is well below the failing stress of duralumin in tension. Since

failure invariably occurred by compression of the base channels, the stress in these members, and not that in the apex channel, is the critical factor in the girders.

Modulus of elasticity in bending. A calculation of the modulus of elasticity of these girders in bending has been made from the deflections of ten typical girders selected at random. The formula connecting the deflection at the center of a uniformly loaded simple beam with the load and the properties of the beam is:

$$d = \frac{5}{384} \frac{W l^3}{EI}$$

$$\text{or } E = \frac{5}{384} \frac{W l^3}{Id}$$

where d is the deflection at the center of the girder, and the other symbols have their usual significance.

The results are shown in Table 24. The comparatively low

TABLE 24. CALCULATION OF E IN BENDING OF GIRDERS

Type	l in.	I in.⁴	W lbs.	d in.	$E = \frac{5}{384} \frac{W l^3}{I d}$ lbs./in.²
54	197	12.08	1,000	1.075	7,670,000
54	197	12.08	1,429	1.774	6,640,000
54	197	12.08	869	.927	7,720,000
54	197	12.08	1,300	1.500	7,150,000
55	197	13.23	1,000	.990	7,600,000
55	197	13.23	1,600	1.502	8,000,000
55	394	13.23	750	4.90	9,200,000
55	394	13.23	680	4.50	9,100,000
21	197	2.725	425	1.53	10,150,000
23	394	2.725	235	7.30	9,400,000

values of E for 5-meter lengths of the 54 and 55 girders are undoubtedly due to the deflection by shear being relatively more important in these short, deep girders, than in the more slender ones. The above formula takes no account of shearing deflection.

Tests as ball-ended columns. The ratio of length to radius of gyration (l/r) of 5-meter lengths of the main longitudinals is only about 48, and is far below the Euler range. The formula of J. B. Johnson should be applicable, as follows:

$$p_i = C\left[1 - \left\{\frac{C}{4N\pi^2 E}\left(\frac{l}{r}\right)^2\right\}\right]$$

Where

p_i = failing load in lbs./in.²
C = compressive strength of the material in lbs./in.²
N = 1 for ball-ended columns

TABLE 25. TESTS OF GIRDERS AS BALL-ENDED COLUMNS

Type	l/r	$(l/r)^2$	a in.²	C lbs./in.²	E lbs./in.²	P lbs.	P/a lbs./in.²	p_i lbs./in.²	p_e lbs./in.²
5 m. LENGTHS:									
21	85.6	7,350	.2115	20,500	9,000,000	2,460	11,610	11,850	12,100
23	85.6	7,350	.2115	21,500	9,000,000	2,500	11,820	12,170	12,100
52	47.0	2,210	.2412	20,050	7,500,000	4,427	18,390	17,100	
53	48.7	2,370	.2584	20,300	7,500,000	4,535	17,550	17,000	
54	47.1	2,220	.2866	19,850	7,500,000	4,928	17,200	16,900	
55	48.5	2,350	.3043	21,200	7,500,000	5,553	18,250	17,650	
71	71.4	5,100	.3360	25,900	9,000,000	5,530	16,450	16,350	17,500
72	71.4	5,100	.3360	26,950	9,000,000	5,640	16,800	16,600	17,500
91	66.5	4,420	.2115	21,400	8,000,000	3,767	17,830	15,000	17,900
92	65.7	4,320	.2584	22,400	8,000,000	4,537	17,500	15,600	18,300
55a	48.5	2,350	.3043	21,200	7,500,000	5,200	17,050	17,650	
55b	48.5	2,350	.3043	18,100	7,500,000	4,843	15,900	15,500	
55d	48.5	2,350	.3043	26,350	7,500,000	6,970	22,900	21,050	
10 m. LENGTHS:									
23	171.2	29,400	.2115	20,550	9,400,000	733	3,470		3,160
55	97.0	9,400	.3043	21,250	9,150,000	3,137	10,300	9,580	9,640
72	141.8	20,400	.3360	25,650	9,500,000	1,697	5,570		4,600
92	131.4	17,280	.2584	21,800	9,000,000	1,463	5,650		5,160

The average values for C and E for each type of girder are taken from the tests as simple beams. The values of p_i, calculated by the formula, are compared in Table 25 with the actual values of P/a, which is the total end load divided by the cross-sectional area of the girder. The Euler failing stress, p_e, is also shown where applicable. The formula for p_e is,

$$p_e = \pi^2 E / \left(\frac{l}{r}\right)^2$$

The values of P/a have been plotted in Figure 70, and the curve of p_e drawn for a uniform value of E = 10,000,000 lbs./in.²,

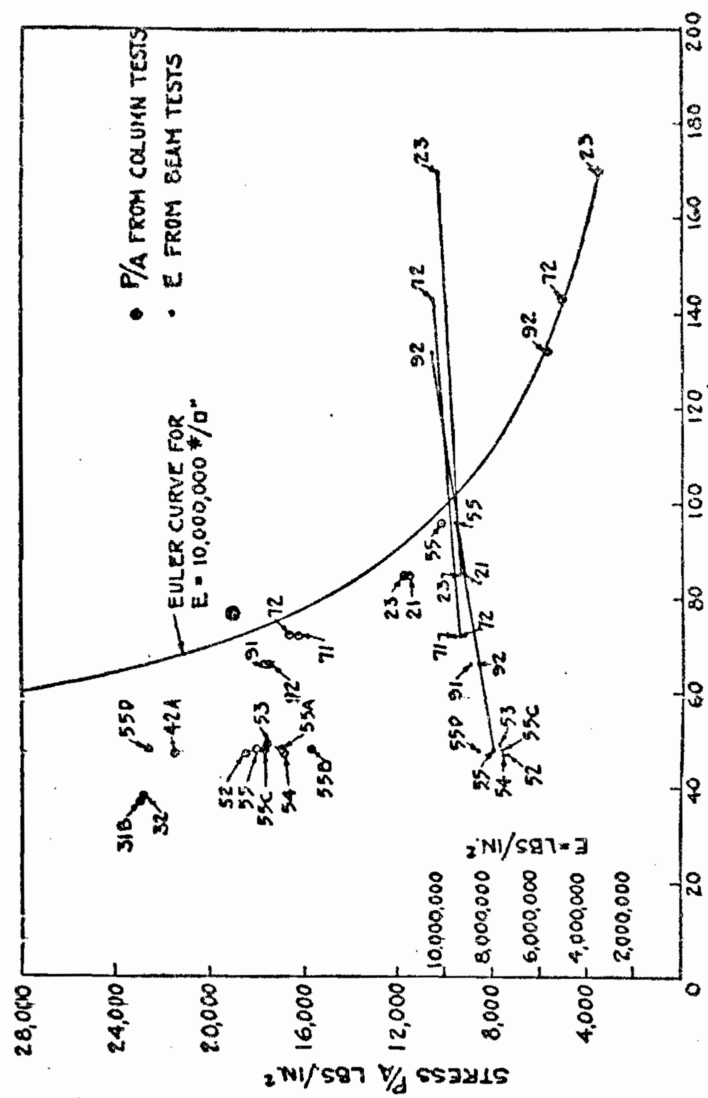


Figure 70. Column Strength of Girders in Values of E from Beam Test

which fits the results of the column tests better than the lower values determined from the beam tests.

Tests with combined beam and column loading. Girders were tested with uniformly distributed running loads, which remained constant during the test of each girder, and an end load was applied and increased until failure occurred. The ends of the girder were ball-supported, and as in all other tests, failure invariably occurred by buckling of a base channel between points of attachment of the lattices.

In the analysis of these tests, the bending moment due to the combined end and transverse loads is calculated from Morley's formula:

$$M = \frac{w E I}{P}\left(\sec \frac{\pi}{2}\sqrt{\frac{P}{P_e}} - 1\right)$$

where

M = the bending moment
w = the running load
E = the modulus of elasticity
I = the moment of inertia about the minor principal axis
P = the actual end failing load
P_e = the Euler failing load calculated about the minor principal axis, instead of about the major axis as in the pure column tests

The failing stress is given by:

$$f = \frac{M}{Z} + \frac{P}{a}$$

The calculation of f is shown in Table 26.

The quantity

$$\sec \frac{\pi}{2}\sqrt{\frac{P}{P_e}} - 1$$

is represented in the table by q. It is assumed that $E = 8{,}000{,}000$ lbs./in.² for the 5-meter lengths of the main longitudinals (types 52 to 54 inclusive) and 9,000,000 lbs./in.² in all other cases.

In the 10-meter lengths, w includes the weight of the girder,

TABLE 26. FAILING STRESSES WITH COMBINED END AND TRANSVERSE LOADS

Type	in.	a in.²	I in.⁴	Z in.³	w lbs./in.	P lbs.	P_e lbs.	q	M in. lbs.	M/Z lbs./in.²	P/a lbs./in.²	f lbs./in.²
21	197	.2115	2.725	.837	2.00	1,167	6,250	.288	12,100	14,430	5,510	19,940
23	197	.2115	2.725	.837	2.00	1,242	6,250	.310	12,250	14,620	5,880	20,500
52	197	.2412	10.035	2.006	5.00	2,447	20,450	.164	26,850	13,380	10,170	23,550
53	197	.2584	11.788	2.019	5.00	2,467	23,000	.149	26,350	13,060	9,550	22,610
54	197	.2866	12.082	2.314	5.00	2,733	24,600	.153	27,050	11,700	9,525	21,225
55	197	.3043	13.233	2.377	5.00	3,440	27,000	.180	27,700	11,670	11,300	22,970
71	197	.3360	4.806	1.289	3.00	3,240	11,220	.506	20,650	16,050	9,650	26,700
72	197	.3360	4.806	1.289	3.00	3,190	11,220	.495	20,500	15,900	9,500	25,400
91	197	.2115	4.617	1.139	2.00	2,090	10,600	.306	12,180	10,700	9,900	20,600
92	197	.2584	5.598	1.404	2.00	3,140	12,850	.400	12,830	9,140	12,130	21,270
23	394	.2115	2.725	.837	.54	180	1,562	.160	11,800	14,100	850	14,950
55	394	.3043	13.233	2.377	1.31	1,563	7,600	.322	32,000	13,500	5,135	18,635
72	394	.3360	4.806	1.289	.81	973	2,805	.600	24,200	18,800	2,900	21,700
92	394	.2584	5.598	1.404	.55	947	3,212	.520	14,800	10,530	3,660	14,190

from .04 to .06 lb./in.; but even with this allowance, the values of f are lower than would be expected from the other tests.

As a result of these tests, it is concluded that there are four different types of critical instability which may cause failure of a girder, as follows:

(1) Flexural failure as a whole, which depends on the slenderness ratios of the girder about its major and minor principal axes, and on the type of connections and loadings.

(2) Flexure and twist of the channel as a whole between lattice points, depending on the slenderness ratios of the channel, its torsional rigidity and the rigidity of the lattices and their connections to the channels.

(3) Buckling of the end lattices due to shear. This can occur only with extreme beam loading such as is not likely to be found in practice.

(4) Spreading and crumpling of the channels.

The nature of the final collapse at failure is not determined by these four factors. The collapse may take various forms, such as crumpling and tearing of lattices, or of channels, or shearing of rivets, which depend upon the yield point and ultimate strength of the various parts, but do not influence the initial cause of the failure.

Application to design. Various formulas for determining girder strength have been proposed. In the design of girders, the failing strength of the longitudinal members, such as the channels and angles used in Zeppelin girders, is a very important criterion of the strength of the girder, and tests of short sections corresponding to the lengths between lattice supports are comparatively inexpensive. For most sections, the strength may be approximated very fairly by calculation of the l/r and application of the usual column formulas. The usual limitation on ratio of thickness to width of material to avoid crumpling failure must be considered in such computations. The weak point in the application of theoretical or tested strengths of the longitudinal members alone is that it is impossible to determine precisely, without tests on actual girders, how much support is given to the longitudinals by the lattice members. Test results or calculations based on a free length of the longitudinal members equal to the length between lattices will usually be on the safe side, but not always, particularly if the lattices are not stiff and well connected to the longitudinals. The design of an airship should never be carried far without tests of girders corresponding closely to those to be actually used in the airship.

German tests of girders. Data obtained by the Schütte-Lanz Company on tests of various sizes and types of wood and duralumin girders have been published in Zeitschrift für Flugtechnik und Motorluftschiffahrt for May, 1924.

It is highly improbable that airships will ever again be constructed with wooden girders; and no space will be devoted here to a consideration of their characteristics. Table 27 summarizes the results of the tests by the Schütte-Lanz Company on the types of girders shown in Figures 72 to 76; and for comparison, the results of the Bureau of Standards tests on typical girders of the *Shenandoah* are similarly summarized in Table 28. The last column in each of these tables gives the "breaking length" which is merely the failing load divided by the weight per foot;

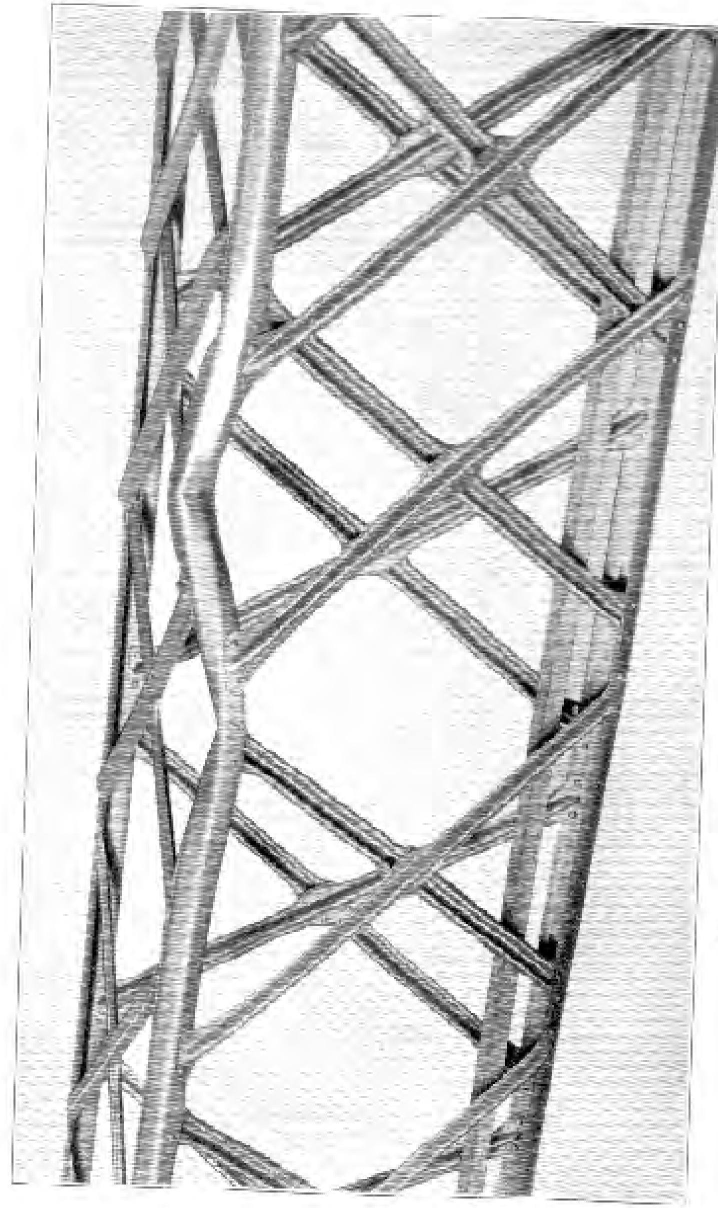


Figure 71. Typical Failure of Compression Member in Beam Test

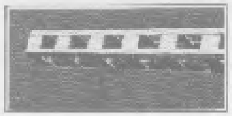


Figure 72

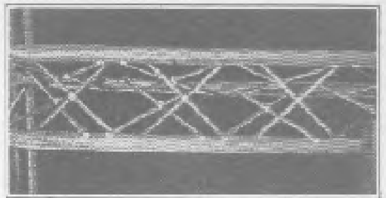


Figure 73

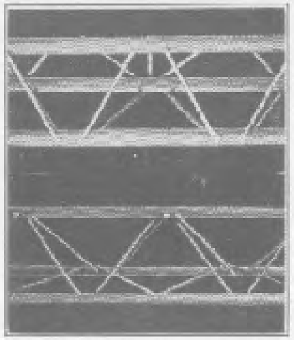


Figure 74

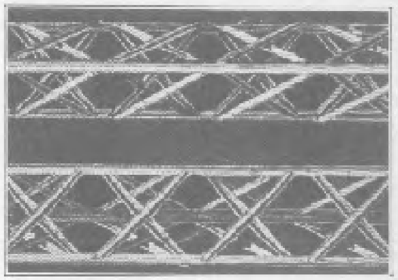


Figure 75

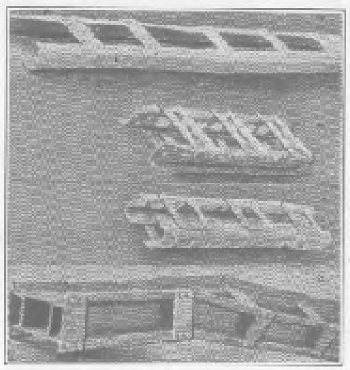


Figure 76

Figures 72–76. Experimental Duralumin Girders by Schütte-Lanz

TABLE 27. CHARACTERISTICS OF VARIOUS TYPES OF DURALUMIN GIRDERS

Type	No. of Longls.	Height in.	Width in.	Length ft.	Lattice Spacing in.	Cross-sectional Area in.²	Length/Radius of Gyration	Total Failing Load lbs.	Maximum Unit Stress lbs./in.²	Weight per Foot lbs.	Total wt./wt of Longls.	Breaking Length ft.
COLUMNS:												
1	2	1.65	.71	1.02	3.5,1.6	.068	39	1,040	28,500	.10	1.22	19,400
2	4	2.76	2.76	3.94	3.04	.260	40	8,800	33,800	.52	1.65	16,900
2a	4	9.85	9.85	13.95	13.8	3.260	40	114,500	35,100	6.7	1.69	17,100
3	2	9.45	2.36	7.87	14.6	.136	22	3,080	22,600	.35	2.12	8,800
4	3	8.66	9.85	8.52	22.8	.418	22	6,600	15,800	.65	1.28	10,150
5	3	9.20	10.60	8.40	10.6	.412	21	9,900	24,000	.78	1.56	12,700
6	3	8.85	7.68	8.40	16.5	.423	22	13,650	32,300	.75	1.46	18,200
7	3	8.66	9.85	13.90	22.8	.688	36	19,800	28,800	1.15	1.38	17,200
8	3	8.66	9.85	8.30	22.8	.688	21	26,400	38,400	1.18	1.41	22,400
SIMPLE BEAMS:							Length/depth					
9	2	9.45	1.97	8.20		.136	10.4	1,035		.44		2,360
10	4	3.94	2.76	3.28		.130	10.0	1,320		.54		2,440
11	3	8.66	9.85	13.90		.465	18.5	2,860		1.20		2,380

TABLE 28. CHARACTERISTICS OF GIRDERS USED IN AIRSHIP *Shenandoah*

Type	No. of Longls.	Height in.	Width in.	Length ft.	Lattice Spacing in.	Sectional Area in.²	Length/Radius of Gyration	Failing Load lbs.	Unit Stress lbs./in.²	Weight per Foot lbs.	Total wt./wt of Longls.	Breaking Length ft.
COLUMNS:												
21	3	7.87	6.10	16.4	5.83	.2115	85.6	2,460	11,610	.446	1.76	5,500
54	3	14.17	10.63	16.4	9.73	.2866	47.1	4,928	17,200	.625	1.79	7,900
91	3	10.04	7.87	16.4	5.83	.2115	66.5	3,767	17,830	.491	1.93	7,660
72	4	8.07	6.10	16.4	5.83	.3360	71.4	5,640	16,800	.674	1.66	8,360
31B	3	7.48	7.87	9.84	5.83	.2226	36.6	5,110	22,000	.470	1.76	10,900
42	3	9.45	10.24	16.4	5.83	.2670	47.3	5,880	22,000	.580	1.77	10,130
SIMPLE BEAMS:							Length/depth					
21	3	7.87	6.10	16.4	5.83	.2115	25.0	697	20,500	.446	1.76	1,560
54	3	14.17	10.63	16.4	9.73	.2866	13.9	1,866	19,850	.625	1.79	2,930
91	3	10.04	7.87	16.4	5.83	.2115	19.6	988	21,400	.491	1.93	2,010
72	4	8.07	6.10	16.4	5.83	.3360	24.4	1,410	26,950	.674	1.66	2,090

and is a rough measure of the efficiency of the girder. Most of the Schütte-Lanz girders, especially the tubular ones, show up very much better than the *Shenandoah* girders; and the difference is due mainly to four advantages of the girders tested by the Schütte-Lanz Company, as follows: (a) smaller l/r ratios, (b) thicker metal, (c) relatively less weight devoted to the lattices, (d) the use of tubular longitudinals instead of open channel sections. Considering each of these four advantages of the Schütte-

Lanz girders, it is obvious that (a) is merely a fortuitous difference between the method of testing; (b) is an inherent advantage of the larger girders possible with larger ships; (c) is also inherent in larger girders because the girders used in the *Shenandoah* were so light that it was not practicable to make the lattices as thin as required by the theoretical stress in them; (d) represents the inherent advantage claimed by the Schütte-Lanz Company for their tubular members in comparison with the open channel sections used by the Zeppelin Company. The reality of this advantage is controversial; but it appears to be a fact that with duralumin tubular sections the very high failing stress of about 38,000 lbs./in.^{2} can be attained with a reasonable l/r. Failing stresses as high as 34,000 lbs./in.^{2} have been reported by the Zeppelin Company for double latticed girders almost identical with the 91 and 92 type of the *Shenandoah* except that the thicknesses of the channels were increased to 1.8 mm. (.071 in.), and the lattices were also thicker and stiffer.

Such advantage in the strength/weight ratio as the Schütte-Lanz tubular girder may possess is heavily paid for by the difficulty of securing satisfactory joints of the girders to one another and of the lattices to the longitudinal members; and also in the impossibility of inspecting the interiors of the tubes for corrosion or other defects.

Zeppelin four-plate box girders. In the airship *Los Angeles*, the Zeppelin Company incorporated a new type of box girder in the cars, fins, and a few other special places; and more extensive use of this new type of girder is contemplated by the Zeppelin Company in their next design. An example of this girder is shown in Figure 77. Its essential features are the use of four perforated plates instead of three channel members with a multiplicity of lattices. Two of the four plates designated "side plates," are bent over near their edges, and the other two plates, called "cover plates" are riveted to them. All four plates are punched with a row of large, round holes, and two rows of smaller

and approximately triangular holes. The metal is bent inward around all the holes, giving great stiffness to the plates. It is claimed that this construction effects an important saving in the cost of construction by greatly reducing the number of parts, and the amount of riveting required; the joint construction is simplified by the ease with which gusset plates may be secured to the girders, the thickness of the four plates may be varied according to the stresses in each, and failure by crumpling and deformation such as occurs in the customary open channels between the joints is minimized.

A practical advantage over the old type of Zeppelin girder is that the plates are much less liable than the old lattice members to damage from the crew and maintenance men walking upon them. An obvious disadvantage is the large amount of scrap material resulting from the punching of the four plates. Improved methods of reusing scrap duralumin may reduce the material cost of perforated plate girders. The effective cross-sectional area of such girders appears rather indeterminate, and no test data are available on the strength or ratio of strength to weight for either columns or beams of this construction.

Minimum economical size of girders. If the longitudinal members of duralumin girders are made of metal less than about .04 in. to .05 in. in thickness, the chances of local damage by movements of the personnel, or other slight causes, become undesirably great; and the resistance of the metal to corrosion is low. If the total cross-sectional area of a girder is to be less than about 0.3 in.^{2}, it is usually preferable to maintain the minimum thickness of .04 in., and reduce the major cross-sectional dimensions, even at the expense of a less favorable ratio of length to radius of gyration, and reduced crippling stress. It follows that the efficiency of duralumin girders falls off when the total cross-sectional area is reduced below about 0.3 in.^{2}

Another limiting figure on the economical size of girders is that the ratio $L/\sqrt{A}$ should not exceed about 350, where L is the length of the girder, and A its cross-sectional area. If this ratio is exceeded, it is impracticable with present types of girders to attain a satisfactory ratio of length to radius of gyration, without excessive weight in the lattices or web members.

Preliminary estimate of girder weights. After the cross-sectional areas of the girders have been chosen from preliminary strength calculations, a very fair approximation to the total weight of the girder, including the lattices, joints, and fittings can be made by adding 75% to the weight of the longitudinal members alone.

Example. Find the total weight, including lattices, joints, and fittings, of a duralumin girder 18.5 ft. in length, with a cross-sectional area of .65 in.^{2}

The weight of a duralumin bar 1 ft. in length by 1 in. square is 1.21 lbs. The total weight is therefore $1.75 \times 1.21 \times 18.5 \times .65 = 25.5$ lbs.

Design of Girder Joints

The design of the intersections of the girders is apt to involve extremely complex problems in descriptive geometry; and it is practically impossible to give any general rules for the procedure to be followed except that the connections must be as neat and light as is consistent with strength. Usually the drawings are prepared by a draughtsman who specializes in these problems; but frequently the most satisfactory and inexpensive method of design is to construct a full size mock-up out of girder sections with tin gusset plates which may be easily cut and bent as required; and to make the actual joints and the drawings by copying the mock-up.

In designing joints it is important to carry the strength through, and this is usually effected by fish-plates with ample rivets. In making butt joints of channel members, the most effective fish-plate is a smaller channel fitting neatly within the bosoms of the channels to be joined. In the joints of girders at

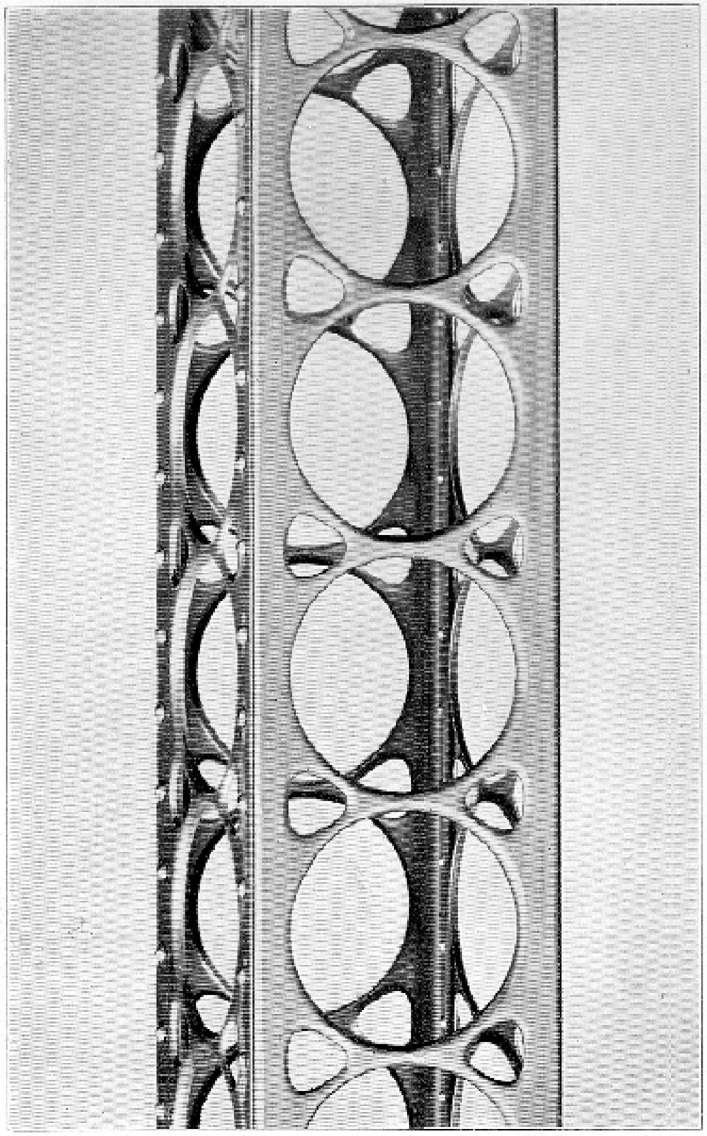


Figure 77. Zeppelin Four-plate Box Girder

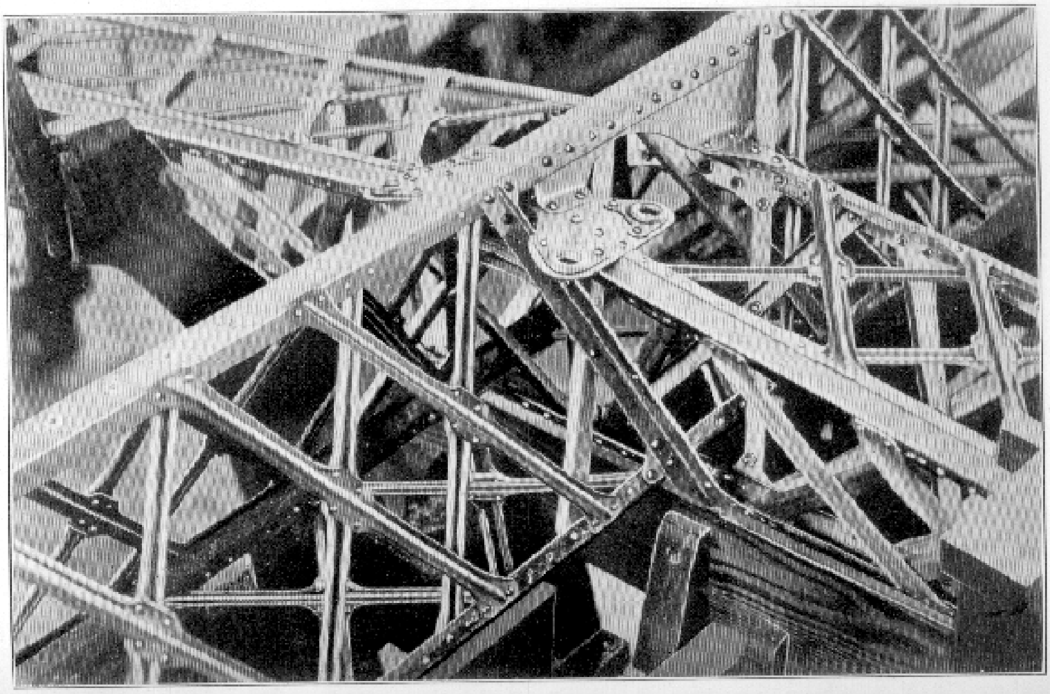


Figure 78. Typical Joints of Zeppelin Type Girders

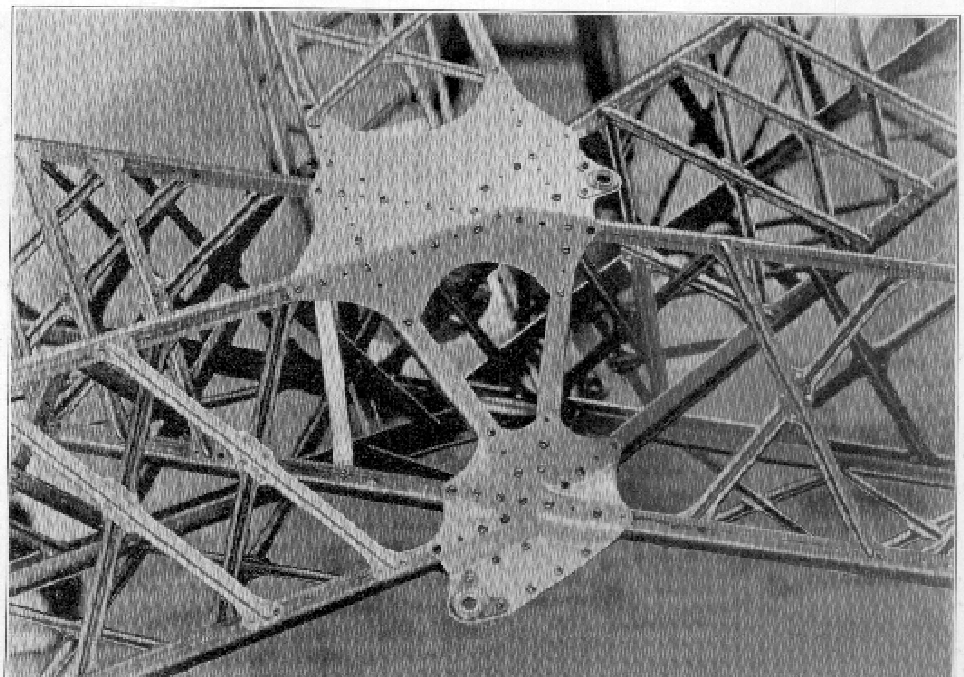


Figure 79. Typical Joints of Zeppelin Type Girders

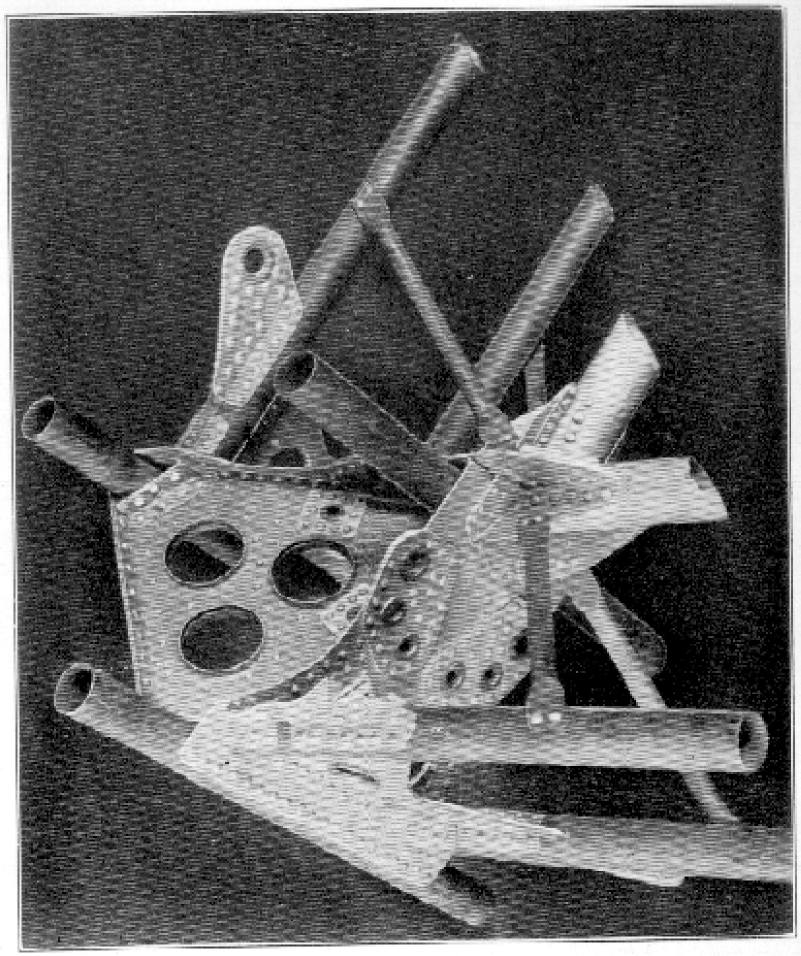


Figure 80. Joint of Schütte-Lanz Tubular Girders

large angles to each other, there is usually but little stress tending to break the connection, so that almost any neat and reasonable arrangement of the gusset plates and riveting will suffice. Such joints usually involve connections for diagonal bracing wires. A thimble set in a gusset plate makes a suitable anchorage for a wire terminal, provided care is taken that the lead-off of the wire lies in the plane of the gusset plate and does not tend to produce bending or rotation of the joint. At the joints of the main frames where many wires come together it is customary to insert a small aluminum alloy casting between the gusset plates in order to provide a secure anchorage with easy curvature for the terminals of the transverse wires.

Typical joints of Zeppelin type girders are shown in Figures 78 and 79. Study of these figures will show the complexity of the problems involved, and the solutions arrived at by designers. A noteworthy point is that little effort is made to attain intersections of the neutral axes of the girders at the joints. Experience shows that the stresses resulting from slightly offset neutral axes are of little importance.

The open channel and angle sections of the Zeppelin type girders lend themselves very readily to neat and reasonably simple joints; the new box girders of the Zeppelin Company consisting of four perforated plates are even more suitable for simple gusset plate connections. A serious objection to the use of girders composed of tubular elements is the obvious difficulty of making satisfactory joints. Figure 80 shows a solution of this problem arrived at by the Schütte-Lanz Company.

It is customary to make butt joints in the longitudinal girders at the intersections with the main and intermediate transverse frames. There would be some advantage in making the longitudinal channel members continuous through the joints with the frames, and making the butt joints at the points of inflection, about one-quarter the panel length either side of the frame. With this arrangement, the butt joints would be at points of little or no bending stress in the girder, and it would be possible to use heavier channels in that section of the girder which passes through the joint with the frame, where the largest bending occurs. This construction is proposed by the Schütte-Lanz Company for their tubular girders. The objection to it is that in the tapered portions of the hull, the longitudinal girders are preferably straight between the frames, with change of inclination to the longitudinal axis of the ship at the frames; obviously it is easier to effect this change at a butt joint than in the mid-length of a girder.

Ball and socket joints. It has frequently been proposed to reduce the secondary stresses in rigid airships by the use of ball and socket joints between the girders, as in the keel construction of the Italian semirigid airships. The ball and socket joints would undoubtedly be much heavier and more costly than the simple riveted gusset and fish-plate connections; and to obtain high efficiency, the girders should be tapered, thus greatly increasing their cost. Moreover, pin-point construction would be likely to result in dangerous instability of the intermediate frames.

Strength of riveted joints. In designing riveted joints it is necessary to guard against the following three principal ways in which failure may occur: (a) tearing the plate between holes, (b) crushing the plate and rivet, (c) shearing the rivet. To obtain the maximum efficiency, the chances of failure by each of these ways should be equally probable. For guidance in designing riveted joints in thin duralumin, the following description and results of tests carried out at the Massachusetts Institute of Technology is taken from the National Advisory Committee for Aeronautics Technical Note No. 165, abstracted and revised by J. G. Lee from the thesis of H. F. Rettew and G. Thumin of the Massachusetts Institute of Technology.

The most surprising results of the work were the unusually high values of crushing and shear found to exist. These values are nearly double what is ordinarily found in shear or com-

pression tests, and are apparently due to the friction of the riveted plates and the reinforcement of the rivet heads. Since this friction and reinforcement are necessarily present in all good riveted joints, the high strength values may properly be used in design.

Following are the values for riveted joints in heat-treated duralumin sheet:

	Commercial Values lbs./sq. in.	Values from Rettew & Thumin lbs./sq. in.	Suggested Values for Design of Joints lbs./sq. in.
Tearing (f_t)	55,000	54,000	50,000
Crushing (f_c)	45,000	105,000	100,000
Shear (f_s)	25,000	43,000	40,000

The recommended values for tearing and shear fall below all of the individual test values found by Rettew and Thumin, but certain specimens of crushing, where the variation was much larger, fell as much as 5% below the recommended value of 100,000 lbs./sq. in. Nevertheless, 100,000 lbs./sq. in. appears to be a reasonably safe average figure.

From the recommended values a chart has been plotted (Figure 81) based on the following formulas for rivet failure:

Shearing

$$F = \pi d^2 f_s/4 = 31{,}400\ d^2 \text{ for single shear}$$
$$F = \pi d^2 f_s/2 = 62{,}800\ d^2 \text{ for double shear}$$

Crushing

$$F = tdf_c = 100{,}000\ td$$

Tearing

$$F = (p - d)tf_t = 50{,}000\ t(p - d)$$

Critical Rivet Diameter

$$100{,}000\ td = 31{,}400\ d^2, \quad d = 3.2\ t$$
$$\text{Double Shear } d = 1.6\ t$$

Critical Pitch

$$50{,}000\ t(p - d) = 100{,}000\ td, \quad p = 3\ d$$
$$\text{For double riveting, } 50{,}000\ t(p - d) = 2(100{,}000\ td), \ p = 5\ d$$
$$\text{For triple riveting, similarly, } p = 7\ d$$

d = driven diameter of rivet
t = thickness of plate
p = pitch of rivets, on centers

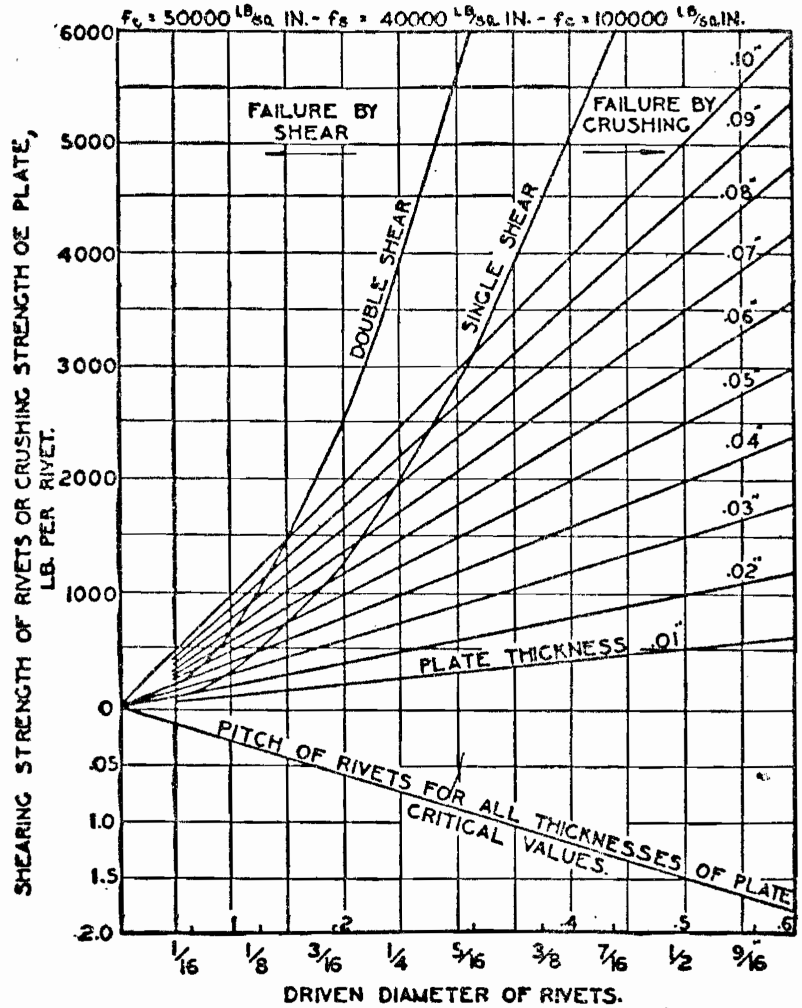


Figure 81. Strength of Single-Riveted Lap Joints of Duralumin Plates

It is good practice to have the lap from 2.5 to $3d$ and the distance between rows in double riveting (staggered) 1 to 1.5 p.

In reading the chart one usually starts with the plate thickness and follows that line along until it intersects one or other of the shear curves. This gives the critical rivet diameter. The nearest available rivet diameter is chosen (preferably larger) and the strength read off on the scale of ordinates at the left, the ordinate of the plate thickness line being used if the rivet diameter is above the critical figure, while the ordinate of the shear parabola is chosen if the diameter is below the critical. To find the pitch, read down from the chosen diameter, and take the nearest convenient pitch (larger than that given by the graph).

The tensile tests gave the following average results:

	Heat-treated		Annealed
Thickness (inches)	.020	.040	.095
Elongation in 8 in. (%)	12.67	19.53	12.55
Elongation in 2 in. (%)	14.83	24.25	17.75
Yield point (lbs./sq. in.)	27,800	26,100	13,750
Tensile strength (lbs./sq. in.)	56,530	56,625	31,750
Modulus of elasticity	11,310,000	11,020,000	9,773,000
Reduction of area (%)	15.33	19.97	34.48
Ratio of Y.P. to T.S.	.491	.456	.437

The riveted joints in the .095 in. annealed sheet showed the following values in lbs./sq. in.:

$$f_t = 32{,}700 \qquad f_s = 42{,}800 \qquad f_c = 62{,}300$$

All of the tension failures both in the riveted joints and in the plain specimens occurred as shear along a 45° plane. The crushing failures appeared to be crushing of the plate and not of the rivets, although the plate, being heat-treated, has the higher theoretical strength of the two. The shearing failures were instantaneous, as opposed to the gradual distortion which preceded a failure by crushing or tearing.

The only noteworthy result of the tests for slippage of the joints is that in all cases a redistribution of load takes place at the lower stresses, as is evidenced by the different amounts of slippage at different points along the joint. In general the slippage is small and unimportant.

CHAPTER X

STEPS IN DESIGN

Principal characteristics. An airship may be designed to fulfill certain performance requirements in which the dimensions of the ship are not specified. Given a problem of this nature, the designer makes his preliminary estimate of the necessary size of the airship by such methods as are described in Chapter II.

It more often happens that the approximate size is determined by consideration of some other ship which must be equalled or surpassed, or by limitations of cost or shed accommodation. In such cases, the designer probably has the volume, and perhaps also the length and diameter, predetermined for him, and he strives to get the best performance he can, having especial regard to the qualities most desired in the problem set before him.

Example of preliminary design calculations. The design of a rigid airship is primarily a structural problem. The preliminary design calculations are largely concerned with the choice of size and arrangements of the members which will give the lightest possible structure consistent with the strength required. The following example of design calculations shows a method of estimating the performance and the principal characteristics for a large rigid airship of conservative design.

The problem is to design a rigid airship of about 6,000,000 cu. ft. gas volume, for service as a naval scout, having a full speed of 70 knots and the utmost practicable range of action when carrying no military load.

An air volume of 6,400,000 cu. ft. is required in order to have ample allowance for air space around the gas cells. If no

arbitrary limitations of maximum length and diameter are imposed by the dimensions of the available airship sheds, the choice of the slenderness ratio is the first decision to be made. Right here, at the very start of the problem, we are on very controversial ground. The principal factors influencing the designer in determining the slenderness ratio are resistance and weight. There is no general agreement as to the ratio which will give the least resistance. Airship designers of recognized ability recommend slenderness ratios all the way from 8.0 to 2.8. Wind tunnel tests indicate the best ratio to be about 4.5, but there is still much doubt as to the scale effect over the long range from wind tunnel models to actual airship.

The smaller the slenderness ratio, the less the weight of the longitudinal structure, but the greater the weight of the transverse frames. The ratio which will result in the least total weight depends upon many variables. For example, the longitudinal strength required depends largely upon the speed so that the faster the airship the less should be the slenderness ratio for minimum weight. Increasing the subdivision of the gas space increases the weight of the transverse structure, so that the greater the number of gas cells required, the greater should be the slenderness ratio for least weight.

Facility and cheapness of construction, and ease of handling the airship upon the ground are best satisfied by a large slenderness ratio; but these requirements are subordinate to least resistance and weight.

Conservative opinion among rigid airship designers at present favors a slenderness ratio of about six. For any given problem, the designer will do well to make two or three preliminary designs with different slenderness ratios and make an approximate weight estimate of each.

The following procedure may be pursued in a preliminary design to which a slenderness ratio of 6.4 is arbitrarily assigned. The next question is how much parallel middle-body, if any, shall be assigned to the ship, and what contours shall be chosen for the bow and stern. It is obvious that practical considerations, apart from resistance, are very much in favor of the use of parallel middle-body, rather than a continuously curved contour. A generous amount of parallel body reduces the time and cost of construction, and reduces the overall dimensions for a given volume.

Aerodynamic considerations also indicate, although the question is controversial, that when the slenderness ratio exceeds about 4.5, the additional length is preferably in the form of parallel middle-body. Good results are obtained with an elliptical fore-body and parabolic after-body; but an experienced designer may obtain the best results with a contour drawn to please his eye, without regard to mathematical formula.

In this preliminary design, let the fore-body be generated by the revolution about its X-axis of a semi-ellipse having the equation:

$$x^2/a^2 + 4\ y^2/D^2 = 1$$

where D is the maximum diameter, and a is the length of the fore-body.

Let the after-body be generated by the revolution about its X-axis of a semi-parabola having the equation:

$$y = D/2 - Dx^2/4a^2$$

where D and a are still the maximum diameter and the length of the fore-body, respectively. From this equation, $y = 0$ when $X = 1.41\ a$, so that the after-body is 1.41 times longer than the fore-body. If there were no parallel middle-body, the fore and after bodies designed to these equations would fit together without change of curvature at their junction.

Integration along the solid generated by the elliptical and parabolic contours shows that their combined volume is .59 times that of a cylinder of the same overall dimensions.

Let $a = 2D$. The length of the tapered portions of the hull is then $2D + 1.41\ (2D) = 4.82\ D$; and the length of the middle-body is $(6.4 - 4.82)\ D = 1.58\ D$. This results in:

$$(1.58D + .59 \times 4.82D) \times \pi D^2/4 = 6{,}400{,}000$$
$$D = 122 \text{ ft.}$$
$$L = 6.4 \times 122 = 780 \text{ ft.}$$

Speed and power. Let a speed of 70 knots, or 118 ft./sec. be required. For safety's sake, this speed will not be attained at a less altitude than 3,000 ft., where the density of the standard atmosphere is $\rho = .00216$ slug/ft.³ The aerodynamic head q used throughout the performance and strength calculations is given by:

$$q = \rho v^2/2$$
$$= .00216 \times 118^2/2 = 15 \text{ lbs./ft.}^2$$

The maximum horse-power required at 3,000 ft. is given by (see page 12),

$$\text{Hp.} = \frac{\rho v^3 V^{2/3}}{550 K}$$

Let $K = 60$

$$\text{Hp.} = \frac{.00216 \times 118^3 \times 6{,}400{,}000^{2/3}}{550 \times 60}$$
$$= 3{,}700$$

The corresponding Hp. at sea level is $3{,}700 \times .00237/.00216 = 4{,}060$.

Preliminary estimate of girder spacing. It will be assumed that in order to support the outer cover effectively, and in accordance with conservative practice, the principle is adopted of spacing the longitudinals about 12 ft. apart amidships, and the transverses at 16 to 20 ft. The number of longitudinals and transverse frames is thus dependent upon the size of the ship; but the number of frames which are main frames is a less variable quantity, and is governed by the principle that about 12 to 15 gas cells are considered good practice.

The circumference of the midship section is $122\ \pi = 383$ ft. 32 longitudinals will give a spacing of 12.0 ft., which is satisfactory; and the number of longitudinals being divisible by 4, permits one to be put at each end of the vertical and horizontal diameters, giving a symmetrical arrangement which facilitates the strength calculations. Frames at 20 ft. apart and every third frame a main frame will give side panels of suitable proportions, and 13 gas cells.

Static and dynamic bending moments. In a large modern airship, the disposable loads should be so distributed that the static bending moment is negligible in a preliminary estimate of the fully loaded condition. The weight of the bow and stern produces an inevitable hogging bending moment in the light load condition. From previous designs, it is found that this moment may be estimated from the expression,

$$M = C \rho g \, VL$$

where M = the bending moment
C = a coefficient = .005
ρg = the unit weight of air in the standard atmosphere at sea level = .07635 lb./ft.^{3}
V = air volume = 6,400,000 ft.^{3}
L = length of ship = 780 ft.

Whence: $M = .005 \times .07635 \times 6{,}400{,}000 \times 780$
$= 1{,}900{,}000$ ft. lbs.

Aerodynamic bending moment. H. Naatz has proposed to estimate the maximum aerodynamic bending moment by the empirical expression:

$$M = .01 \, q \, V^{2/3} L$$

This expression was derived from calculations of the maximum bending moment resulting from the maximum estimated aerodynamic loads on the tail surfaces opposed only by the inertia of the ship. Naatz found that it agreed very well with the bending moments calculated from the pressure required to prevent buckling of the large Parseval nonrigid airship *PL-27* in flight in squally weather in the Baltic; and it has also checked up well with bending moments estimated from observations with strain gages on the U. S. Navy's airships *Shenandoah* and *Los Angeles.* (See Chapter V.)

Since bending may result from combined rudder and elevator action, the maximum occurs in the longitudinal plane inclined at approximately 45° to the vertical and horizontal. For either the vertical or horizontal plane, Naatz's coefficient should be reduced from .01 to $.01/\sqrt{2} = .007$. Applying this method to the problem under consideration, the maximum aerodynamic bending moment in either the vertical or horizontal plane is given by:

$$M = .007 \times 15 \times 6{,}400{,}000^{2/3} \times 780$$
$$= 2{,}800{,}000 \text{ ft. lbs.}$$

Limiting conditions. The limiting, or most severe, condition for the upper longitudinals occurs with the ship fully loaded and maximum aerodynamic sagging bending moment, combined with ~~maximum~~ gas pressure and minimum tension in the outer cover.

The limiting condition for the lower longitudinals occurs with minimum load, and maximum hogging bending moment from static and aerodynamic forces, combined with no gas pressure in the bottom of the ship, and maximum tension in the outer cover.

Both conditions should be combined with the maximum aerodynamic bending moment in the horizontal plane.

It may be noted that these limiting conditions are for compressive forces which are the hardest to take care of. The maximum tensile forces occur in the same conditions as the maximum compressive forces, but at the opposite side of the ship.

The limiting bending conditions in this example are, therefore:

Hogging, $M = 2{,}800{,}000 + 1{,}900{,}000$
$= 4{,}700{,}000$ ft. lbs.
Sagging, $M = 2{,}800{,}000$ ft. lbs.

Both conditions must be combined with 2,800,000 ft. lbs. bending moment in the horizontal plane.

Maximum stresses in the longitudinals. For a first approximation, the end loads in the longitudinals may be estimated from the simple expression derived in Chapter VII:

$$S = \frac{2\ M \sin \theta}{Rn}$$

For the top longitudinal,

$$M = 2{,}800{,}000 \text{ ft. lbs.}$$
$$\sin \theta = 1.0$$
$$R = 60.5 \text{ ft.}$$
$$n = 32$$

Whence $S = 2{,}890$ lbs., compression

For the bottom longitudinal, $M = 4{,}700{,}000$ ft. lbs., and the other quantities are the same as for the top longitudinal, whence,

$$S = 4{,}850 \text{ lbs., compression}$$

The mid-height longitudinals are affected only by the horizontal bending, which is of the same magnitude as the maximum sagging bending moment. Therefore, for the mid-height longitudinals,

$$S = 2{,}890 \text{ lbs., compression}$$

For all longitudinals except those at the ends of the vertical or horizontal diameters, S must be calculated from the combined effects of the vertical and horizontal bending. For example, in the longitudinal at the lower end of the diameter inclined at 30° to the vertical, the limiting end load is given by:

$$S = \frac{4{,}700{,}000 \sin 60^\circ}{60.5 \times 16} + \frac{2{,}800{,}000 \sin 30^\circ}{60.5 \times 16}$$
$$= 5{,}650 \text{ lbs., compression}$$

Gas pressure and outer cover stresses. Chapter VIII contains a sample calculation of the gas pressure stresses on a ship of the dimensions of our preliminary design. These stresses result from bending of the longitudinals by gas pressure load between the frames, plus the bending by the rise and distortion of the intermediate frames with respect to the main frames.

The effect of the outer cover tension considered on page 267 must be added to the stresses on the longitudinals from the bending of the ship and the gas pressure.

The designer should prepare several studies of various systems of gas-cell wiring in a typical midship frame space. Only by this means can the best compromise be found between the conflicting requirements of a tight netting to reduce the loads on the longitudinals and a fairly loose netting to obtain the maximum gas space by bulging between the longitudinals. This phase of the study should also include the effect of longitudinal girders of different depths and types of construction. Within reasonable limits, increasing the depth of the longitudinals saves weight at the expense of gas space, and the saving must be set against the loss, every case on its own merits.

Transverse framing. The transverse framing is very little affected by aerodynamic forces. The sizes of the transverse girders are therefore determined by calculation of the static forces upon them, as described in Chapter VIII.

Order of procedure. A general rule for working up an airship design is to begin with a general arrangement plan, showing scarcely any detail; and then proceed to detail the midship frame bay. The characteristic features of construction chosen for this bay will determine the nature of the entire design; it is unwise to proceed further until the design problems for this section of the ship have been fully solved. These problems are mainly concerned with the structural strength and may be dealt with by the methods of calculation described in previous chapters. From amidships, the designer works towards the bow and stern.

The complete design of a large airship involves the preparation of several thousand drawings, covering an immense amount of detail work which can be learned only by practical experience. At best a book can be only a useful guide. Many details of airship design are described in other volumes of the Ronald Aeronautic Library.[1] In this chapter we shall deal only with some of the broader aspects of design and arrangement of airship parts. Some promising lines for future development will also be discussed.

Keel or corridor. Large airships of the rigid and semirigid types have a keel or longitudinal corridor giving a passageway through the hull, and access to the various cars. The crew's quarters, fuel and ballast, and other items of the useful load are usually carried in the keel. The keel was the principal source of the longitudinal strength in the early Zeppelins; and it also contributes very largely to the strength of semirigid airships (see Part II of "Pressure Airships"). In modern rigid airships, the keel is of comparatively minor importance in the primary strength; but it acts as a bridge to carry the useful loads between the main frames.

In the early rigid airships, the keel was placed outside and below the hull. The cross-sections were isosceles triangles, apex down. This type of keel is also found in the semirigid airship *Norge*, (Figure 2) but without the re-entrant angle between keel and hull which disfigured the early Zeppelins.

The Schütte-Lanz Company introduced the isosceles triangular keel, apex up, placed inside the hull. This construction was adopted by other designers, and became standard practice for several years. The *Shenandoah's* keel (Figure 49, page 174) was a typical example. The *Los Angeles* has a five-sided keel (Figure 1, page 4), the bottom of which projects slightly below the circumscribing circle through the remainder of the hull. This keel is much more roomy and convenient than the triangular keel of the *Shenandoah*, but it involves a rather unnecessarily large waste of space which might be devoted to gas. An advantage of the *Los Angeles* type of keel is that the walkway girder, instead of being a special and rather limber longitudinal as in the *Shenandoah*, is the largest, strongest, and

[1] "Pressure Airships" by Blakemore and Pagon.

most important longitudinal girder in the hull. The flat top of the keel permits of placing the fuel tanks and ballast bags at a sufficient distance apart to provide ample passage way along the keel.

In order to reduce the local stresses in the keel, the heavy weights such as fuel tanks and ballast bags should be suspended as close to the main frames as possible. When there are two intermediate frames between successive main frames, no heavy loads should be suspended between the two intermediates.

In order to obtain the most efficient slope for the keel shear wires (about 45°), additional frames known as quarter frames are provided in the keel between the regular main and intermediate transverse frames which are common to both the hull and the keel. In addition to carrying its local load, the keel also takes part in the primary strength of the hull, and its members are included in the primary strength calculations.

Future development in corridor design. The stresses in the main transverse frames would be considerably relieved if the loads which are normally carried in the keel were disposed further out from the center line. Reduction of the transverse stresses becomes particularly important in unwired frames, such as are discussed on page 208. It is therefore probable that in the future, the useful loads will be carried in the main frames instead of in the keel, or else the single center-line keel will give way to two keels, disposed one on either side, perhaps at the lower ends of the 45° diameters.

A corridor of small dimensions and light construction along the top center line of the hull would greatly facilitate access to the structure and the surface of the gas cells in flight. It would be essential if a system of combined automatic and maneuvering valves at the top of the ship were adopted, as discussed on page 278.

Design of power cars. The power cars of a rigid airship should be small, light and of good streamline form, so far as consistent with providing a firm bed for the power plant and sufficient space for the engineers and engine accessories. Probably it will always be controversial just how much room the engineers require to perform their duties efficiently. They will ask for more space than the designer is inclined to allow them. The overall dimensions of the power cars of the *Shenandoah* are 16 ft. 2 in. in length by 6 ft. 8 in. in depth, by 5 ft. 10 in. in width; each is designed to hold one 300 Hp. straight six-cylinder engine. The maximum cross-sectional area is about 34 ft.^{2}, a wind tunnel test of a tenth-scale model showed a resistance coefficient $C = .15$, where the coefficient is of the form $C = 2R/A\rho v^2$ in absolute units, and A is the maximum cross-sectional area of the car. This coefficient was obtained with a closed visor over the radiator in the front of the car. With the visor wide open, the resistance was increased 60%.

A car placed forward of amidships increases the total resistance of the airship much more than the amount of its own resistance, because it breaks up the smooth flow of the air around the ship. It is therefore especially important to make the forward cars as small and cleanly lined as possible, or better still—eliminate them altogether.

The bottom of the power car forms the bed for the engine, and must be of substantial construction, usually of duralumin plates, angles and channels. Aft of the engine the strong bottom of the car must be continued to provide support for the propeller bearings, also perhaps for reduction and reverse gearing between the engine and propeller. Forward of the engine, support must be provided for the radiator. Radial air-cooled tractor engines have been suggested for airships instead of the conventional water-cooled pusher type; if this change should be made, an important saving in the length and weight of the power cars could probably be effected.

The sides and top of the power cars can be of very light framework covered with fabric or thin duralumin plating. Experience shows that the plating has a serious tendency to crack

under the vibration, consequently fabric is probably preferable, but it should be protected by a fireproof dope. Near the forward and after ends of the car, struts extend upward from the strong bottom, meeting at fittings to which the car suspension members are attached.

Figure 82 shows a power car of the U.S.S. *Los Angeles* under construction. Each car without the engine or any equipment weighs about 600 lbs.

A possible development in power car construction which would reduce the weight and resistance is to make each car practically nothing more than a streamline hood over the enclosed engine, with a suspension system permitting the car to be swung against or into the hull for access to the engine.

Power car suspensions. The side power cars are usually connected to the hull by three struts forming a tripod having its base upon the car, and its vertex at a joint of a main transverse frame of the hull. This tripod does not take the weight of the car, but acts as a spacer, assisting alignment between the car and the hull. The weight of the car is taken chiefly by two steel cables secured to a joint of the main frame above the center of gravity of the car. Additional wires lead forward and aft from the car to the frame next ahead and astern to take the longitudinal forces resulting from the propeller thrust and fore-and-aft inclination of the ship. These inclined cables or guys also act with the tripod struts to resist the torque of the propellers or similar rolling forces which may be produced by rolling of the airship.

The after center-line engine car is usually connected to the hull by from four to six struts secured to the keel at the main frame immediately over the car and to the quarter frames next forward and aft. These struts are designed primarily to take the shock of landing, and are intended to break under a shock less than that which would damage the structure of the hull of the airship. The bumper bag and its supports under the car are in turn intended to fail at a load less than that which would break the struts of the car. There is an advantage in having considerable weight in the after center-line car, because when the car strikes the ground and the ship is relieved of its weight, the effect upon the buoyancy of the ship is equal to the exertion upon the hull of an upward thrust equal to the weight of the car, without actually applying that force to the structure. For this reason, it is a common practice to place two engines geared to one propeller in the after car, instead of only a single engine as in the other cars.

A small fire in a power car is by no means a rare occurrence; and in hydrogen inflated airships it is important that there be no closed ducts or passages between the power cars and the hull, through which fire might be communicated. With helium, this precaution seems unnecessary; and a possible future development is the elimination of externally suspended cars. Cars may be rigidly connected to the hull with streamline fairing enclosing the connecting structure; or the engines may be placed inside the hull with shaft-drive to the propellers.

Control and passenger cars. The control car of a rigid airship is on the center line about 15% of the length of the ship aft of the bow. Sometimes the control car is suspended several feet below the hull, and the various controls are carried up to the hull through streamlined cases. It seems preferable to have the control car close up under the hull to provide greater strength and safety, and to minimize resistance. In the passenger airship *Los Angeles*, the control and passenger cars are built integrally, close up to the hull, and extend from 15% to 25% of the length aft of the bow. The best position for a large passenger car is controversial. A saving of weight and resistance is obtained by making control and passenger cars integral. Obviously, the best position for the control car is well forward, and this is also a good position for the passenger car in order to escape the noise of the engines. Handling the airship on the ground is facilitated

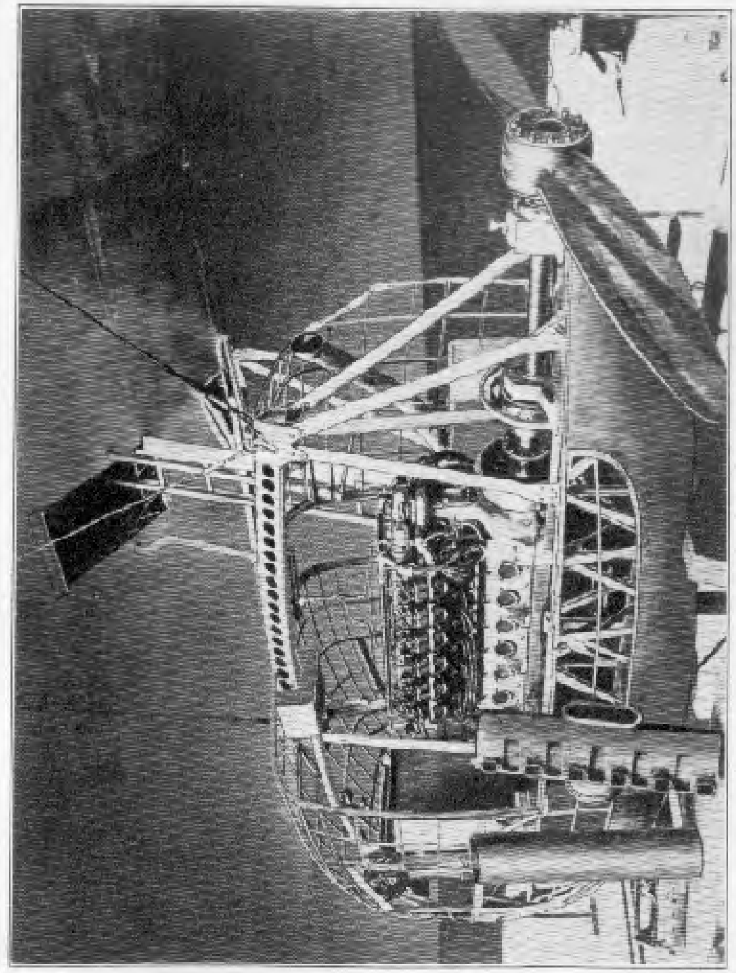


Figure 82. Power Car of U.S.S. *Los Angeles* Under Construction

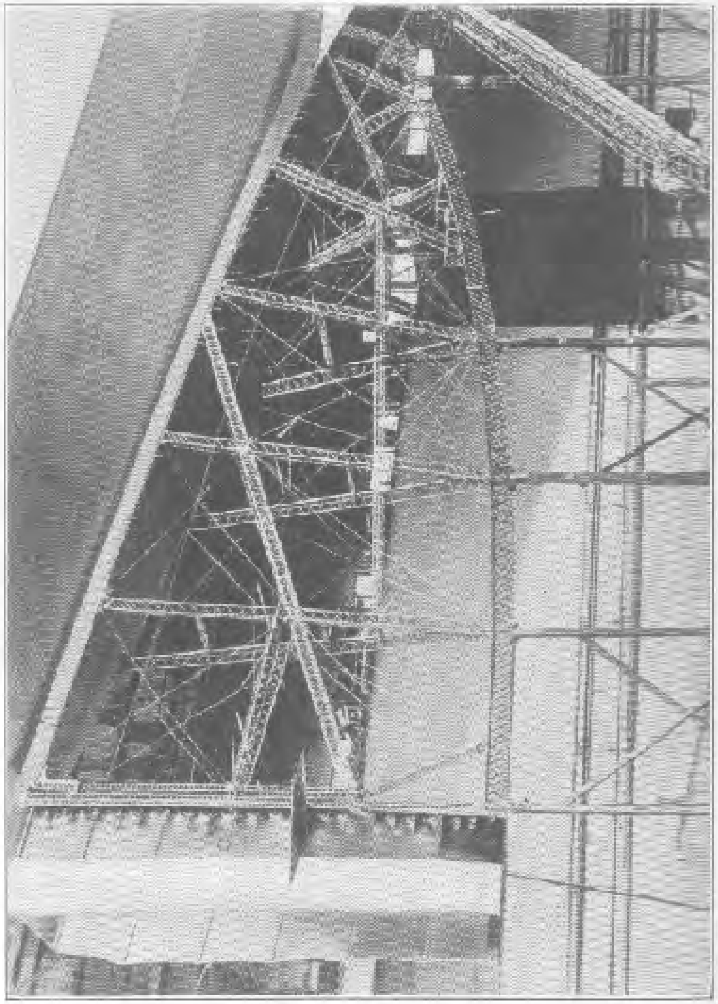


Figure 84. Lower Fin and Rudder of U.S.S. *Los Angeles* with Fabric Removed from Fin

if the landing points are bumpers on cars well forward and aft on the center line, with no intermediate positions that might strike upon uneven ground. Zeppelin practice is to put a large bumping bag under the control car and another one under the rear center-line power car.

The argument for having the passenger car amidships is that variable weights such as the number of passengers carried should be balanced about the center of buoyancy, and that aerodynamically it is better to have a large car amidships rather than forward where it may have an undesirable forward fin effect, also increasing the total resistance more than if it is placed further aft. In the small German airships *Bodensee* and *Nordstern*, the passenger and control car interfered with the directional control of the ship and caused excessive rolling. It was rather large and placed well forward, and its sides were vertical, giving it a very decided fin effect.

The weight of the bare structure of airship cars usually runs 1 to 1.5 lbs. per cu. ft. volume. About half of the total weight is devoted to obtaining a strong substructure under the floor.

Tail cone. The stern of a rigid airship, aft of the cruciform girder, is finished off with a light structure known as the tail cone, having from six to eight sides. Usually there is no gas cell within the tail cone; but in the *Los Angeles* there is a cell in this part of the ship. It is customary to carry the tail cone out to a fine point, sometimes with a bucket for a lookout at the tip of the stern. A finely drawn out stern has no appreciable effect on the aerodynamic qualities of an airship, but its weight is slight and it improves the appearance. The designer may save a few pounds of weight and sacrifice beauty by rounding off the tail cone. The total weight of the tail cone and its outer cover on the *Shenandoah* was about 350 lbs. Its length was 34 ft. and its diameter at the cruciform 18 ft.

Outer cover. A smooth and taut outer cover is essential for satisfactory performance of a rigid airship. It is a comparatively easy matter to make the cover taut when it is first applied to the ship, but this tautness has a disappointing way of disappearing very rapidly in service. It has been estimated that the slackness which frequently occurs after only six months' service may cause a 10% loss of speed, or nearly 30% increase in fuel consumption at a given speed.

The means of holding the outer cover when the longitudinals are too far apart to give adequate support presents one of the most difficult technical problems confronting the airship designer. It is easy to use wiring for additional support to the outer cover between the longitudinals, but wires are not very effective in preventing fluttering, and they put undesirable compressive loads upon the girders. One solution of the problem is the use of light secondary longitudinals for the sole purpose of supporting the outer cover, and possibly to take some of the gas pressure force. These secondary longitudinals are preferably quite resilient, so that they may act to some extent as springs to take up slackness in the cover.

Another method of securing a taut, nonfluttering outer cover in flight is to maintain a slight excess of interior air pressure in the hull by means of scoops or by a hole in the high pressure region at the bow of the ship. This scheme is very attractive in some respects; but the objection to it lies in the narrow limits within which the air pressure must be maintained. Maximum and minimum limits of the transverse tension in the cover may be placed at 150 lbs./ft. and 30 lbs./ft., respectively. Taking the minimum radius of curvature as 80 ft., the corresponding air pressures required are 2.0 and 0.4 lb./ft.^{2}, or 0.385 in. and 0.077 in. of water, giving a range of 0.308 in. of water. Near sea level, the absolute air pressure changes by this amount within a height of only 21 ft., and such narrow limits of excess of internal over external air pressure can be maintained only by having large air scoops in conjunction with ample openings for the escape of air. The air outlets might be controlled by some kind of very light valve, in which the springs

are accurately adjusted; but only approximate air tightness is required.

Weight of outer cover. The outer cover is made of single-ply cloth weighing from 2.7 to 4.5 oz./yd.^{2}, and the dope and attachments increase the total weight by about 75%. It is good practice to have stronger cloth on the top of the ship and over the bow and fins than on the less exposed parts along the side and underneath the hull.

Load on the longitudinals from tension in the outer cover. The calculation of the radially inward loads on the longitudinals from the tension in the outer cover is a problem of a similar nature to the calculation of the outward loads from the gas pressure acting on the netting, and the same methods will be applied to it. The cover is assumed to have a constant tension all around the circumference of the cross-section so that there are no tangential loads to be considered.

Let β equal half the exterior angle between adjacent sides of the hull (Figure 84). When there is no difference between the air pressure inside and outside of the hull, and the cover is drawn tight with a tension t per unit width of the cloth, it will lie in a straight line between successive longitudinals, and will put upon the girders a uniform running load equal to $2t \sin \beta$. In the *ZR*-1, $\beta = 7°7.5'$, and $\sin \beta = .1240$.

The tension produced in the outer cover by a given difference between the external and internal air pressures cannot be computed exactly because of the lack of definite stress-strain relations in doped fabric. Let it be assumed that a difference of pressure equal to .5 in. of water (2.6 lbs./ft.^{2}) produces a bulge e of 3 in. in the outer cover in a panel 9.76 ft. in width. Then in Figure 84,

$$\text{the angle } \gamma = 4\,e/c = (4 \times .25)/9.76 = .1024 \text{ radians} = 5°52'$$
$$\psi = \pi/2 + \beta \pm \gamma$$
$$= 90° + 7°7.5' \pm 5°52'$$

The tension is given by

$$t = pc^2/8e = (2.6 \times 9.76^2)/(8 \times .25) = 124 \text{ lbs./ft.}$$

If the excess of pressure is external, γ is positive, and $\psi = 103°0'$, and the radial load $= 2 \times 124 \times \cos 103° = 55.9$

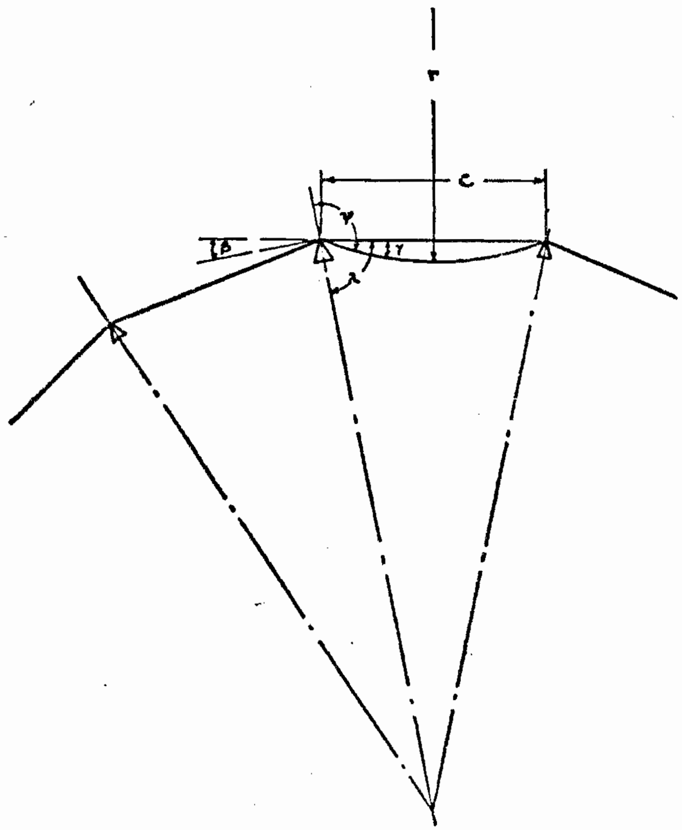


Figure 84. Illustrating Load on Longitudinals Due to Tension in Outer Cover

lbs./ft. inward, and the maximum compressive stress produced in the main longitudinals is 6,480 lbs./in.² in the base channels at the joints with the transverses. If the excess pressure is internal, γ is negative, $\psi = 91°15'$, and the radial load $= 2 \times 124 \times \cos 91°15' = 5.4$ lbs./ft. inward.

The example shows the importance of preventing appreciable differences between the internal and external air pressure, except in the special case discussed in the last section when internal pressure is used to keep the outer cover from fluttering; and in that case, the initial tension or slackness of the cover must be so arranged that when bulging under full air pressure, the angle ψ is as nearly 90° as practicable. The example shows also that excess of external pressure produces much greater loads upon the longitudinals than excess of internal pressure, so that the former is especially to be avoided. The load upon the longitudinals is always inward, even with excess of internal pressure, unless the cover is so extraordinarily slack that it bulges beyond the circumscribing circle through the apexes of the longitudinals, and such a condition is very unlikely to occur.

Openings in the outer cover. A doped outer cover is fairly air-tight, and a sufficient area of openings must be provided to prevent excessive internal or external air pressure when the airship is ascending or descending at the maximum rate. The rate of flow through the openings is given by the expression:

$$V = CA\sqrt{2p/\rho}$$

where

- V = the volume of air passing per second
- A = the total area of the openings
- p = the difference between the external and internal air pressures
- ρ = the density of the air
- C = the coefficient of discharge, determined by experience

As an example, find the area of openings required to prevent p exceeding .5 lbs./ft.² in an airship of 2,000,000 cu. ft. volume, descending at 20 ft./sec., assuming that the air weighs .072 lbs./ft.³, and the rate of change of density is 3.3%, per 1,000 ft. altitude.

Assume C = .5

$$V = 20/1{,}000 \times .033 \times 2{,}000{,}000$$
$$= 1{,}320 \text{ ft.}^3\text{/sec.}$$

$$.5A\sqrt{2 \times 32.2 \times .5/.072} = 1{,}320$$
$$A = 125 \text{ ft.}^2$$

This is the area of openings required.

The exhaust trunks usually provide sufficient openings for the escape of air from the hull, but additional openings are necessary for the admission of air to the hull. Two large hatches with inward opening flaps are placed in the region of high pressure under the bow; and several smaller openings with similar flaps are placed along the keel. An alternative to the flap doors is the use of fine mesh netting over the openings.

Gas cells. The gas cells of rigid airships are usually constructed of single-ply cotton cloth, lined with goldbeater's skin to give gas-tightness. This skin is the outer casing of the cæcum of the ox. Three-quarters of a million animals may contribute to the gas cells of a single airship. Because of the great cost of goldbeater's skin, extensive researches have been made toward the development of a satisfactory artificial substitute to be prepared in sheets, or spread mechanically on rubberized cloth. A completely satisfactory solution of the problem has not yet (1927) been attained; but there is every indication that it is not far off. The chief advantage of the natural product lies in the toughness resulting from its fibrous nature, which cannot well be reproduced in the artificial substitutes. The skins are stuck to the cloth with a special glue, or by a thin spreading of gum rubber. Gas cells are sometimes constructed of rubberized cloth.[2] This material makes rugged cells of fair gas-tightness, but undesirably heavy. Rubberized cloth is dangerous for hydrogen containers because of its electrostatic properties.

It is accepted practice to construct gas cells about 3% longer and 1.5% greater diameter than the space they are designed to fit, as an allowance for bulging between the girders and wires of the airship, and to prevent any considerable tension being put upon the fabric. A typical specification for goldbeater's skin gas-cell fabric is as follows:

[2] See Part III of "Free and Captive Balloons" by C. deF. Chandler, a volume of the Ronald Aeronautic Library.

Cotton cloth, type HH	2.00 oz./yd.²
Spread rubber	.50 "
Rubber cement	1.00 "
Goldbeater's skin	.70 "
Varnish and chalk	.35 "
Total weight	4.55 oz./yd.²

In weight estimating, an allowance of about 10% should be made to cover the weights of seams, suspension patches, etc.

Stabilizing surfaces. The stabilizing or tail surfaces of an airship comprise the fixed vertical and horizontal fins and also the movable horizontal and vertical surfaces or rudders. The horizontal rudders are called elevators.

The early airships usually had multiplane stabilizing surfaces resembling box kites. This type of horizontal surface persisted in Italy until after the war. In the early days bow rudders were sometimes used, but were soon found to be a source of instability and were abandoned. With the exception of the peculiar box-kite surfaces used in Italy, the monoplane type of fin and rudder has been practically universal since about 1910. At first the monoplane surfaces consisted of a flat framework of wood or metal construction over which cloth was tightly stretched and doped. These flat surfaces could obviously have but little strength in themselves and required much external bracing wire to prevent collapse. This external wiring presented a very considerable amount of parasite resistance. A notable improvement in the stabilizing surfaces was the internally braced *V* type surface of the *L*-70 class of Zeppelin airship constructed shortly before the Armistice. The cross-sections of these fins were isosceles triangles with their bases against the hull of the airship. With this shape it is very easy to make the forward portion of the fin strong and light without any external bracing, but the after part tapered out to a thin stern post forward of the rudders. The stern post received large loads from the rudder, and on account of its shape it was impossible to avoid a considerable amount of external bracing. This type of tail surface was used in the *R*-38, the *Bodensee* and the *Shenandoah*, with various types of rudders. The next notable improvement in stabilizing surfaces was incorporated in the *ZR*-3. In this airship, the after end of the fin is about 3 ft. in width at the base; the fore-and-aft lines of the fin are carried out in the rudder. The stern post is a narrow *A* frame instead of a single box girder as in the earlier type of internally braced fins. It is possible to dispense with all external bracing except a single set of streamlined wires from the center of each stern post to the hull.

No less important than the strength of the fins in themselves is the manner in which the fin loads are introduced into the hull. There are two principal alternatives open to the designer in this problem. He may choose to construct his fins so that they will form an integral part of the hull structure, assisting the hull to resist shear and bending; or he may prefer to construct the fins as separate units secured to the hull structure. If the latter course is chosen, care should be taken that the fin loads are transmitted to the hull chiefly through the main frames, which should be specially reinforced to take such loads. It is obviously important that large fin loads should not be applied through the intermediate transverse frames which are without transverse bracing to withstand such loads.

Coefficients for the areas and aerodynamic loading of the fins and rudders are given on pages 56 and 106.

The frame of an airship fin usually consists of girders running in approximately longitudinal directions along the outer edge of the fin and along the intersections of the fin with the hull, and transverse girders at from 8 to 16 feet apart. The fabric cover is supported between the girders by the internal diagonal bracing wires of the fin structure, and by longitudinal wires about 3 ft. apart. (See Figure 83, facing page 265).

The fin structure must be strong enough to sustain not only the transverse aerodynamic pressure upon the surfaces, but also the compressive loads due to the tension in the cover; this tension is not likely to exceed 100 lbs./ft.

The cruciform girder. The cruciform girder is of great assistance in transferring loads from the fins and rudders to the hull. In its simplest form this girder is merely an extension of the vertical and horizontal stern-posts of the fins through the hull, the two posts crossing at the center of the cross-section. When each stern-post consists of two girders, as in the *Los Angeles*, the cruciform becomes a double-cross; and in that airship the cruciform girder idea is further extended by having another and much wider double-cross group of girders through the hull, supporting the fins at 10 meters forward of the stern-post.

Rudders and elevators. The rudders and elevators of rigid airships are each constructed with a stout rudder-post, usually

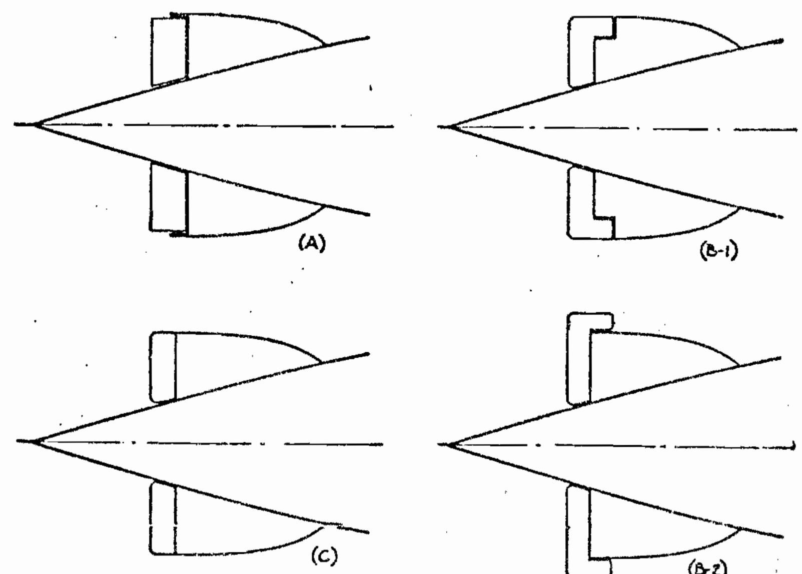


Figure 85. Various Types of Airship Rudders

a box-shaped girder, about which the rudder turns. This rudder-post receives the principal bending and torsional loads in the rudder. The framework is completed by light fore-and-aft girders secured to the rudder-post and tapering aft to a fine trailing edge, and usually some light auxiliary members parallel to the rudder-post. The structure is wire braced and covered with fabric.

Airship rudders may be classified as belonging to three types shown in Figure 85, (A) internally balanced, (B) externally balanced, and (C) unbalanced. The internally balanced rudders were common with the old type of flat fins. It had the disadvantage that the rudder-post could only be supported at its inboard and outboard extremities, and because of the gap between the fin and rudder, the latter acted alone as a flat plate when inclined to the wind. In the externally balanced and unbalanced types, the rudder-post is hinged to the stern-post of the fins and may be supported by three or four hinges. These types have also the advantage that rudder and fin act together to some extent as a curved aerofoil so that a much narrower rudder than with the old internally balanced type suffices for control of the ship. The external balancing area is very effective in balancing the rudder because of its position and the large pressures which come upon it. In the experiments on the *C-7* the maximum pressure on the balanced portion of the rudder was about twice as intense as the maximum on any other portion. The external balance has the structural disadvantage that owing to its position, it throws large torsional forces into the stern-post of the rudder. The unbalanced rudder is structurally the best of all, but puts considerable physical labor upon the helmsman. For the vertical rudder this is not very important, because it is never held long at one side, and with the modern type of narrow rudder, the moment about the rudder hinge is not very great. It is more serious for the elevators, because an airship when statically unbalanced may be for a considerable period at an angle of pitch, requiring a constantly applied force upon the elevators. It therefore seems reasonable to follow the practice adopted in the *Los Angeles*, which has balanced elevators but unbalanced rudders.

The *Shenandoah* had the Breguet type of balance on both rudders and elevators. In this design, the control surfaces are structurally of the unbalanced type, and the balancing area consists of a small auxiliary surface on each side of the rudder or elevator, forward of the hinge point and supported by the rudder quadrant and special struts. This type of balance appears to be very effective, but when the rudder is at a large angle, one of the balancing surfaces is almost in contact with the fin, and probably produces undesirable disturbances in the air flow.

The Flettner type of aileron controlled rudder has not yet been tried upon airships, but appears to hold possibilities for the future.

Rudder and elevator controls. The rudders and elevators are operated by long wire leads running through the corridor of the ship from near the cruciform girder to the control car. The leads are designed with a large factor of safety, usually six or even more, based upon the maximum force calculated upon the rudders from the pressure coefficients considered in Chapter V. This large factor of safety is required in order to eliminate lost motion between the rudders and the control wheels. General rules for designing control leads are to have them readily accessible throughout their length for inspection and repair, also to avoid sharp turns as much as possible. The leads are of solid wire throughout the greater part of their length in the corridor, but extra flexible cable must be used where passing over pulleys or drums. The leads are connected to the rudders and elevators by large quadrants, usually of about 3 to 4 ft. radius, placed midway between the outboard and inboard ends of the rudders. These quadrants are also of importance in increasing the stiffness of the movable surfaces.

There has been a wide variation in the number of turns provided for the rudder and elevator control wheels in moving the rudder from hard-over to hard-over. Practice has varied from four to twelve turns, and it is still very controversial as to whether the helmsman should be required to exert much physical force and "feel" the operation of the rudders, or whether he should be given a powerful gearing and have the movement of the rudder slowed down by the fact that, with such gearing, many turns must be given to the wheel to obtain large rudder movements. Complicated control systems have been suggested in which by hydraulic or other means a time element is introduced into the movement of the control surfaces, making it impossible for them to be turned too rapidly. Such systems, if developed, will serve the double purpose of relieving the helmsman of hard physical effort and at the same time preventing too rapid or powerful operation of the rudders.

In addition to the control positions in the control car, emergency control positions for the rudders and elevators are placed aft in the ship, either in the corridor within the hull or within the bottom fin.

Exhaust trunks. The exhaust trunks are ventilating shafts extending from the bottom to the top of the airship at alternate main frames, designed primarily to conduct out of the ship as directly and rapidly as possible the gas escaping from the automatic gas valves which are placed near the bottoms of the cells, in positions readily accessible from the corridor. In airships inflated with hydrogen, the exhaust trunks are of vital importance in order to prevent the formation of explosive mixtures of hydrogen and air within the hull. With helium the exhaust trunk is less important than with hydrogen, but nevertheless it is a desirable feature in order to avoid the possibly stifling effects of large quantities of gas suddenly discharged into the corridor. The trunk is also useful in preventing accumulation of gasoline vapor within the hull, although a much smaller trunk than the present type would suffice for this purpose. It is possible that in the near future, the present system of automatic gas valves near the bottom of the cells and manually operated maneuvering valves at the top will be abandoned in favor of combined automatic and controlled valves at the top of the cells; exhaust trunks will then be unnecessary, except in a modified form as a ventilating shaft, chiefly for gasoline vapor.

An exhaust trunk consists of a rectangular lower portion against which the automatic gas valves rest, surmounted by a shaft of semicircular or elliptical cross-section. If the section is semicircular, the flat side rests against the wiring of a main transverse frame of the ship. The rectangular lower part usually consists of a bamboo framework covered with cordage netting; and the shaft consists of horizontal wooden hoops connected by vertical cords and netting. The trunks must be strong enough to sustain the maximum gas pressure in the cells, and particular care must be taken that the vertical cords are sufficiently strong and well secured to prevent collapse of the trunk by the simultaneous rotation of the horizontal hoops into a vertical position.

In the parallel middle-body of the *Shenandoah* each trunk served two gas cells of about 150,000 cu. ft. volume each; its shaft was of semicircular cross-section with a radius of 16 in. The total weight of each exhaust trunk was about 47 lbs.

Each trunk is surmounted by a light plywood hood opening aft to keep out rain and snow, and to create a draft up the trunk when the airship is in motion.

Gas valves. Rigid airships are fitted with two sets of gas valves having different functions. Near the bottom of each gas cell, in a position readily accessible from the keel, one or more gas valves are fitted, opening automatically at a predetermined gas pressure, usually from .4 in. to .6 in. of water. Owing to the low pressures at which these valves operate, they must necessarily be of large size, usually 3 ft. or more in diameter with an opening of 1 in. to 2 in. The valve has a spun aluminum frame with a conical seat of rubberized fabric. The disc consists of a circular ring and frame covered with rubberized fabric. The valve is closed by a spiral spring around the stem. With this simple type of mechanism the force required to open the valve increases about 25% per inch of opening. At the top of each cell an inwardly opening, manually controlled, maneuvering valve is usually provided, although this valve may be omitted from some of the cells, especially those cells over large fixed weights such as the power cars. The maneuvering valves are only of about two-thirds the diameter of the automatic valves, and because of the greater pressure at the top of the ship, their construction is more rugged.

The reasons for having two distinct types of valves are best understood by a consideration of the function of each type. The purpose of the automatic valve is to relieve the cell of excessive pressure when the airship rises above pressure height where the cell is completely distended by the gas, and when further rising of the ship or heating of the gas would cause dangerous pressure. In order to obviate freezing of these valves it is preferred to have them within the ship near the bottom of the cell where they may be quickly attended to if they fail to function properly. The maneuvering valves are used chiefly when it is necessary to reduce the buoyancy of the airship in landing; and since the cells are never completely full of gas when landing, the maneuvering valves must be placed at a considerable height in the cell, preferably at the very top.

Combined automatic and manually operated valves have been designed to replace the present separate automatic and maneuvering valves, taking the position at the top of the cells, now occupied by the maneuvering valves. The use of such valves would effect an important saving in weight because not only is one valve eliminated, but owing to the greater pressure at the top of the ship the valve may be much smaller than the present type of automatic valve placed in the region of least gas pressure; and the exhaust trunk would also be eliminated or at least very much reduced in size. Another important feature of this type of valve is that the springs should be con-

nected to the disc by a system of levers so designed that the force to open the valve is practically independent of the extent of the opening. The objections to the top valve are inaccessibility and the danger of freezing. To obviate this danger it is essential that both the valve disc and its seat have rubber lips; but it is not certain that even this precaution would suffice to prevent freezing in all conditions of weather. Such valves are still in the development stage.

Size and weight of gas valves. The automatic gas valves must be of sufficient area to prevent appreciable rise of the gas pressure beyond the blowing pressure of the valves when the airship is climbing at the maximum rate of ascent, usually about 1,200 ft./min. The rate of flow of the gas through the valve opening is given by

$$V = CA\sqrt{2p/\rho}$$

where

V = the volume of gas to be discharged per second
A = the area of the valve opening
p = the gas pressure at the valve
ρ = the density of the gas
C = a coefficient for the valve

Example. Find the valve area required for a gas cell of 150,000 cu. ft. volume to permit of an ascent of 1,200 ft./min. with a maximum gas pressure of 0.4 in. of water, using helium weighing .014 lb./ft.^{3}, and assuming the rate of change of the atmospheric pressure is 3.3% per 1,000 ft. altitude. Let C = .5.

$$V = \frac{1{,}200 \times .033 \times 150{,}000}{60 \times 1{,}000}$$
$$= 99 \text{ ft.}^3/\text{sec.}$$
$$p = .4 \times 5.2 = 2.08 \text{ lbs./ft.}^2$$
$$\rho = .014/32.2 = .000435 \text{ slug/ft.}^3$$
$$A = \frac{V}{.5\sqrt{2p/\rho}} = \frac{2 \times 99}{\sqrt{4.16/.000435}}$$
$$= 2.02 \text{ ft.}^2$$

The maneuvering valves are usually designed for about the same rate of discharge as the automatic valves.

The automatic gas valves of the *Shenandoah* had discs 31.5 in. diameter; each weighed about 14 lbs. complete. The maneuvering valves had discs 19.7 in. diameter, and weighed complete with their supports and protecting hoods about 22 lbs. each.

Valve control mechanism. The maneuvering valves are opened by light wires leading to the control car where provision is made for opening all valves simultaneously or separately as may be required. The control wires should be carefully adjusted so that when the valves are opened simultaneously the openings are equal.

Ballast system. Water is the only suitable ballast for rigid airships. It is contained in bags of rubberized fabric suspended in the keel. These bags are of two types, known as ordinary and emergency water ballast bags. The ordinary water ballast bags are usually of about one ton capacity each, and are distributed along the keel. Under each bag there is a discharge valve operated by a wire leading to a handle in the control car.[3] The emergency or landing ballast is contained in bags of about 500 lbs. to a half-ton capacity each, located near the bow and stern of the airship. The bottom of each bag is fitted with a wide sleeve which is ordinarily tucked up into the bag with its opening above the water level where it is held in position by a trigger operated by a wire leading to the control car. To discharge the ballast, this trigger is pulled, allowing the sleeve to drop down and form a large opening through which all the water in the bag is discharged almost instantaneously. The emergency ballast should amount to about 3% of the gross lift of the ship; and provision should be made for total ballast amounting to not less than 15% of the gross lift. In a modern rigid airship fitted for mast mooring, a ballast pipe line should

[3] One type of valve for water ballast bags is illustrated in "Pressure Airships," a volume of the Ronald Aeronautic Library.

be provided in the keel of sufficient capacity for receiving water equivalent to 10% of the gross lift of the ship per hour.

The provision of water recovery apparatus on the engine exhaust has introduced a new phase into the water ballast problem in airships.[4] Ordinary water ballast bags must be provided for ballast equal in weight to the fuel which may be consumed, with pumps to distribute the ballast along the keel as required to keep the ship in trim, and avoid large bending moments. Hand-operated pumps usually suffice for this purpose.

Provision must be made against freezing of the water ballast. Before the days of water recovery, it was a simple matter to place sufficient calcium chloride or other nonfreezing mixture in the water ballast bags to prevent all danger of the water freezing; but with water recovery, provision of sufficient calcium chloride to make a nonfreezing mixture of all the ballast which an airship may be carrying at the end of a long voyage becomes a serious problem and has not yet been satisfactorily solved. The girders must be protected by varnish or other suitable coating against the corrosive action of the antifreezing material. The use of noncorroding antifreeze substances is better still, but is expensive.

Mooring and handling system. There are two principal mooring or anchorage points on an airship; one is the mast mooring point at the extreme tip of the bow, designed for securing the airship to a mooring tower or mast; the other is the anchoring point at which the drag-ropes are attached to the keel, below the turn of the bow about 7% to 8% of the ship's length aft of the extreme tip. Wire strops for attachment of the handling lines are fitted at most of the joints of the main frames with the keel from near the bow to near the forward end of the bottom fin. In addition, the center-line cars, which usually are placed well forward and aft on the ship, have hand-

[4] Apparatus for water ballast recovery from engine exhaust gas is described in Part III of "Aircraft Power Plants," a volume of the Ronald Aeronautic Library.

rails where they can be easily grasped by the handling crew when the airship is resting on the ground. The hand-rails on the cars are sufficiently long to afford holding room for only a small part of the total ground crew, and in order to provide additional points for grasping the ship, light handling-frames of tubing with wire bracing are provided as part of the ground equipment, and provision is made for securing them to the underside of the keel when taking the ship in and out of the shed.

Mast mooring gear. Connection between the airship and the mooring mast is effected by means of a hanging cone which fits snugly into the female cone at the top of the ram of the mooring mast. The ship's cone is a steel casting 21 in. in height by 15 in. extreme diameter with 3/8 in. thickness walls. The shape and dimensions of the cone will undoubtedly become standard so that all airships may be moored to all mooring masts throughout the world. The top of this cone is secured by a horizontal transverse pin to a steel tubular spindle between 4 and 5 ft. in length, projecting through the bow of the ship on the longitudinal axis of the hull. Freedom for the ship to swing in the horizontal plane is provided by motion of the female cone on the mast. Freedom to swing in the vertical plane is afforded by the transverse pin connecting the cone to the spindle. Freedom to roll is provided by the spindle which is held in strong bearings within the ship's structure. The break-away of the *Shenandoah* from her mooring mast in January, 1924, was due primarily to the jamming of the spindle in its bearings so that the ship was deprived of its freedom to roll, and the large twisting force caused in part by the collapse of the top fin resulted in the longitudinals being torn from the bow cap which carried the bearings of the spindle. The bow mooring line of the *Shenandoah* was a flexible steel cable 9/16 in. diameter and 500 ft. in length, having a minimum breaking strength of 21,000 lbs. This cable was rove through the spindle and mooring cone, and its bitter-

end had a plug stop which fitted against a stop ring on the mooring cone. A windlass is provided in the keel for hauling the mooring cable into the ship. In addition to the mooring cable through the cone, there are two steadying guys attached to the spindle. These guys are also of steel cable, each about 550 ft. in length and about two-thirds the strength of the bow mooring cable. Windlasses are also provided to haul these guys into the ship.

Anchoring point. The first main transverse frame aft of the pronounced turn of the bow is specially reinforced with wire to make a strong mooring or anchoring point in the keel. Two drag-ropes of soft braided hemp are secured to this frame and are held in coils which may be dropped through hatches provided for the purpose. In the *Shenandoah* these drag-ropes were 500 ft. in length and 4¼ in. circumference. The anchoring point is also used for mooring the airship by the three-wire system.

Strength of the mooring and handling systems. There is not yet any very definite data available as to the actual forces on an airship riding to a mooring mast; but it is believed an approximation to the maximum transverse force at the bow is given by

$$F = C\, q\, V^{2/3}$$

where
F = the transverse force
C = a coefficient = .12
q = the aerodynamic pressure = $\rho v^2/2$
V = the air volume of the ship

Example. Find the maximum transverse force on the bow of the *Shenandoah* when riding to a mooring mast near sea level in a wind blowing 60 m.p.h.

$$V^{2/3} = 2{,}290{,}000^{2/3} = 17{,}400 \text{ ft.}^2$$

$$q = \frac{.00236 \times 88^2}{2} = 9.15 \text{ lbs./ft.}^2$$

$$F = .12 \times 9.15 \times 17{,}400 = 19{,}200 \text{ lbs.}$$

The maximum bending moment resulting from the transverse force at the bow is about 25% of the length aft of the bow, and its magnitude is given approximately by

$$M = .075\,FL$$

where L is the length of the ship.

When held at the anchoring point instead of at the extreme bow, the transverse force may be about 25% greater because of the larger angles of yaw likely to occur; but the maximum bending moment is rather less due to the more favorable position at which the load is applied.

The maximum longitudinal force at the bow or the anchoring point is only about half the corresponding transverse force, and hardly has to be considered, since the ship is enormously strong in longitudinal tension. It is customary to design the bow mooring strong enough to hold the airship in a wind of 60 m.p.h., and the anchoring point for a wind of about 40 m.p.h., with a factor of safety of two in both cases.

The handling points along the keel are designed for the transverse forces when taking the airship into or out of the shed in a cross-wind not exceeding about 15 m.p.h. The total force on the ship in a cross-wind is assumed to be given by

$$F = .2\,LD\rho v^2$$

where L and D are the length and diameter, respectively, and ρ and v are the air density and wind velocity in absolute units. This force is divided between four points, all on main frames, two forward and two aft.

CHAPTER XI

COMMON AIRSHIP FALLACIES

The following six inventions constantly recur, and have been submitted to the Navy Department many times by aspiring inventors. It is hoped that the explanations of the fallacies of these inventions may save some future inventors much wasted efforts and subsequent heart-burnings. The six inventions are:

(1) The vacuum airship
(2) Compressing gas or air for ballast
(3) Artificial control of superheat
(4) Combined heavier and lighter-than-air craft
(5) Channel through hull to reduce resistance
(6) Wind screen at mooring mast

Stress in a vacuum airship. Inventors frequently propose the construction of a vacuum balloon, or airship, to secure buoyancy without the use of gas. The following treatment of this problem has been presented by Dr. Zahm:[1]

The unit stress in the wall of a thin, hollow, spherical balloon subject to uniform hydrostatic pressure, which is prevented from buckling, is given by equating the total stress on a diametral section of the shell to the total hydrostatic pressure across a diametral section of the sphere, thus:

$$2\,\pi\,r\,t\,S = \pi\,p\,r^2$$

in which S may be the stress in pounds per square inch, p the resultant hydrostatic pressure in pounds per square inch, r the radius of the sphere, t the wall thickness.

The greatest allowable mass of the shell is found by equating it to the mass of the displaced air, thus:

$$4\,\pi\,r^2\,t\,\rho_1 = \frac{4\,\pi\,r^3\,\rho_2}{3}$$

[1] "Aerial Navigation."

in which ρ_1 is the density of the wall material, ρ_2 the density of the atmosphere outside.

Now, assuming $p = 15$, $\rho_1/\rho_2 = 6{,}000$, for steel and air, the equations give:

$$S = 3p\,\rho_1/2\rho_2 = 45 \times 6{,}000/2 = 135{,}000 \text{ lbs. /sq. in.}$$

as the stress in a steel vacuum balloon.

For aluminum ρ_1 is less, but the permissible value of S is also less in about the same proportion.

The last equation shows that for a given material and atmospheric environment, the stress in the shell or wall of the spherical balloon is independent of the radius of the surface. It is also well known that the stress is less for the sphere than for any other surface. Hence, no surface can be constructed in which S will be less than $3\,p\,\rho_1/2\rho_2$. The argument is easily seen to apply to a partial vacuum balloon, since a balloon of one nth vacuum will float a cover of but one nth the mass and strength.

The above result was obtained on the assumption that the shell was prevented from buckling. As a matter of fact, it would buckle long before the crushing stress could be attained. We must conclude, therefore, that while a vacuum balloon is alluring as an idea, the materials of engineering are not strong enough to favor such a structure. Perhaps, it is nearer the truth to say that such a project is visionary, with the materials now available.

Compressing air or gas. A very common idea with inventors is to reduce the lift of an airship by compressing some of the gas from the cells into high pressure containers, or else to obtain the same effect by compressing air from the atmosphere into flasks, thereby increasing the weight of the ship. Both of these schemes split on the same rock as the vacuum airship, the pressure forces entering into the problem are far too great to be controlled within the permissible limits of weight. The

following figures show the utter impracticability of controlling the buoyancy by pressure even if the weight of the compressor is neglected.

A steel flask such as is used for the transportation of gases weighs from 115 to 130 lbs. and contains 180 to 200 cu. ft. of free gas compressed to between 1,800 and 2,000 lbs./in.^{2} The maximum lift of 200 cu. ft. of hydrogen is only about 14.4 lbs., and is rarely more than 13.6 lbs. The lift of the gas is thus only about 10% to 12% of the weight of its container. Changing the pressure makes practically no difference in the result of the problem. If the pressure were halved, the necessary increase in the dimensions of the containers would almost exactly balance the saving in weight made possible by thinner walls.

Compressing air into high pressure flasks is equally futile with compressing the gas. The 180 to 200 cu. ft. of free air which might be compressed into the steel flask considered above for gas storage would weigh from 14 to 16 lbs., or only 11% to 13% of the weight of the container.

Lift independent of altitude. Inventors of means for controlling the lift of airships are very frequently unaware that as long as an airship is below pressure height (i.e., the altitude of complete fullness of the gas space), the buoyancy is independent of the altitude. On the contrary, there is a very general belief that as an airship ascends, its buoyancy is decreased by the decreasing density of the air in which it floats. Actually, the varying density of the air is balanced by the varying volume of the gas cells, so that the buoyancy is unaltered so long as the air and gas are equal to each other in respect to pressure and temperature.

Owing to the limitations of strength in an airship, the absolute pressures of the air and gas are necessarily almost equal at all times; and it follows that when no gas is discharged, change of altitude can produce change of lift only by creating temporary differences in the relative temperature of the gas and the atmosphere. Such disturbances in the buoyancy are easily taken care of by the dynamic lift while the airship is in motion.

When an airship rises above pressure height, gas is discharged to prevent destructive internal pressure, and at the same time, there is a loss of lift because diminishing density of the air is no longer balanced by increasing volume of displacement. Obviously this is a permanent loss which cannot be restored by descending to lower altitudes, because the gas which has been discharged cannot be replaced during flight.

The difficulty of landing a light airship has been the inspiration of most of the suggestions for compressing gas or air. This difficulty is not the result of increasing buoyancy as the airship descends, but results from the loss of dynamic control as the ship slows down for the landing. The lightness of the ship (i.e., the excess of buoyancy) is usually due to the consumption of fuel during flight, and while this may be easily compensated by heading the ship downward while in motion at a fair rate of speed, the compensating power which we have called "dynamic control," disappears with loss of speed.

Artificial control of gas temperature. The weight of air displaced by a given mass of gas varies directly as the absolute temperature of the gas, given that the temperature and pressure of the air are constant. Since the normal temperature of the air is about 520° F. absolute, the temperature of gas must be raised about 26° F. in order to increase the lift 5%.

In an airship of two million cubic feet volume about 36,000 B.t.u. are required to raise the temperature 1°F., and 5% increase in lift would therefore require about 935,000 B.t.u. No very satisfactory data on the rate of heat loss from airship envelopes are available, but the rapidity with which superheat is lost when passing under a cloud shows that the exchange of temperature between the air and gas within the ship and the outside atmosphere is rapid. It is probable that starting with 25° F. superheat, half of it would be lost in 13 min. if there were

no source of heat to make up the loss by radiation. In other words, about 36,000 B.t.u. must be supplied per minute. This is equivalent to 850 Hp. in energy, and coupled with the weight of apparatus required makes the scheme impracticable. A new type of gas cell construction in which convection losses were greatly reduced might possibly result in a practical solution of the problem.

Planes to increase the lift of airships. The use of planes to give dynamic lift to airships has been a favorite idea with inventors for the past thirty years; and some of the early airships, notably Santos-Dumont's No. 10, built in 1903, were actually fitted with such planes. An airship with lifting planes may be regarded as a combination airship and airplane, and a consideration of the characteristics of such a craft shows that it combines the disadvantages, and loses the merits of both types in a peculiarly complete manner.

The combination craft is heavier than air, and depends, therefore, upon the power of its engines to keep it aloft. It must leave the ground and alight at high speed, because the lift of the planes is proportional to the square of the speed, and at low speed they would afford no appreciable lift. All these disadvantages the combination craft shares with the airplane; but on the other hand, because of the great bulk of its gas filled hull, it is not a handy and easily maneuverable craft like an airplane. It is impossible to conceive a large airship running at high speed along the flying field before taking the air; and still less would it be possible to construct an airship capable of sustaining the shock of striking the ground at high speed when landing.

Even if the difficulties of starting and landing the combination craft could be overcome, it would still be inefficient in flight. For every 1,000 lbs. lift carried by the planes, approximately 60 lbs. resistance must be overcome by the thrust of the propellers. On the other hand, a 5,000,000 cu. ft. airship flying at

60 m.p.h. experiences only about 20 lbs. resistance per 1,000 lbs. lift, and the relative resistance decreases with increasing size and diminishing speed. It is apparent, therefore, that the increase of lift obtained by the use of planes on an airship would require a disproportionate increase in engine power and fuel consumption.

Finally, there is the objection that the lifting forces of the planes would necessarily be far more localized than the gas lift, so that the hull would have to be much stronger and heavier to take care of the concentrated forces.

Summing up, it may be said that the increased weight of structure, engines, and fuel would considerably exceed the lift derived from the planes; and there would still remain the apparently insurmountable difficulties in starting and landing the combination craft.

Channel to reduce resistance. Innumerable inventors have proposed to reduce the resistance of both air and water craft by providing ducts or channels to permit some of the air or water to flow from the bow to the stern by a more direct passage through the ship instead of around the outside. The idea sounds attractive; but tests have proved that such channels actually increase the resistance, due probably to the increased surface for frictional resistance to act upon, and losses due to eddies as the air or water enters or leaves the channel. The outside of a fair streamlined body appears to present the most efficient course for the flow of the fluid from bow to stern.

Proposals for flow channels through ships are frequently combined with schemes of propulsion by propellers or blowers within the channels. These additions in no way alter the basic inefficiency of the arrangement.

Wind screen at mooring mast. After the break-away of the *Shenandoah* from the mooring mast at Lakehurst, in January, 1924, many proposals were advanced to protect the airship when riding to a mooring mast by some form of windshield

around the bow. By far the most important stresses in an airship lying to a mast result from the transverse wind forces upon the ship, and in fact longitudinal tensions are usually desirable because they reduce the compressive forces resulting from the lateral bending due to the transverse air forces. A wind screen around the bow of the ship would undoubtedly reduce the longitudinal tension, but owing to the increased turbulence which it would cause in the air flow it would probably increase, rather than decrease, the transverse forces along the greater part of the ship.

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